Tuesday, april 9

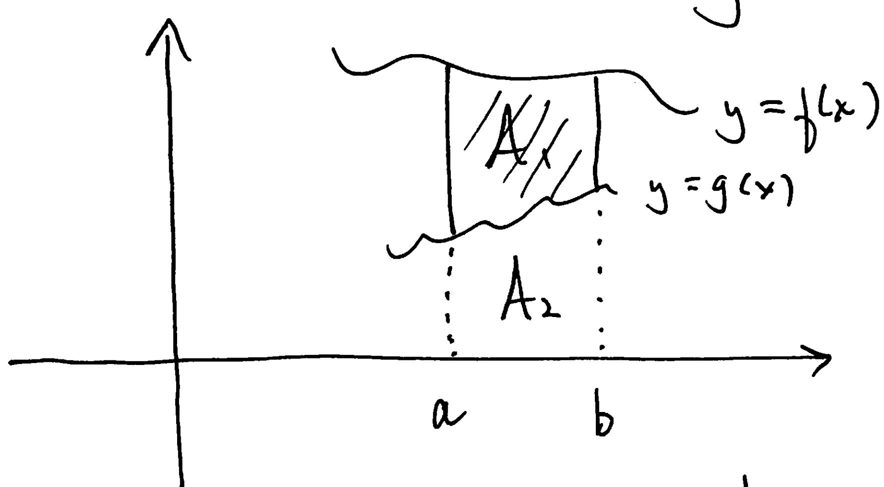
this week:

<u>5.1</u>: areas

<u>5.2</u>: volumes

- TEST #4: Thursday, april 18th
- FINAL EXAM: Tuesday, may 7th 1:00-4:00pm
  <u>or</u> ... (ALT. DATE: Sat, may 4th 8:00-11:00am)

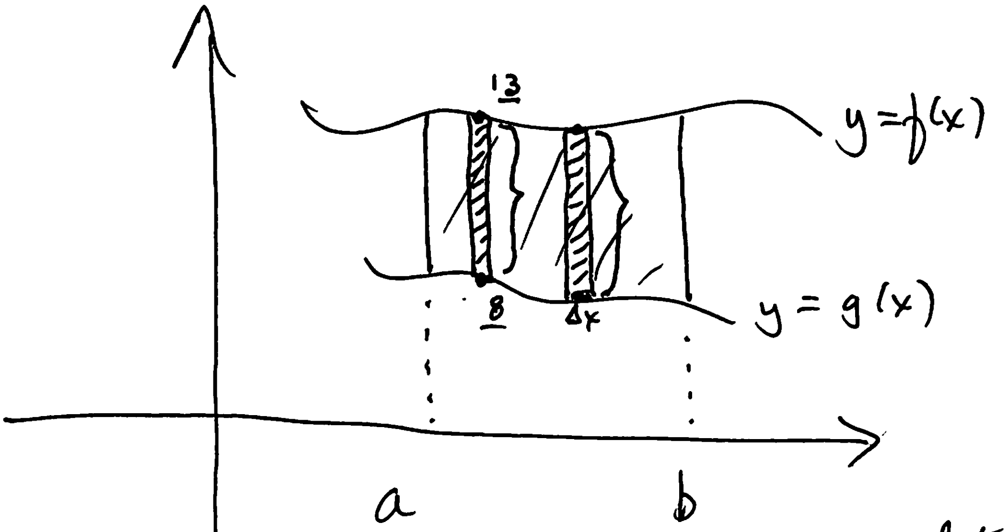
<u>5.1</u>: areas bounded by 2 curves



$$\int_a^b f(x)\,dx - \int_a^b g(x)\,dx = A_1$$

$$(A_1+A_2) \qquad (A_2) \qquad = A_1$$

$$A_1 = \int_a^b \left(f(x) - g(x)\right)dx$$

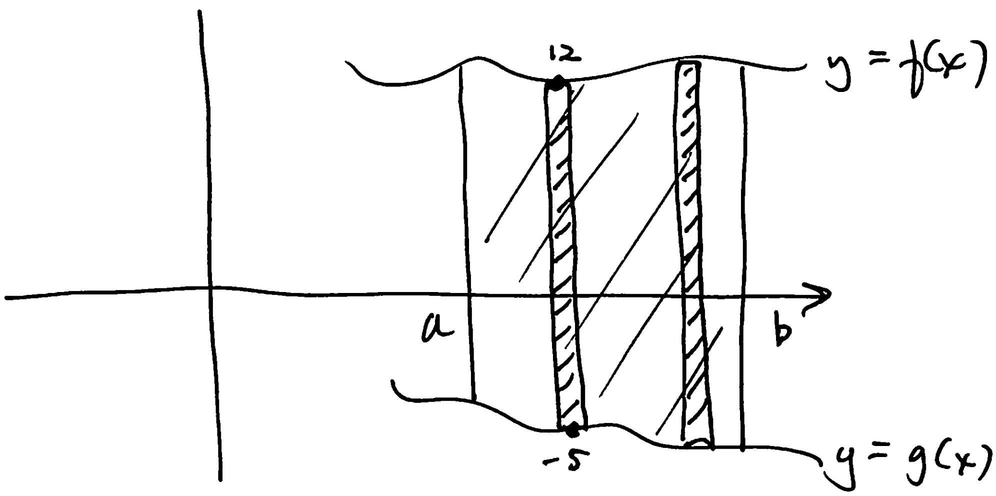


*

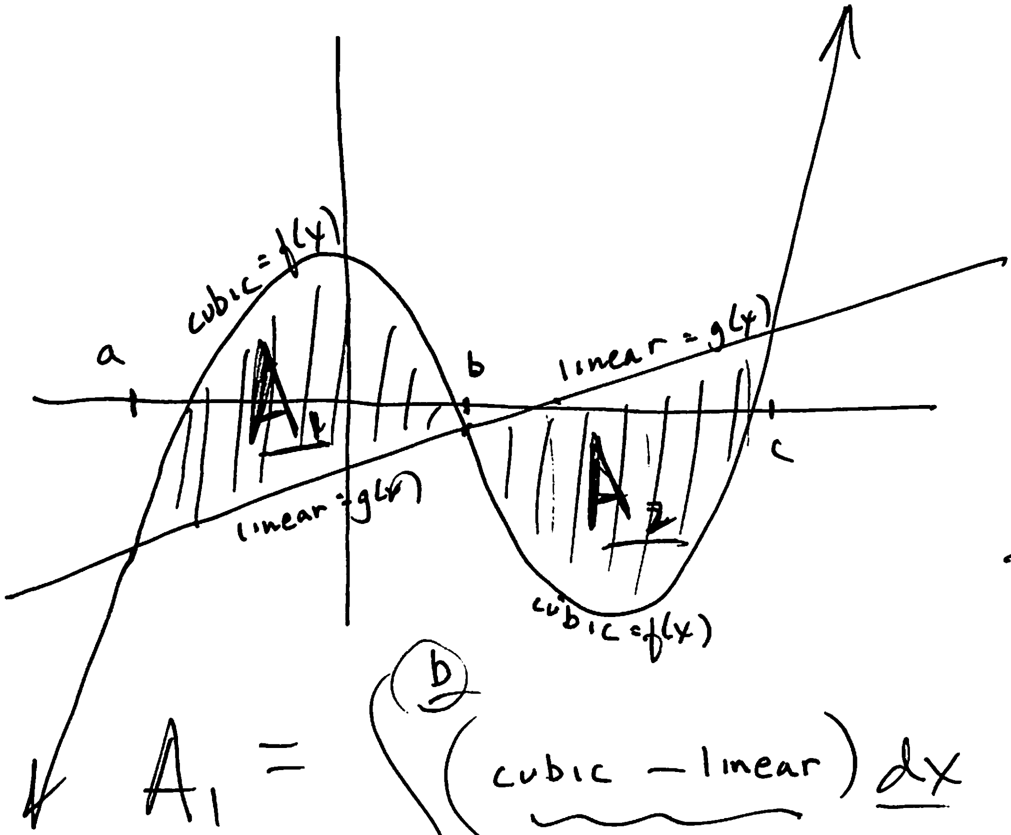
y-value on the LOWER CURVE

$$\int_a^b \underbrace{\left[f(x) - g(x)\right]}_{\text{HT}} \cdot \underbrace{dx}_{\text{WIDTH}}$$

y-value on the UPPER CURVE



$$\text{AREA} = \int_a^b \left[f(x) - g(x)\right] dx$$



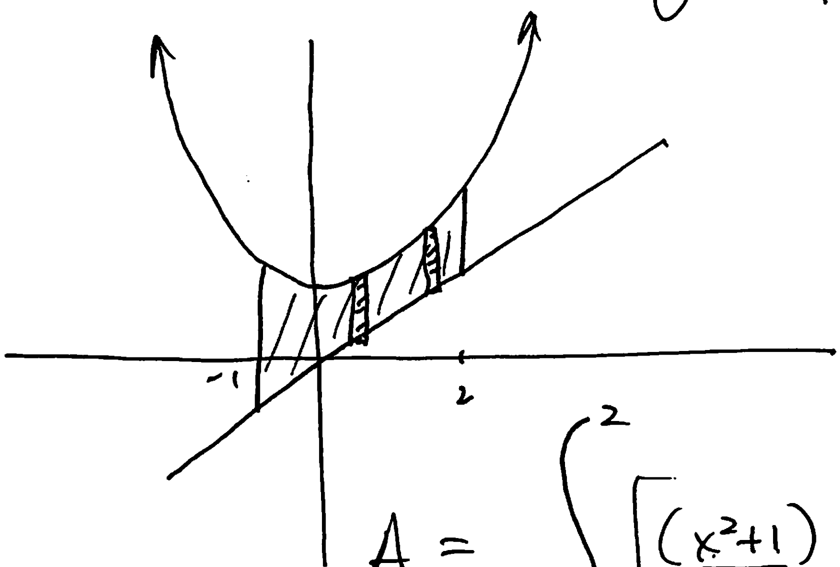
$f(x) = g(x)$

$\vdots$

$$A_1 = \int_a^b (\text{cubic} - \text{linear})\, dx$$

$$A_2 = \int_b^c (\text{linear} - \text{cubic})\, dx$$

$y = x^2+1 \qquad y = x \qquad$ from $x = -1$ to $x = 2$



$$A = \int_{-1}^{2} \left[(x^2+1) - (x) \right] dx$$

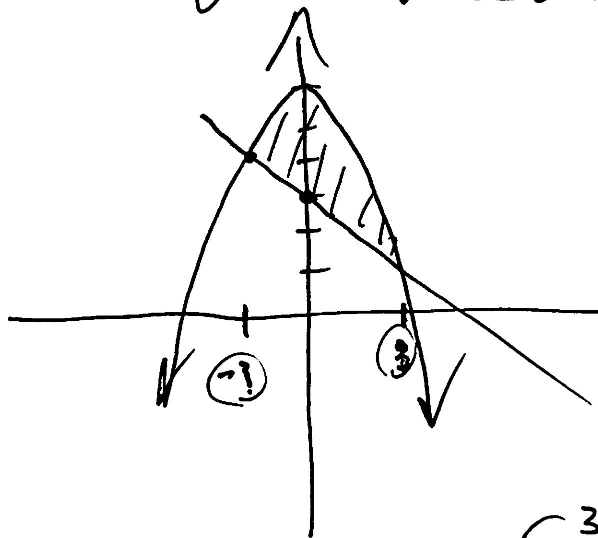
$$A = \left[\frac{x^3}{3} + x - \frac{x^2}{2} \right]_{-1}^{2}$$

$$A = \left[\left(\frac{2^3}{3} + 2 - \frac{2^2}{2} \right) - \left(\frac{(-1)^3}{3} + (-1) - \frac{(-1)^2}{2} \right) \right]$$

$A =$ ______

$y = 6 - x^2$ $\quad$ $y = -2x + 3$

find the area of the bounded region



$$6 - x^2 = -2x + 3$$

$$0 = x^2 - 2x - 3$$

$$0 = (x-3)(x+1)$$

$x = 3$ $\quad$ $x = -1$

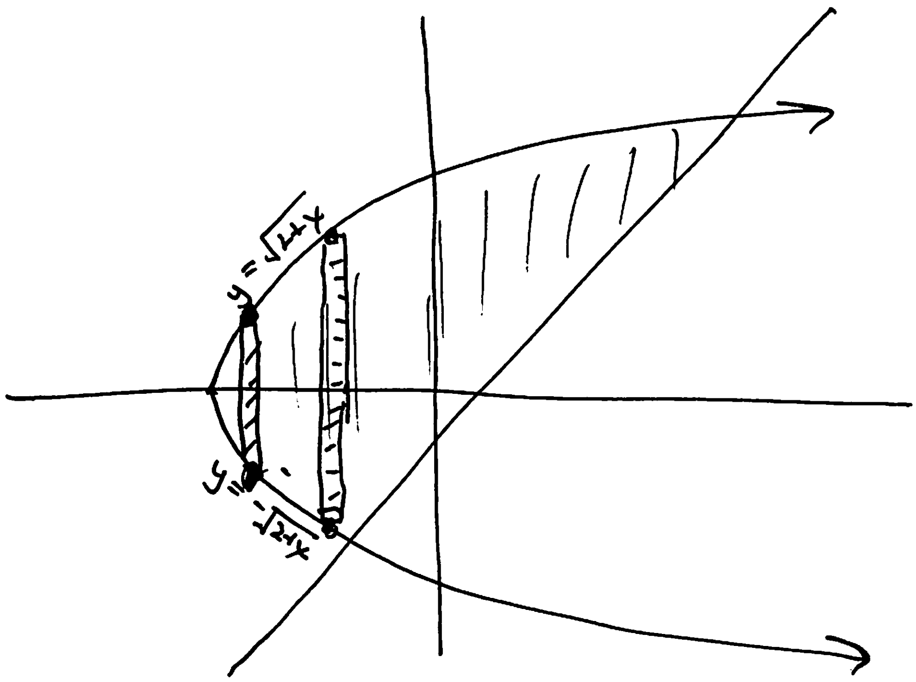
$$A = \int_{-1}^{3} \left[(6 - x^2) - (-2x + 3)\right] dx$$

$$A = \int_{-1}^{3} (3 - x^2 + 2x)\, dx$$

$$A = \left[3x - \frac{x^3}{3} + x^2\right]_{-1}^{3}$$

$$A = \left[\left(3(3) - \frac{(3)^3}{3} + (3)^2\right) - \left(3(-1) - \frac{(-1)^3}{3} + (-1)^2\right)\right]$$

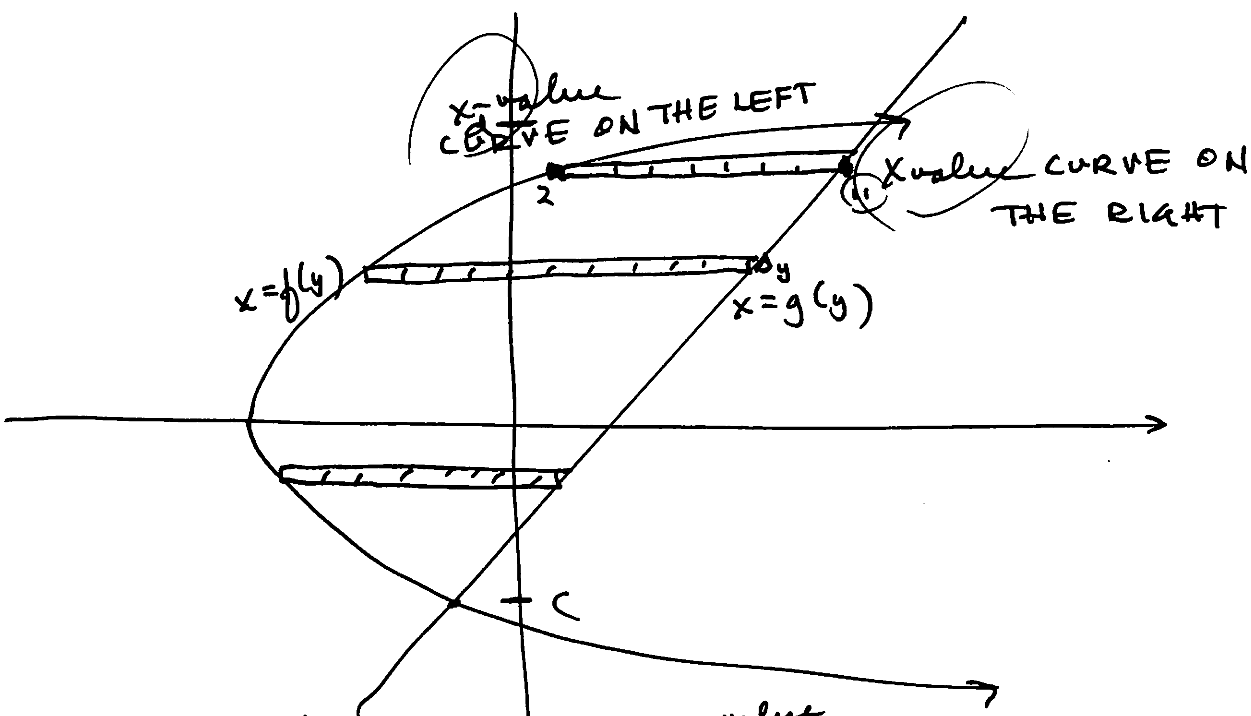
$A =$ ______



parabola:

$$y^2 = 2 + x$$

$$y = \pm\sqrt{2+x}$$



x-value CURVE ON LEFT

$$A = \int_{y=c}^{y=d} \left[g(y) - f(y) \right] dy$$

x-value CURVE ON RIGHT

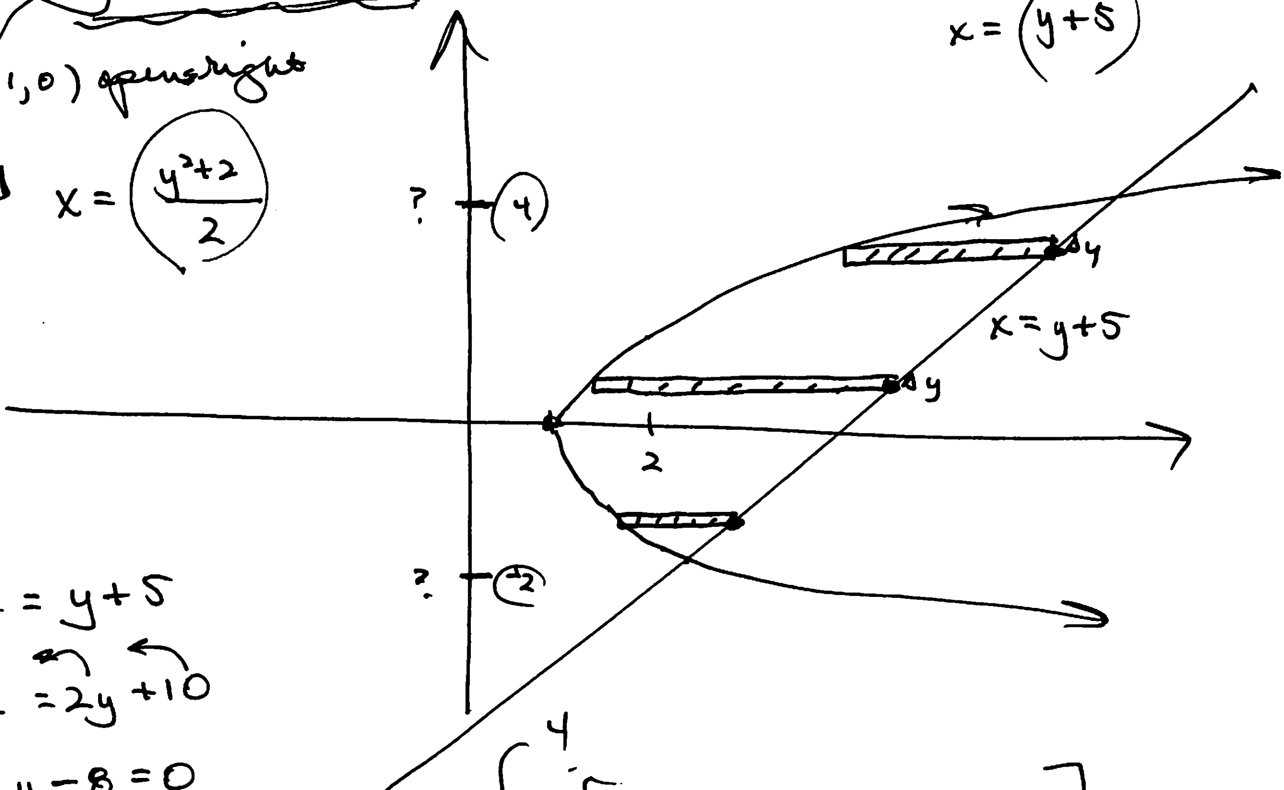
$y = \pm\sqrt{2x-2}$

$y^2 = 2x - 2$ and $y = x - 5$

$x = y + 5$

(1,0) opens right

$x = \dfrac{y^2+2}{2}$



$\dfrac{y^2+2}{2} = y + 5$

$y^2 + 2 = 2y + 10$

$y^2 - 2y - 8 = 0$

$(y-4)\ (y+2) = 0$

$y = 4 \quad y = -2$

$$A = \int_{-2}^{4} \left[(y+5) - \left(\frac{y^2+2}{2} \right) \right] dy$$

$$A = \int_{-2}^{4} \left(y + 4 - \tfrac{1}{2} y^2 \right) dy$$

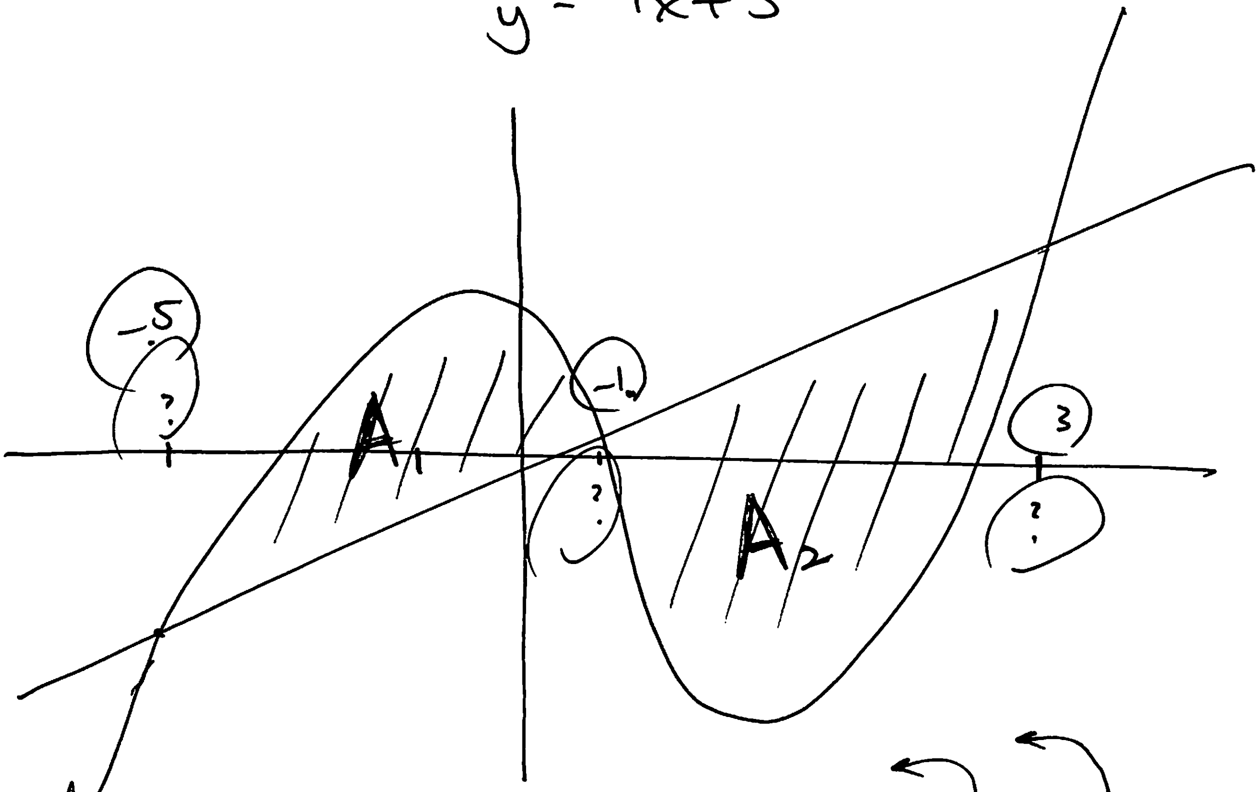
$$A = \left[\frac{y^2}{2} + 4y - \frac{1}{2}\frac{y^3}{3} \right]_{-2}^{4}$$

$$A = \left[\left(\frac{4^2}{2} + 4(4) - \frac{1}{6}(4)^3 \right) - \left(\frac{(-2)^2}{2} + 4(-2) - \frac{1}{6}(-2)^3 \right) \right]$$

$A =$ ____________

$y = x^3 + 3x^2 - 9x - 12$ <u>and</u>

$y = 4x + 3$



$x^3 + 3x^2 - 9x - 12 = 4x + 3$

$\pm 1, \pm 3, \pm 5, \pm 15$ $\boxed{x^3 + 3x^2 - 13x - 15 = 0}$

$(x \;\_\_)(x \;\_\_)(x \;\_\_) = 0$

$f(1) = (1)^3 + 3(1)^2 - 13(1) - 15 \overset{??}{\neq} 0$

$f(-1) = (-1)^3 + 3(-1)^2 - 13(-1) - 15 \overset{??}{=} 0$

-1 is a <u>root</u>; $(x - (-1))$ is a factor

$(x+1)$

$$x^2+2x-15$$

$$x+1 \;\big)\; x^3+3x^2-13x-15$$

$$-(x^3+1\cdot x^2)$$

$$2x^2-13x$$

$$-(2x^2+2x)$$

$$-15x-15$$

$$-(-15x-15)$$

$$0$$

$$(x+1)(x^2+2x-15)$$

$$(x+1)(x+5)(x-3)$$

$x=-1$ $\quad$ $x=-5$ $\quad$ $x=3$

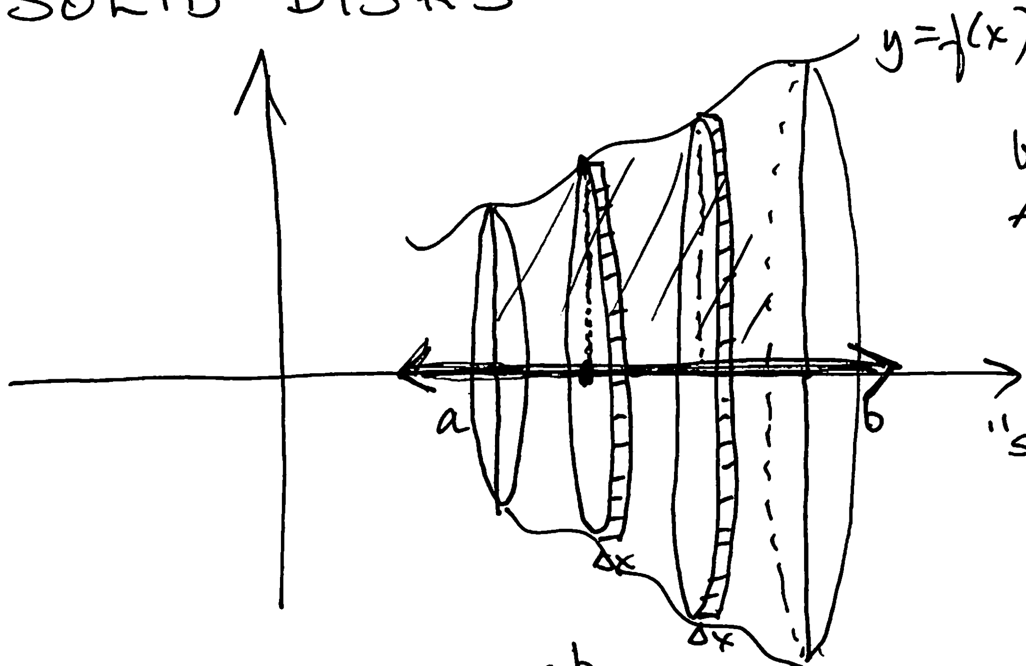
$$A_1=\int_{-5}^{-1}\left[(x^3+3x^2-9x-12)-(4x+3)\right]dx$$

$$A_2=\int_{-1}^{3}\left[(4x+3)-(x^3+3x^2-9x-12)\right]dx$$

5.2:

VOLUMES OF SOLIDS OF REVOLUTION

1 SOLID DISKS

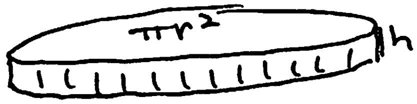


REVOLVE the bounded region ABOUT the x-axis

"slice up" $\perp$ to the axis of REV.

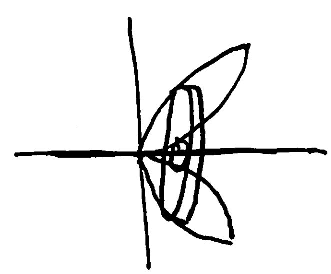
$r = y = f(x)$

$h = \Delta x \to dx$

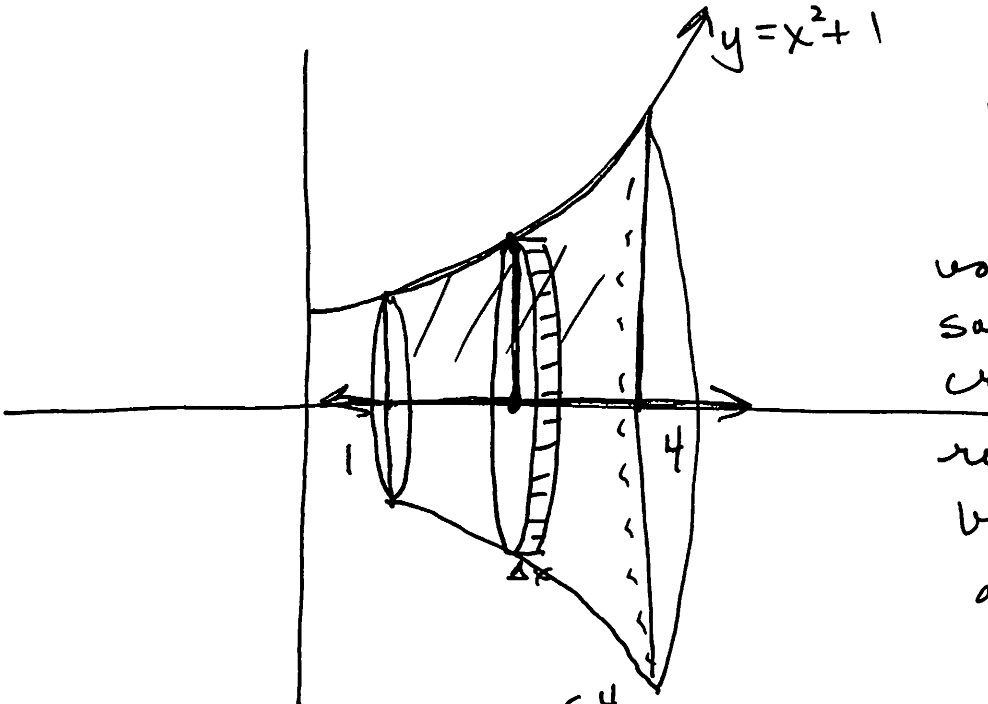


CYLINDER:
VOL = $\pi r^2 \cdot h$

$$V = \int_a^b \underbrace{\pi \cdot r^2 \cdot h}$$



$$V = \pi \int_{x=a}^{x=b} (f(x))^2 \, dx$$



$y = x^2+1$ $x=1$ to $x=4$

volume of the solid of revol. created by revolving the bounded region about the x-axis

$r = y = x^2+1$

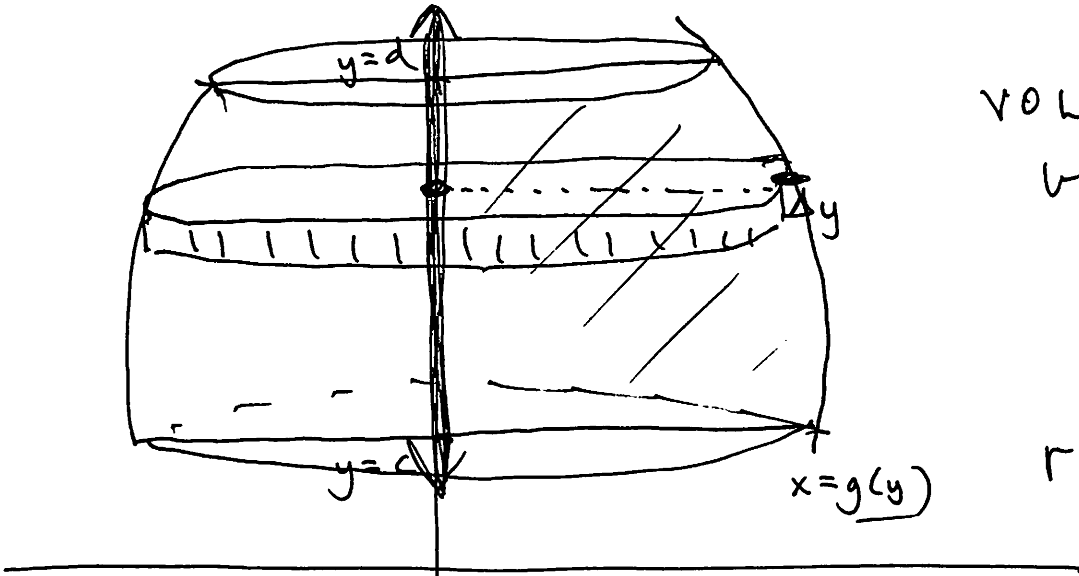
$h = \Delta x \to dx$

$$V = \int_1^4 \pi r^2 h$$

$$V = \pi \int_1^4 (x^2+1)^2 \, dx$$

$$V = \pi \int_1^4 (x^4+2x^2+1)\, dx$$

$$V = \pi \left[\frac{x^5}{5} + \frac{2x^3}{3} + x \right]_1^4$$



VOL:
bounded
region
about the
y-axis

$r = x = g(y)$

$$V = \int_c^d \pi \left(g(y)\right)^2 \cdot dy$$