MA141-004

Tuesday, April 2

4.4: INTEGRATION USING SUBSTITUTION

(change of variable method)

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C$$

$(n \neq -1)$

subst ...

let $u =$

$du =$

$$\int u^n \, du = \frac{u^{n+1}}{n+1} + C$$

$\vdots$

$$\int x^{-1} \, dx = \int \frac{1}{x} \, dx = \ln|x| + C$$

subst ....

let $u =$

$du =$

$$\int \frac{1}{u} \, du = \ln|u| + C$$

$\vdots$

$$\int e^x \, dx = e^x + C$$

subst ...

let $u =$

$du =$

$$\int e^u \, du = e^u + C$$

$\vdots$

$F' = f$ chain rule

$$\int d\left[F(g(x))\right] = \int F'(g(x)) \cdot g'(x)$$

$$F(g(x)) = \boxed{\int f(g(x)) \cdot g'(x)}$$ **

ex:

$$\int x \cdot \sqrt[3]{6-8x^2}\, dx$$

let $\boxed{u = 6-8x^2}$

$$\frac{du}{dx} = 0 - 16x$$

$$\cancel{dx}\,\frac{du}{\cancel{dx}} = -16x \cdot dx$$

$$\boxed{du = \boxed{-16}\,x\,dx}$$

$$= \frac{-1}{16}\int (6-8x^2)^{1/3}\ \boxed{x \cdot dx\ (-16)}\left(\cancel{\frac{-1}{16}}\right)$$

$$\frac{-1}{16}\int u^{1/3} \cdot du = \frac{-1}{16} \cdot \frac{u^{4/3}}{4/3} + C$$

$$= \frac{-1}{16} \cdot \frac{3}{4}\, u^{4/3} + C$$

$$= \frac{-3}{64}\, u^{4/3} + C$$

$$= \frac{-3}{64}(6-8x^2)^{4/3} + C$$

$$\frac{1}{12} \cdot 11 \cdot \int \sec^2(4x^3+1) \cdot \boxed{x^2 \cdot dx \, (12)}$$

let $\boxed{u = 4x^3+1}$

$$dx \, \frac{du}{dx} = 12x^2 \cdot dx$$

$$\boxed{du = 12 x^2 dx}$$

$$\frac{11}{12} \int \sec^2 u \cdot du$$

$$= \frac{11}{12} \tan u + C$$

$$= \frac{11}{12} \tan(4x^3+1) + C$$

$$\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx = 2 \int e^{\sqrt{x}} \cdot \boxed{\frac{1}{\sqrt{x}} dx \cdot \frac{1}{2}}$$

$\int e^u du$

let $\boxed{u = \sqrt{x}} = x^{1/2}$

$$du = \frac{1}{2} \cdot x^{-1/2} \cdot dx$$

$$\boxed{du = \frac{1}{2} \cdot \frac{1}{\sqrt{x}} dx}$$

$$= 2 \int e^u \cdot du = 2 \cdot e^u + C$$

$$= 2 \cdot e^{\sqrt{x}} + C$$

$$\int \tan x \, dx$$

rewrite

$\cancel{d(???) = \tan x}$

$$-1 \int \boxed{\frac{\sin x}{\cos x} dx (-1)}$$

let $u = \cos x$

$du = -\sin x \cdot dx$

$$-1 \int \frac{du}{u} = -1 \int \frac{1}{u} du$$

$$= -1 \cdot \ln|u| + C$$

$$= \boxed{-1} \cdot \ln|\cos x| + C$$

$\ln a^b = b \cdot \ln a$

$$= \ln|\cos x|^{-1} + C$$

$$= \ln|\sec x| + C$$ *

$$\int \tan x \, dx = \ln|\sec x| + C$$

$$\int x \cdot \sqrt{x+3}\, dx$$

let $\boxed{u = x+3} \Rightarrow x = u - 3$

$du = 1 \cdot dx$

$$\int (u-3)\, u^{1/2} \cdot du$$

$$= \int \left(u^{3/2} - 3u^{1/2}\right) du$$

$-\cancel{3} \cdot \frac{2}{\cancel{3}}$

$$= \frac{u^{5/2}}{5/2} - 3 \cdot \frac{u^{3/2}}{3/2} + C$$

$$= \frac{2}{5}(u)^{5/2} - 2(u)^{3/2} + C$$

$$= \frac{2}{5}(x+3)^{5/2} - 2(x+3)^{3/2} + C$$

$$\int x\cdot\sqrt{x+3}\cdot dx$$

let $\boxed{u=\sqrt{x+3}}$

$$u^2 = x+3 \Rightarrow x = u^2-3$$

$$\boxed{2u\,du = 1\cdot dx}$$

$$\int (u^2-3)\cdot u\cdot 2u\,du$$

$$2\int (u^2-3)\cdot u^2\cdot du$$

$$2\int (u^4-3u^2)\,du$$

$$2\left[\frac{u^5}{5} - \cancel{3}\cdot\frac{u^3}{\cancel{3}}\right] + C$$

$$= \frac{2}{5}(u)^5 - 2(u)^3 + C$$

$$= \frac{2}{5}(\sqrt{x+3})^5 - 2(\sqrt{x+3})^3 + C$$

$$\frac{2}{5}(x+3)^{5/2} - 2(x+3)^{3/2} + C$$

⑧

$$2\int_{x=1}^{x=4} e^{\sqrt{x}} \boxed{\frac{1}{\sqrt{x}} \cdot dx \cdot \frac{1}{2}}$$

let $u=\sqrt{x}$ $\quad (x^{1/2})$

$du = \frac{1}{2}x^{-1/2} \cdot dx$

$du = \frac{1}{2} \cdot \frac{1}{\sqrt{x}} \cdot dx$

$$2\int_{u=1}^{u=2} e^{u}\, du$$

$x=1 \quad (u=\sqrt{x}) \quad u=1$

$x=4 \quad (u=\sqrt{x}) \quad u=2$

$$2\left[e^{u}\right]_1^2 = 2\left[e^2 - e^1\right] \approx 9.34$$

OR ....

$$2\int e^{u}\, du = 2\left[e^{u}\right]$$

$$= 2\left[e^{\sqrt{x}}\right]_1^4 = 2\left[e^{\sqrt{4}} - e^{\sqrt{1}}\right]$$

$$= 2\left[e^2 - e^1\right]$$

$$\int e^{\tan\theta}\cdot\boxed{\sec^2\theta\cdot d\theta} \qquad \int e^u\,du$$

let $u = \tan\theta$

$du = \sec^2\theta\, d\theta$

$$\int e^u\,du = e^u + C$$

$$= e^{\tan\theta} + C$$

$$\int \cos^3(5\theta)\cdot\sin(5\theta)\cdot d\theta$$

$$-\frac{1}{5}\int \left[\boxed{\cos(5\theta)}\right]^3\cdot\boxed{\sin(5\theta)\cdot d\theta\,(-5)}$$

~~let $u = \sin(5\theta)$~~

~~$du = \cos(5\theta)\cdot 5\cdot d\theta$~~

let $u = \cos(5\theta)$

$du = -\sin(5\theta)\cdot 5\cdot d\theta$

$$-\frac{1}{5}\int u^3\cdot du = -\frac{1}{5}\,\frac{u^4}{4} + C$$

$$= \frac{-1}{20}\cdot(\cos(5\theta))^4 + C$$

$$\frac{1}{2}\int_{t=0}^{t=1} \frac{4t+1}{4t^2+2t+3}\, dt \cdot 2$$

let $u = 4t^2+2t+3$

$du = (8t+2)\,dt$

$du = 2(4t+1)\,dt$

$t=0 \quad (u = 4t^2+2t+3) \quad u=3$

$t=1 \quad (u = 4t^2+2t+3) \quad u=9$

$$\frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\int \frac{1}{u}\cdot du$$

$$= \frac{1}{2}\ln|u|\Big]_3^9$$

$$= \frac{1}{2}\left[\ln 9 - \ln 3\right] = \frac{1}{2}\left[\ln\frac{9}{3}\right]$$

$$= \frac{1}{2}\ln 3 \approx \underline{\qquad}$$