

Please put all work and answers in the stamped blue book provided. Do not put work or answers on the test itself. Show all simplifications and substitutions. Do **not** use a calculator that does calculus; do not use a graphing calculator. Please include your **name, row and seat** on the front of the blue book. Put the test copy in the blue book.

1.) Integrate: $\int \frac{5 \sec^2 x}{1 + 3 \tan x} dx$

2.) Integrate and evaluate: $\int_0^6 (6x - x^2) dx$

3.) Integrate using substitution and evaluate: $\int_0^1 7x^3(1 + 4x^4)^2 dx$

4.) Integrate by parts: $\int 3x^4 \ln x dx$

5.) Evaluate using the Fundamental Theorem of Calculus: $\frac{d}{dx} \left(\int_0^{x^3} \sqrt{1+r^2} dr \right)$

6.) Using a Riemann Sum and the sigma notation formulas (provided), find the exact area under the curve $f(x) = 6x - x^2$ from $x = 0$ to $x = 6$. (compare with #2 above)

$$\sum_{i=1}^n i = \frac{n(n+1)}{2} \quad \sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$$

7.) Find the area of the region bounded by the two curves:

$$y = 4x - x^2 \quad \text{and} \quad y = x^2 - 6x + 8$$

Bonus (5 points): Integrate $\int \sec^3 t dt$

TEST SOLUTIONS (14 pts each)

$$1.) \int \frac{5 \sec^2 x}{1 + 3 \tan x} dx = \frac{1}{3} \cdot 5 \int \frac{\sec^2 x}{1 + 3 \tan x} dx \cdot 3$$

$$\text{let } u = 1 + 3 \tan x$$

$$du = 3 \sec^2 x dx$$

$$= \frac{5}{3} \int \frac{du}{u} = \frac{5}{3} \int \frac{1}{u} du = \frac{5}{3} \ln |u| + C$$

$$= \boxed{\frac{5}{3} \ln |1 + 3 \tan x| + C}$$

$$2.) \int_0^6 (6x - x^2) dx = \left[6 \cdot \frac{x^2}{2} - \frac{x^3}{3} \right]_0^6$$

$$= \left[3x^2 - \frac{x^3}{3} \right]_0^6 = \left(3 \cdot 6^2 - \frac{6^3}{3} \right) - (0)$$

$$= 108 - 72 = \boxed{36}$$

$$3.) \int_0^1 7x^3 (1 + 4x^4)^2 dx \quad \left\{ \begin{array}{l} x=0 \xrightarrow{u=1+4x^4} u=1 \\ x=1 \longrightarrow u=5 \end{array} \right.$$
$$\text{let } u = 1 + 4x^4$$
$$du = 16x^3 dx$$

$$= 7 \cdot \frac{1}{16} \int_0^1 (1 + 4x^4)^2 \cdot \underbrace{x^3 \cdot dx \cdot 16}$$

(page 2)

$$\begin{aligned} &= \frac{7}{16} \int_1^5 u^2 \cdot du = \frac{7}{16} \cdot \frac{u^3}{3} \Big|_1^5 = \frac{7}{48} u^3 \Big|_1^5 \\ &= \frac{7}{48} [5^3 - 1^3] = \frac{7}{48} [125 - 1] = \frac{7}{48} (124) = \frac{7(31)}{12} \\ &= \frac{217}{12} \approx 18.083 \end{aligned}$$

$$\begin{aligned} 4.) \quad &\int 3x^4 \cdot \ln x \, dx \\ &\text{let } u = \ln x \quad v = 3 \cdot \frac{x^5}{5} \\ &du = \frac{1}{x} dx \quad dv = 3x^4 \cdot dx \\ &= (u)(v) - \int v \cdot du \\ &= (\ln x) \left(\frac{3}{5} \cdot x^5 \right) - \int \left(\frac{3}{5} x^5 \right) \left(\frac{1}{x} \right) dx \\ &= \frac{3}{5} x^5 \cdot \ln x - \frac{3}{5} \int x^4 dx \\ &= \frac{3}{5} x^5 \cdot \ln x - \frac{3}{5} \cdot \frac{x^5}{5} + C \\ &= \frac{3}{5} x^5 \cdot \ln x - \frac{3}{25} x^5 + C \end{aligned}$$

$$\begin{aligned} 5.) \quad &\frac{d}{dx} \left[\int_0^{x^3} \sqrt{1+r^2} \, dr \right] = \sqrt{1+(x^3)^2} \cdot d(x^3) \\ &= 3x^2 \sqrt{1+x^6} \end{aligned}$$

(page 3)

$$6.) \text{ AREA} = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(a+i \cdot \Delta x) \cdot \Delta x$$

$$a=0 \quad b=6$$

$$\Delta x = \frac{b-a}{n} = \frac{6-0}{n} = \frac{6}{n}$$

$$f(x) = 6x - x^2$$

$$A = \lim_{n \rightarrow \infty} \sum_{i=1}^n f\left(0 + i \cdot \frac{6}{n}\right) \cdot \frac{6}{n}$$

$$= \lim_{n \rightarrow \infty} \sum_{i=1}^n f\left(\frac{6i}{n}\right) \cdot \frac{6}{n} = \lim_{n \rightarrow \infty} \sum_{i=1}^n \left(6 \cdot \left(\frac{6i}{n}\right) - \left(\frac{6i}{n}\right)^2\right) \cdot \frac{6}{n}$$

$$= \lim_{n \rightarrow \infty} \sum_{i=1}^n \left(\frac{36i}{n} - \frac{36i^2}{n^2}\right) \cdot \frac{6}{n}$$

$$= \lim_{n \rightarrow \infty} \sum_{i=1}^n \left(\frac{216i}{n^2} - \frac{216i^2}{n^3}\right)$$

$$= \lim_{n \rightarrow \infty} \left[\frac{216}{n^2} \left(\sum_{i=1}^n i \right) - \frac{216}{n^3} \left(\sum_{i=1}^n i^2 \right) \right]$$

$$= \lim_{n \rightarrow \infty} \left[\frac{216}{n^2} \cdot \frac{n(n+1)}{2} - \frac{216}{n^3} \cdot \frac{n(n+1)(2n+1)}{6} \right]$$

$$= \lim_{n \rightarrow \infty} \left[\frac{216}{2} \cdot \frac{n^2+n}{n^2} - \frac{216}{6} \cdot \frac{(2n^3 + \dots)}{n^3} \right]$$

$$= \frac{216}{2} (1) - \frac{216}{6} (2) = 108 - 72 = 36$$

7.) area of the bounded region

14 pts

$$y = 4x - x^2$$

$$y = x^2 - 6x + 8$$

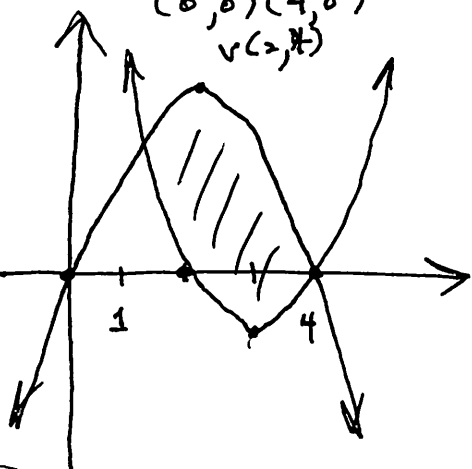
$(0,0)(4,0)$
 $v(2,4)$

$$y = (x-4)(x-2)$$

$(4,0)(2,0)$

$$v(3, 9-18+8) = (3, -1)$$

graph
not
needed



$$\begin{array}{r} 4x - x^2 = x^2 - 6x + 8 \\ -4x + x^2 \quad + x^2 - 4x \\ \hline \end{array}$$

$$0 = 2x^2 - 10x + 8$$

$$0 = 2(x^2 - 5x + 4)$$

$$0 = 2(x-4)(x-1)$$

$$x=4, x=1$$

$b \uparrow$

$a \uparrow$

$$A = \int_1^4 [(4x - x^2) - (x^2 - 6x + 8)] dx$$

$$A = \int_1^4 (-2x^2 + 10x - 8) dx = \left[\frac{-2x^3}{3} + \frac{10x^2}{2} - 8x \right]_1^4$$

$$A = \left[\frac{-2(4)^3}{3} + 5(4)^2 - 8(4) \right] - \left[\frac{-2(1)^3}{3} + 5(1)^2 - 8(1) \right]$$

$$A = \frac{-128}{3} + 80 - 32 + \frac{2}{3} - 5 + 8$$

$$A = \frac{-126}{3} + 51 = -42 + 51 = 9$$

(page 4)

BONUS : 5 pts

$$\int \sec^3 t \, dt = \int \sec t \cdot \sec^2 t \, dt$$

$$u = \sec t \quad v = \tan t$$

$$du = \sec t \tan t \, dt \quad dv = \sec^2 t \, dt$$

$$\int \sec^3 t \, dt = (\sec t)(\tan t) - \int \sec t \cdot \tan^2 t \, dt$$

$\tan^2 t = \sec^2 t - 1$

$$\int \sec^3 t \, dt = \sec t \tan t - \int \sec t (\sec^2 t - 1) \, dt$$

$$\int \sec^3 t \, dt = \sec t \tan t - \int \sec^3 t \, dt + \int \sec t \, dt$$
$$+ \int \sec^3 t \, dt$$

$$2 \int \sec^3 t \, dt = \sec t \tan t + \int \sec t \, dt$$

$$\int \sec^3 t \, dt = \frac{1}{2} \left[\sec t \tan t + \ln |\sec t + \tan t| \right] + C$$