MA141-004 ①

Thursday, April 11

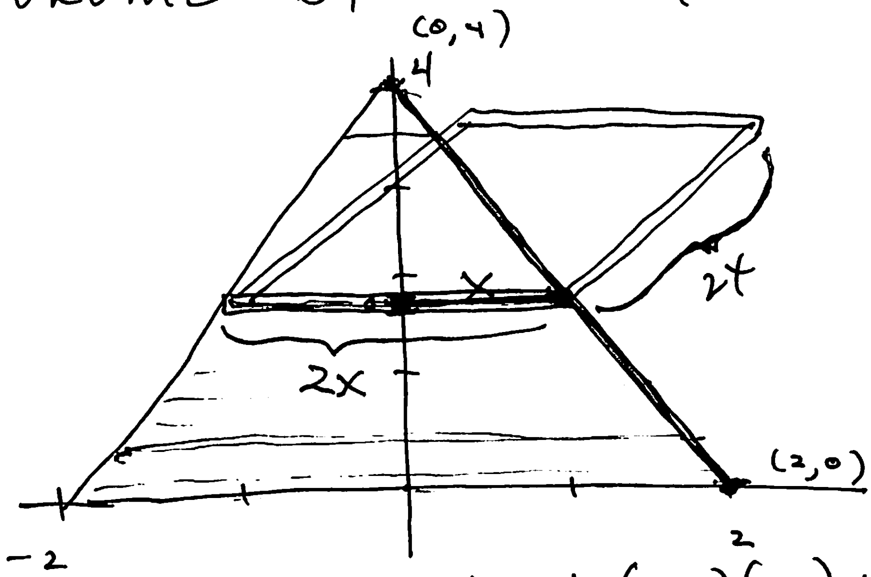
today: 5.2: (VOLUMES: washer method; cylindrical shells)

next week:

TUES, 4/16: finish course content; review

THURS, 4/18: TEST #4 (4.1 → 4.5; 5.1)

VOLUME BY "SLICING":



EQN OF LINE (EDGE):

(2,0) & (0,4)

$m = \frac{4-0}{0-2} = \frac{4}{-2}$

$m = -2$

$y - y_1 = m(x - x_1)$

$y - 0 = -2(x-2)$

$y = -2x + 4$

$\frac{y-4}{-2} = \frac{-\cancel{2}x}{-\cancel{2}}$

$\frac{4-y}{2} = x$

VOL: $(2x)(2x)\,dy$ (slice)

$$V = \int_0^4 (2x)(2x)\,dy$$

$$V = \int_0^4 \left[\cancel{2}\left(\frac{4-y}{\cancel{2}}\right)\right] \cdot \left[\cancel{2}\left(\frac{4-y}{\cancel{2}}\right)\right] dy$$

$$V = \int_0^4 (4-y)^2 \, dy$$

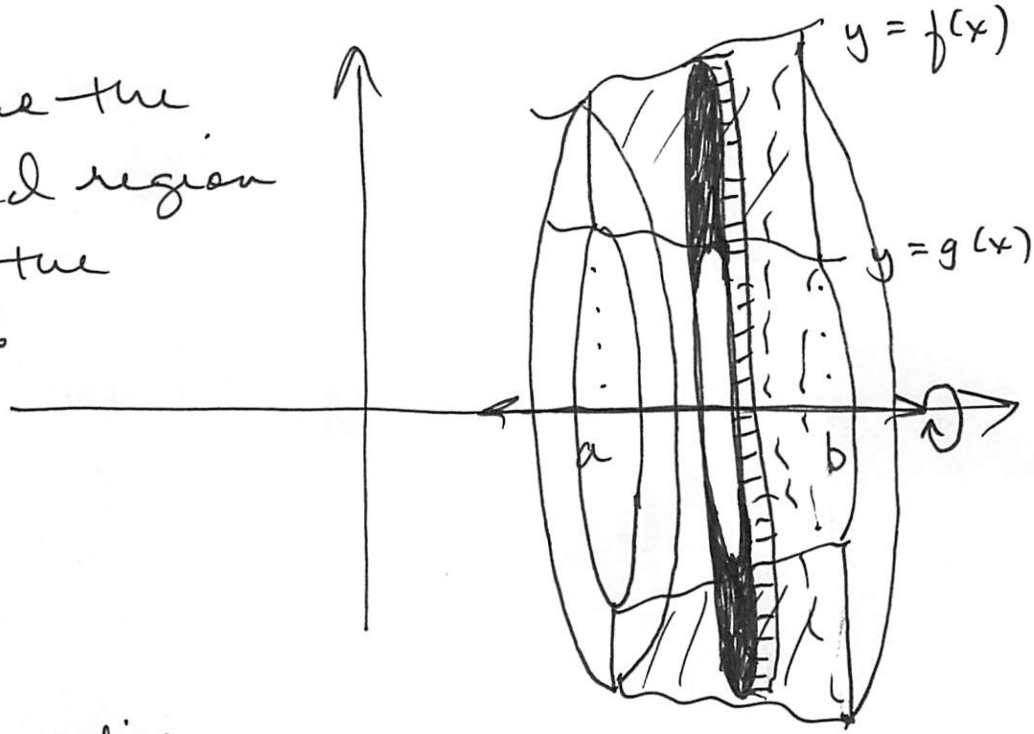
$$V = \int_0^4 (16 - 8y + y^2) \, dy$$

$$V = \left[16y - 8\frac{y^2}{2} + \frac{y^3}{3} \right]_0^4$$

$$V = \left[16(4) - 4(4)^2 + \frac{(4)^3}{3} \right] = \underline{\qquad\qquad}$$

WASHER METHOD:

revolve the bounded region about the x-axis



R: outer radius
r: inner radius

VOL OF A WASHER:

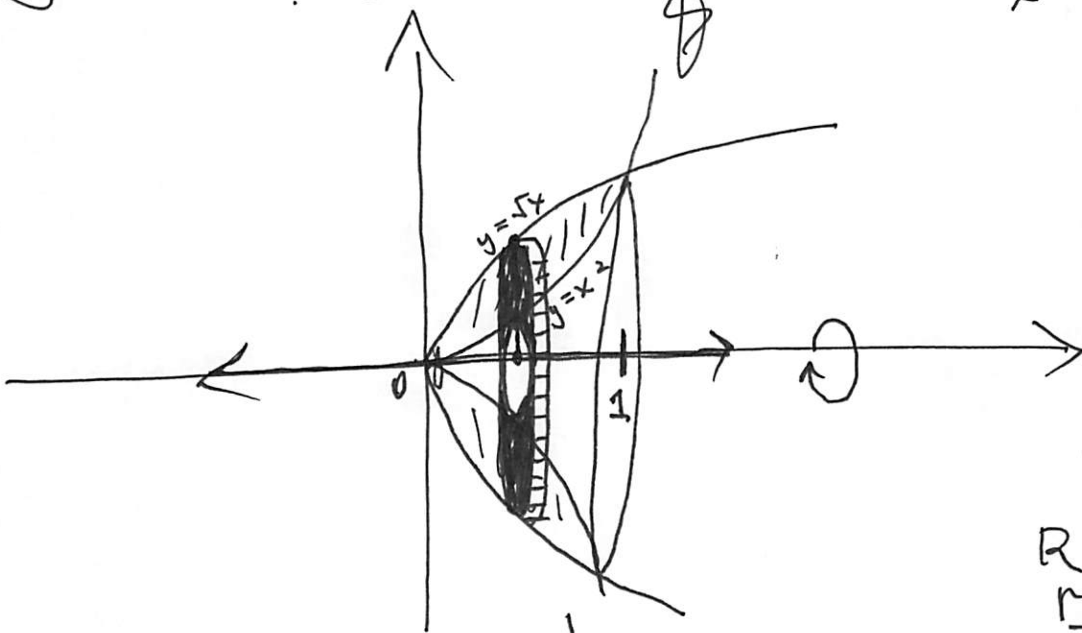
$$V = \underbrace{\pi \cdot R^2 \cdot h} - \underbrace{\pi \cdot r^2 \cdot h}$$

$$V = \pi \cdot h \left[R^2 - r^2\right]$$

$$V = \pi \left[R^2 - r^2\right] \cdot h \quad \text{(one washer)}$$

$$\text{VOL} = \int \pi \left[R^2 - r^2\right] \cdot h$$

$y = x^2$ $\&$ $y = \sqrt{x}$ about the x-axis



$y = x^2$ $\&$ $y = \sqrt{x}$

$x^2 = \sqrt{x}$

$x^4 = x$

$x^4 - x = 0$

$x(x^3 - 1) = 0$

$h = dx$

$R = \sqrt{x}$

$r = x^2$

$$\text{Vol} = \int_0^1 \pi \left[R^2 - r^2\right] \cdot h$$

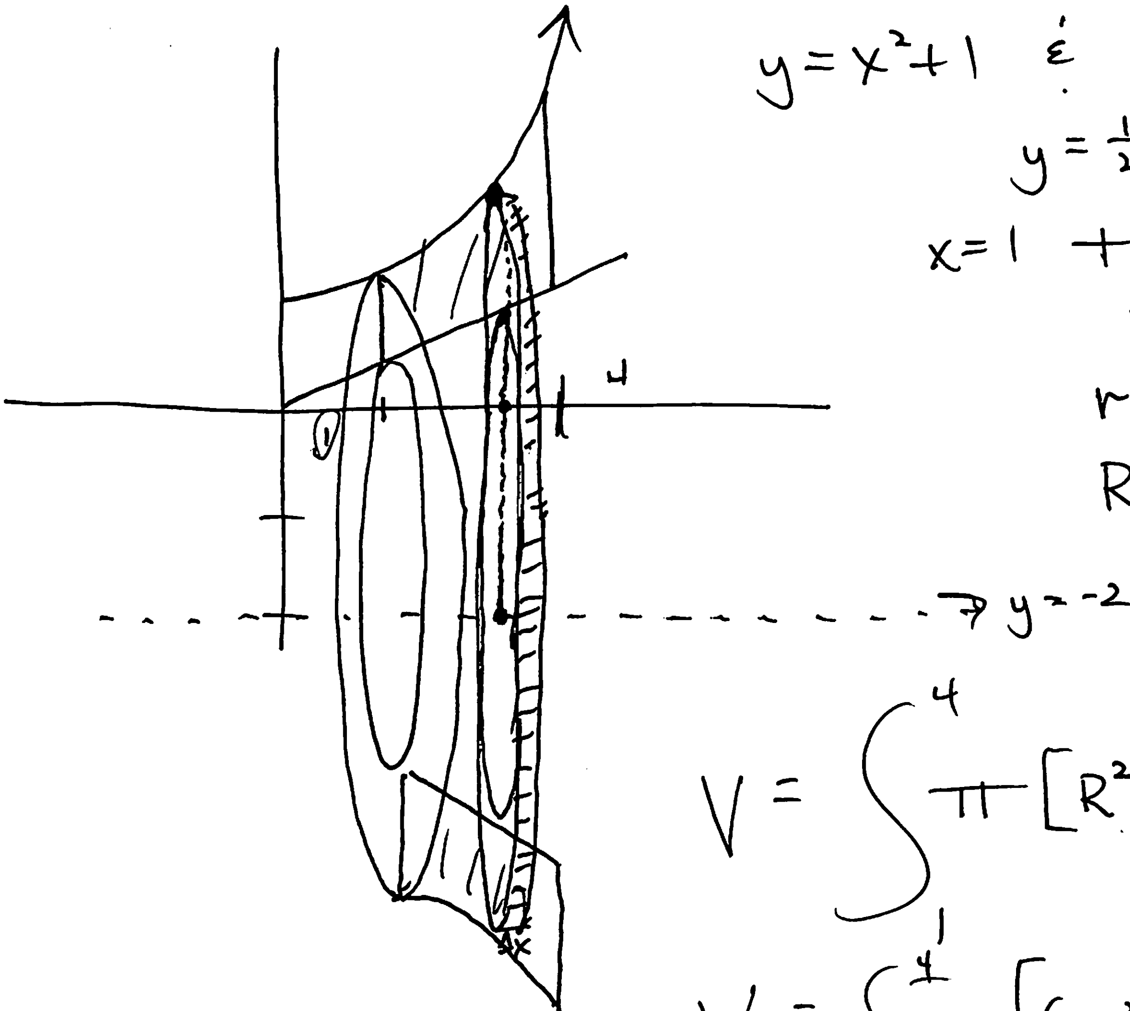
$$= \int_0^1 \pi \left[(\sqrt{x})^2 - (x^2)^2\right] dx$$

$$= \pi \int_0^1 (x - x^4)\, dx$$

$$= \pi \left[\frac{x^2}{2} - \frac{x^5}{5}\right]_0^1$$

$$= \pi \left[\left(\frac{1}{2} - \frac{1}{5}\right) - (0)\right]$$

$$= \pi \left[\frac{5}{10} - \frac{2}{10}\right] = \frac{3\pi}{10}$$



$y = x^2 + 1$ &

$y = \frac{1}{2}x$

$x = 1$ to $x = 4$

$h = dx$

$r = 2 + \frac{1}{2}x$

$R = 2 + x^2 + 1$

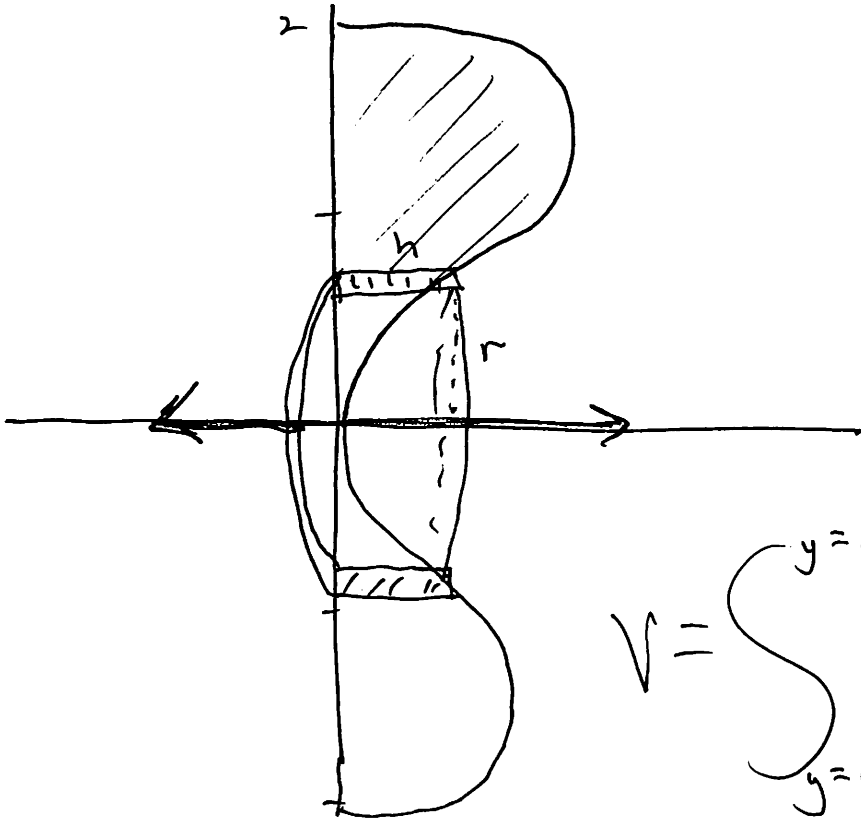
$R = \underline{3 + x^2}$

$$V = \int_1^4 \pi \left[R^2 - r^2\right] \cdot h$$

$$V = \int_1^4 \pi \left[(3+x^2)^2 - \left(2+\tfrac{1}{2}x\right)^2\right] dx$$

CYLINDRICAL SHELL:

$x = 2y^3 - y^4$



$$V = \int_{y=0}^{y=2} 2\pi r h \, (th)$$

$r = y \quad h = x$

$(th) = \Delta y$

$$V = \int_0^2 2\pi (y)(x)\, dy$$

$$V = \int_0^2 2\pi \cdot y \, (2y^3 - y^4)\, dy$$

$$V = 2\pi \int_0^2 (2y^4 - y^5)\, dy$$

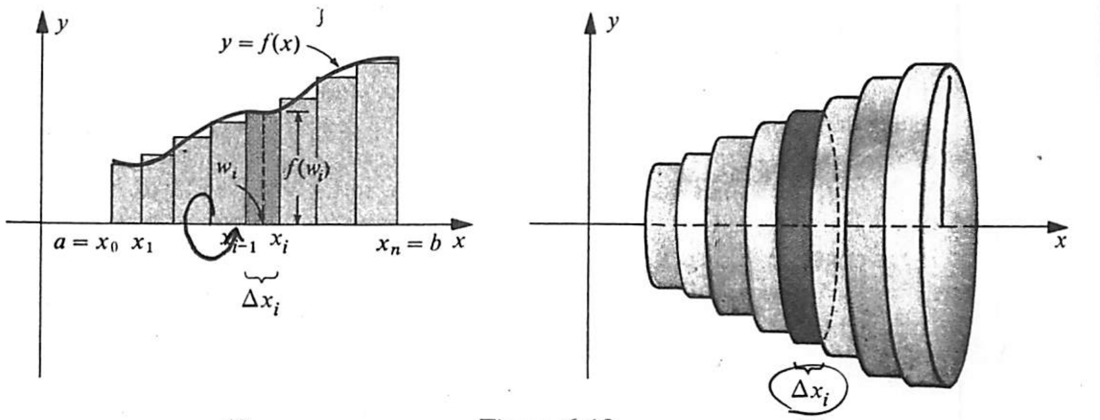
$$V = 2\pi \left[2\frac{y^5}{5} - \frac{y^6}{6} \right]_0^2$$

$$V = 2\pi\left[\left(\frac{2(2)^5}{5} - \frac{(2)^6}{6}\right) - (0)\right]$$

$$V = 2\pi\left[\frac{64}{5} - \frac{64}{6}\right]$$

$$V = 2\pi\left[\frac{64 \cdot 6}{5 \cdot 6} - \frac{64 \cdot 5}{6 \cdot 5}\right]$$

$$V = 2\pi\left[\frac{64}{30}\right] \cdots$$

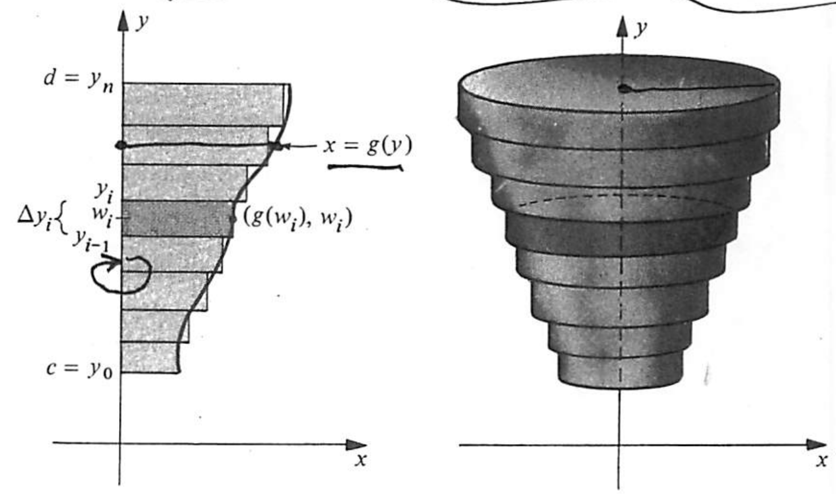


(i) *Figure 6.12* (ii)

solid disk: $V = \int_a^b \pi r^2 h$

$r = f(x)$

$h = \Delta x \to dx$



(i) (ii)

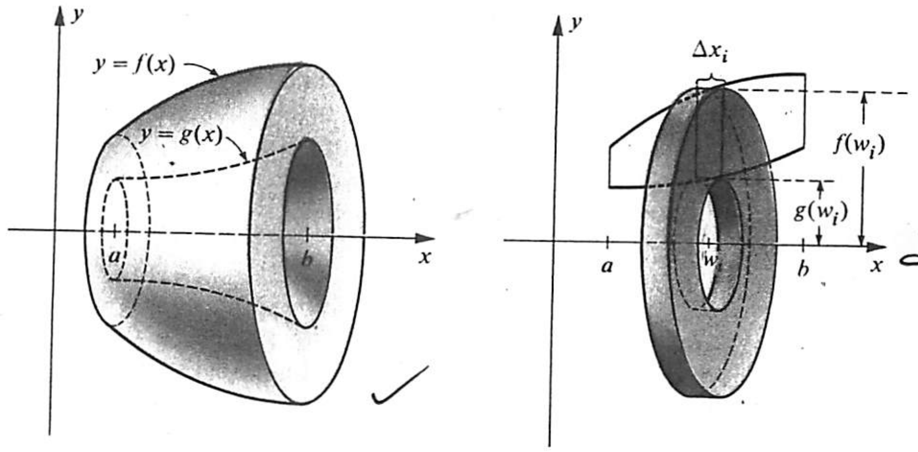
Figure 6.15

solid disk:

$V = \int_c^d \pi r^2 h$

$r = g(y)$

$h = \Delta y \to dy$



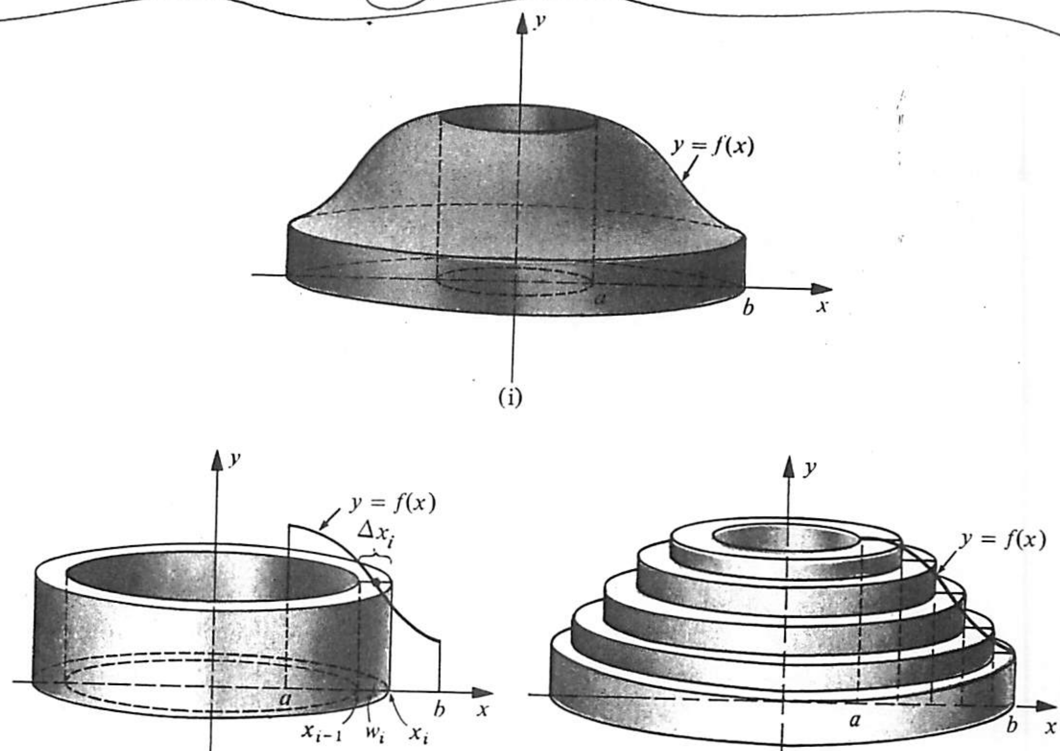
(i) (ii)

Figure 6.17

WASHER:

$$V = \int_a^b \pi (R^2 - r^2) h$$

$R = f(x)$
$r = g(x)$
$h = dx$



(ii) (iii)

Figure 6.22

CYLINDRICAL SHELL:

$$V = \int_a^b 2\pi r h \, th$$

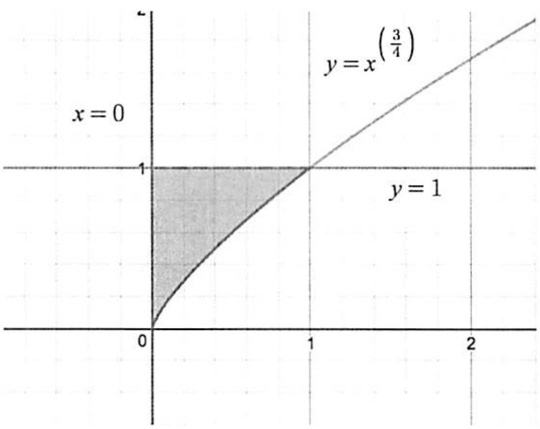
MA 141-004 S19 Recitation 13, April 10

1. Use integration by parts to evaluate the integral $\int_0^{\frac{\pi}{8}} x\sin(4x)dx$.

2. Use integration by parts (and possibly other integration methods) to evaluate the integral $\int \theta\csc^2(\theta)d\theta$.

3. Use integration by parts (and possibly other integration methods) to evaluate the integral $\int \frac{x^3}{\sqrt{x^2+1}}dx$.

4. Find the area of the shaded region bounded by $y = x^{\frac{3}{4}}, y = 1$, and $x = 0$ (in the first quadrant). Then determine the volume of the solid of revolution obtained by rotating the region about the y-axis.



5. Find the volume of the solid of revolution obtained by rotating the region bounded by $y = -|x-2|+2$ and $y = 0$ about the x- axis.