MA 141 - 004

Thursday, april 4

4.5: INTEGRATION BY PARTS

4.4:

$$\frac{1}{8}\cdot 3\int \frac{\cancel{3}x\cdot 8}{4x^2+7}\,dx$$

let $u = 4x^2+7$

$du = 8x\,dx$

$$\frac{3}{8}\int \frac{du}{u} =$$

$$\frac{3}{8}\ln|u| + C$$

$$\frac{3}{8}\ln|4x^2+7| + C$$

$$3\int \frac{1\cdot \cancel{3}\cdot \cancel{x}\cdot\cancel{\frac{1}{x}}}{4x^2+7}\,dx$$

~~let $u = 4x^2+7$~~

~~$du = 8x\,dx$~~

$\tan^{-1}$

$$\int \frac{1}{x^2+1}\,dx = \tan^{-1}x$$

2

$$\int \frac{1}{u^2+a^2}\,du \dashrightarrow \int \frac{1}{x^2+1}\,dx$$

$$\int \frac{1/a^2}{\frac{u^2}{a^2}+\frac{a^2}{a^2}}\,du = \frac{1}{a^2}\cdot a \int \frac{1}{\left(\frac{u}{a}\right)^2+1}\,\frac{1}{a}du$$

let $w = \frac{u}{a}$

$dw = \frac{1}{a}du$

$\frac{u}{3}$ or $\frac{u}{7}$

$\frac{1}{3}u \quad \frac{1}{7}u$

$$= \frac{1}{a}\int \frac{1}{w^2+1}\,dw$$

$$= \frac{1}{a}\tan^{-1} w$$

$$= \frac{1}{a}\tan\left(\frac{u}{a}\right)+C$$

$$\int \frac{1}{u^2+a^2}\,du = \frac{1}{a}\cdot\tan^{-1}\left(\frac{u}{a}\right)+C$$

$$\int \frac{3}{\underbrace{4x^2}_{u^2}+\underbrace{7}_{a^2}}\,dx = 3\cdot\frac{1}{2}\int \frac{1}{(2x)^2+(\sqrt{7})^2}\,dx(2)$$

$\boxed{u=2x}$ $du=2\cdot dx$

$a=\sqrt{7}$

$$= \frac{3}{2}\left[\frac{1}{\sqrt{7}}\tan^{-1}\frac{2x}{\sqrt{7}}\right]+C$$

4.5: INTEGRATION BY PARTS

(reverse product rule)

$$d(u\cdot v) = \underbrace{u\cdot dv}_{\text{isolate this}} + v\cdot du$$

$$\int d(u\cdot v) - v\cdot du = \int u\cdot dv$$

$$\int d(u\cdot v) - \int v\cdot du = \int u\cdot dv$$

$$\boxed{u \cdot v - \int v\,du = \int u\,dv}$$

$$\int \boxed{u}\,\underline{dv} = \underbrace{u \cdot v - \int v \cdot du}$$

"by parts"

ex:

$$\int \underbrace{x}\cdot\underbrace{\cos x\,dx}$$

$d(???) = x\cos x$

let $u = x$, $du = 1 \cdot dx$

$v = \sin x$, $dv = \cos x \cdot dx$

$$\int x\cos x\,dx = (x)(\sin x) - \underbrace{\int \sin x\,dx}$$

$$= x\sin x + \cos x + C$$

check:

$$d(x \cdot \sin x + \cos x + C) =$$

$$[x \cdot \cos x + \cancel{\sin x\,(1)}] + (\cancel{-\sin x}) + \cancel{0}$$

$$\int \underline{u} \cdot dv = u \cdot v - \int \underline{v} \cdot \underline{du}$$

ex:
$$\int \underline{x^3} \cdot \ln x \cdot \underline{dx}$$

let $u = \ln x$ $\qquad v = \frac{x^4}{4}$

$du = \frac{1}{x} dx \qquad dv = x^3 \cdot dx$

$$\int \underline{x^3 \cdot \ln x} \, dx = (\ln x)\left(\frac{x^4}{4}\right) - \int \frac{x^4}{4} \cdot \frac{1}{x} \, dx$$

$$= (\ln x)\left(\frac{x^4}{4}\right) - \frac{1}{4} \boxed{\int x^3 \, dx}$$

$$= (\ln x)\left(\frac{x^4}{4}\right) - \frac{1}{4} \cdot \frac{x^4}{4} + C$$

$$= (\ln x)\left(\frac{x^4}{4}\right) - \frac{1}{16} x^4 + C$$

$$\int u \cdot dv = u \cdot v - \int v \cdot du$$

ex: $\displaystyle\int 5x^2 \cdot e^x \cdot dx$

let $u = 5x^2$ $\quad v = e^x$

$du = 10x\,dx$ $\quad dv = e^x\,dx$

$$\int 5x^2 \cdot e^x \cdot dx = (5x^2)(e^x) - \int e^x \cdot 10x\,dx$$

by parts AGAIN ....

$$= (5x^2)(e^x) - \int 10x \cdot e^x\,dx$$

$U = 10x$ $\quad V = e^x$

$dU = 10dx$ $\quad dV = e^x \cdot dx$

$$= (5x^2)(e^x) - \left[(10x)(e^x) - \int e^x \cdot 10\,dx\right]$$

$$(5x^2)(e^x) - (10x)(e^x) + 10 \cdot e^x + C$$

$$\int \tan^{-1} x \, dx$$

let $u = \tan^{-1} x$ $\qquad v = x$

$du = \frac{1}{x^2+1} dx$ $\qquad dv = dx$

$$= (x)(\tan^{-1} x) - \int x \cdot \frac{1}{x^2+1} dx$$

$$= (x)(\tan^{-1} x) - \frac{1}{2} \int \frac{2 \cdot x \, dx}{x^2+1}$$

SUBST.

$u = x^2+1$

$du = 2 x \, dx$

$$= x \cdot \tan^{-1} x - \frac{1}{2} \ln |u| + C$$

$$= x \cdot \tan^{-1} x - \frac{1}{2} \ln |x^2+1| + C$$

$$\int e^x \cdot \sin x \, dx$$

let $u = e^x$ $\qquad v = -\cos x$

$du = e^x dx \qquad dv = \sin x \, dx$

$$\int e^x \cdot \sin x \, dx = (e^x)(-\cos x) - \int -\cos x \cdot e^x dx$$

$$\int e^x \cdot \sin x \, dx = -e^x \cdot \cos x + \left[\int e^x \cdot \cos x \cdot dx\right]$$

by parts AGAIN

$u = e^x \qquad v = \sin x$

$du = e^x dx \qquad dv = \cos x dx$

$$\int e^x \sin x \, dx = -e^x \cdot \cos x + (e^x)(\sin x) - \cancel{\int e^x \sin x dx}$$

$$+\int e^x \sin x dx \qquad\qquad + \cancel{\int e^x \sin x dx}$$

$$\frac{2\int e^x \sin x dx}{2} = \frac{-e^x \cos x}{2} + \frac{e^x \sin x}{2}$$

$$\int e^x \sin x dx = \frac{1}{2}\left(-e^x \cdot \cos x + e^x \sin x\right) + C$$

$$\int_2^3 \frac{\ln x}{3x}\,dx$$

$$= \int_2^3 \frac{1}{3}\cdot \ln x \cdot \frac{1}{x}\cdot dx$$

$$= \frac{1}{3}\int_2^3 \overset{u}{\underline{\ln x}} \cdot \overset{du}{\left(\frac{1}{x}\cdot dx\right)} \quad \text{SUBST.}$$

$$u = \ln x$$
$$du = \frac{1}{x}\cdot dx$$

$$= \frac{1}{3}\int \underline{u}\cdot \underline{du} = \frac{1}{3}\left[\frac{u^2}{2}\right]$$

$$= \frac{1}{6}\left[(\ln x)^2\right]_2^3$$

$$= \frac{1}{6}\left[(\ln 3)^2 - (\ln 2)^2\right] \approx \_\_\_\_\_\_$$

$$\int \sec x \, dx$$

$$\int \frac{\sec x(\sec x + \tan x)}{(\sec x + \tan x)} dx$$

$$= \int \frac{\sec^2 x + \sec x \cdot \tan x}{(\sec x + \tan x)} dx$$

let $u = \sec x + \tan x$

$du = (\sec x \cdot \tan x + \sec^2 x)dx$

$$= \int \frac{du}{u} = \ln|u| + C$$

$$\boxed{\int \sec x \, dx = \ln|\sec x + \tan x| + C}$$

$$\int \sec^3 x\,dx$$ (challenging)

$$\int \sec x \cdot \underline{\sec^2 x\,dx}$$

let $u = \sec x$ $\qquad v = \tan x$

$du = \sec x \cdot \tan x\,dx$ $\quad dv = \sec^2 x\,dx$

$$\int \sec^3 x\,dx = (\sec x)(\tan x) - \int \tan x \cdot \sec x \cdot \tan x\,dx$$

$$\int \sec^3 x\,dx = (\sec x)(\tan x) - \int \sec x \cdot \underline{\tan^2 x} \cdot dx$$

$$\int \sec^3 x\,dx = (\sec x)(\tan x) - \int \sec x\,(\sec^2 x - 1)\,dx$$

$$\int \sec^3 x\,dx = (\sec x)(\tan x) - \cancel{\int \sec^3 x\,dx} + \underline{\int \sec x\,dx}$$

$$+\int \sec^3 x\,dx \qquad\qquad + \cancel{\int \sec^3 x\,dx}$$

$$2\int \sec^3 x\,dx = (\sec x)(\tan x) + \ln|\sec x + \tan x| + C$$

$$\int \sec^3 x\,dx = \frac{1}{2}\Big[(\sec x)(\tan x) + \ln|\sec x + \tan x|\Big] + C$$

MA 141-004 S19 Recitation 12, April 3

1. Use The Fundamental Theorem of Calculus to find $\frac{dG}{dx}$ given

$$G(x) = \int_x^{\pi} 3\cos t dt.$$

2. Use The Fundamental Theorem of Calculus to find $F'(x)$ given

$$F(x) = \int_2^{x^4} 5\sqrt{t} dt.$$

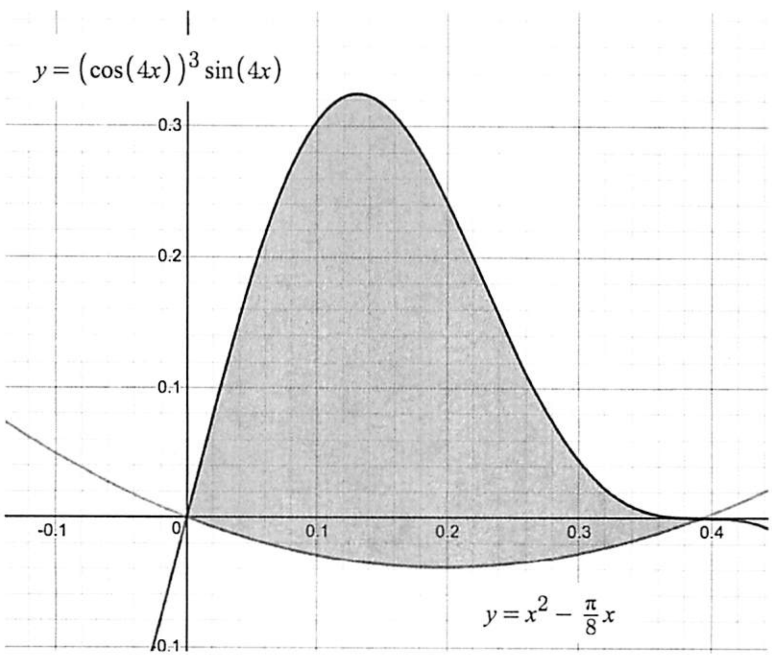
3. Evaluate the integrals using substitution.

(a) $\int x(3x^2-1)^3 dx$

(b) $\int \frac{9x^2+9}{x^3+3x} dx$

(c) $\int \sec(4x - \frac{\pi}{2}) \tan(4x - \frac{\pi}{2}) dx$

4. The graph of the equations $y = \cos^3(4x)\sin(4x)$ and $y = x^2 - \frac{\pi}{8}x$ is given below. These equations intersect at the points $(0,0)$ and $(\frac{\pi}{8}, 0)$. Use substitution to find the area of the enclosed region.



Theorem 4. The Fundamental Theorem of Calculus

Let f be continuous on $[a,b]$.

Part 1: The function G defined by

$$G(x) = \int_a^x f(t)\,dt \qquad \text{for all } x \text{ in } [a,b]$$

is an antiderivative of f on $[a,b]$.

Part 2: If F is any antiderivative of f on $[a,b]$, then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$