NORTH CAROLINA STATE UNIVERSITY Department of Mathematics
MA121-002 Test #4 Form B Thursday, April 18, 2019 Dr. John Griggs

Please put all work and solutions in the stamped blue book provided. Begin each new problem on a new page (the back of the sheet can be a new page). Put your **name, form of test** (A or B), **row letter**, and **seat number** on the outside of the blue book. Be sure to include proper units on the answers when appropriate. Fold the test copy and turn it in with your blue book.
(14 points per question – 2 points for following directions – show all work)

1.) Solve for y: $\frac{dy}{dx} = 6xy$

2.) Evaluate the improper integral; is the integral convergent or divergent? $\int_0^\infty 4e^{-3x}\,dx$

3.) Find the volume generated when the region bounded by $y = 3\sqrt{x}$, the x-axis, x=2 and x=4 is revolved about the x-axis.

4.) Find the equilibrium point, the consumer surplus (labeled CS) and producer surplus (labeled PS) for the given supply and demand functions: $D(x) = -.05x + 50$ $S(x) = .1x + 20$

5.) John and Laura LaMont have a new grandson, Johnny, and want to create a college trust fund that will yield $85,000 on his 18th birthday. What lump sum do they need to deposit now (one time), at 2.2% interest, compounded continuously, to grow to $85,000?

6.) Using the problem above (5.), John and Laura decide that their one-time deposit is more than they can afford at the present time, and want to know how much they would need to deposit every year for 18 years such that the accumulated future value of their continuous money stream would be $85,000 (at 2.2% interest, compounded continuously).

7.) Integrate (using substitution) and evaluate: $\int_0^2 3x^2 e^{x^3}\,dx$

Bonus: (5 points) Cesium-137 has a decay rate of 2.3% per year. Suppose an accident causes cesium-137 to be released into the atmosphere **perpetually** at the rate of 2 pounds per year. What is the limiting value of the radioactive buildup?

MA 121 TEST #4 FORM B J. GRIGGS

(14 points each)

1.) $\frac{dy}{dx} = 6xy$ solve for y ...

14 PTS

$$\frac{1}{y}\cdot \cancel{dx}\cdot \frac{dy}{\cancel{dx}} = 6x\cancel{y}\cdot dx \cdot \frac{1}{\cancel{y}}$$

$$\frac{1}{y}\,dy = 6x\,dx$$

$$\int \frac{1}{y}\,dy = \int 6x\,dx$$

$$\ln y = 6\left(\frac{x^2}{2}\right) + C$$

$$\ln y = 3x^2 + C$$

$$e^{\ln y} = e^{3x^2+C}$$

$$\boxed{y = e^{3x^2}\cdot e^{c}} \quad (e^{c} = A)$$

$$\boxed{y = A\cdot e^{3x^2}}$$

2.) $\int_0^\infty 4\cdot e^{-3x}\,dx = \lim_{A\to\infty}\int_0^A 4\cdot e^{-3x}\,dx$

14 PTS

$$= \lim_{A\to\infty} 4\cdot\int_0^A e^{-3x}\,dx = 4\cdot\lim_{A\to\infty}\left[\frac{1}{-3}e^{-3x}\right]_0^A$$

$$= \frac{4}{-3}\cdot\lim_{A\to\infty}\left[\frac{1}{e^{3x}}\right]_0^A = -\frac{4}{3}\lim_{A\to\infty}\left[\frac{1}{e^{3A}} - \frac{1}{e^{3(0)}}\right]$$

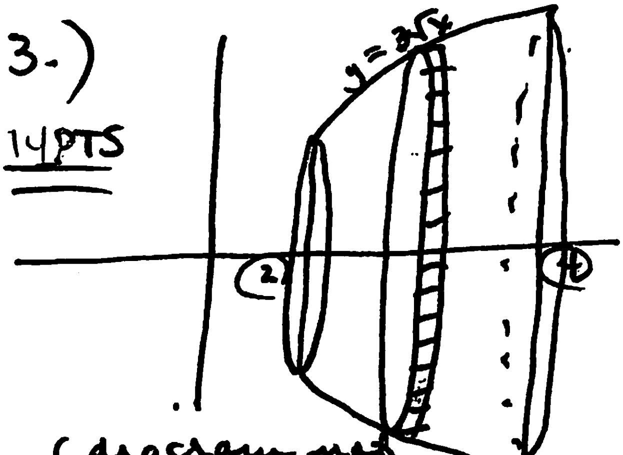
(page 2)

$$= \frac{-4}{3} \lim_{A \to \infty} \left[\cancel{\frac{1}{e^{3A}}}^{\to 0} - 1 \right] = \frac{-4}{3} \cdot (0-1) = \boxed{\frac{4}{3}}$$

∴ integral converges

3.)

14PTS



(diagram not necessary)

$$\int \pi r^2 h \to \pi \int (f(x))^2 \cdot dx$$

$r = y = f(x)$

$h = \Delta x \to dx$

$$\pi \int_2^4 (3\sqrt{x})^2 dx = \pi \int_2^4 9x\,dx = 9\pi \left(\frac{x^2}{2}\right)\Bigg]_2^4$$

$$= \frac{9\pi}{2}\left[4^2 - 2^2\right] = \frac{9\pi}{2}\left[16-4\right] = \frac{9(\cancel{12}^{6})\pi}{\cancel{2}_{1}} = 54\pi$$

$$\approx 169.65$$

4.) $D(x) = -.05x + 50$

14PTS

$S(x) = .1x + 20$

EQ. PT.:

$$-.\cancel{05x} + 50 = .1x + \cancel{20}$$

$$+\cancel{.05x} - 20 \quad +.05x - \cancel{20}$$

$$30 = .15x \qquad x = \frac{30}{.15} = 200$$

$D(200) = -.05(200) + 50 = 40$

$S(200) = .1(20) + 20 = 40$

$(200, 40^{00})$

C.S.: $\int_0^{200}(-.05x+50)\,dx - \left[(200)(40)\right]$

C.S. = $\left[-.05\frac{x^2}{2}+50x\right]_0^{200} - 8000$

C.S. = $\left[\frac{-.05}{2}(40,000)+50(200)\right] - 8000$

C.S. = $\left[-1,000 + 10,000\right] - 8000 = 10,000 - 9,000$

$\boxed{C.S. = 1000^{\underline{00}}}$

P.S.: $\left[(200)(40)\right] - \int_0^{200}(.1x+20)\,dx$

P.S. = $8,000 - \left[\frac{.1x^2}{2}+20x\right]_0^{200}$

P.S. = $8,000 - \left[\frac{.1}{2}(200)^2+20(200)\right]$

P.S. = $8,000 - \left[2,000 + 4,000\right] = 8,000 - 6,000$

$\boxed{P.S. = 2000^{\underline{00}}}$

5.) $P = P_0 \cdot e^{kt}$

14pts

$$85{,}000 = P_0 \cdot e^{.022(18)}$$

$$\frac{85{,}000}{e^{(.022)(18)}} = P_0 \approx \boxed{\$57{,}205.^{\underline{57}}}$$

6.) $85{,}000 = \int_0^{18} P_0 \cdot e^{.022} \, dt$

14pts

$$85{,}000 = P_0 \cdot \left[\frac{1}{.022} e^{.022t}\right]_0^{18}$$

$$85{,}000 = \frac{P_0}{.022}\left[e^{.022(18)} - e^{(.022)(0)}\right]$$

$$85{,}000 = \frac{P_0\left[e^{.022(18)} - 1\right]}{.022}$$

$$P_0 = \frac{(85{,}000)(.022)}{e^{(.022)(18)} - 1} \approx \boxed{\$3848.^{\underline{77}}}$$

7.) $\int_0^2 3x^2 \cdot e^{x^3} \, dx$

14pts

$u = x^3$

$du = 3x^2 dx$

$x=0 \xrightarrow{u=x^3} u=0$

$x=2 \longrightarrow u=8$

$$\int_0^8 e^u \, du = e^u\Big]_0^8 = e^8 - e^0 = \boxed{e^8 - 1}$$

$$\approx \boxed{2979.96}$$

BONUS (5 points)

$$\int_0^{\infty} 2 \cdot e^{-.023t}\,dt$$

$$= \lim_{A\to\infty} \int_0^{A} 2\,e^{-.023t}\,dt$$

$$= \lim_{A\to\infty} 2 \cdot \frac{1}{-.023}\left[e^{-.023t}\right]_0^A$$

$$= \lim_{A\to\infty} \frac{2}{-.023}\left[e^{(-.023)(A)} - e^{(-.023)(0)}\right]$$

$$= \lim_{A\to\infty} \frac{2}{-.023}\left[\cancelto{0}{\frac{1}{e^{.023(A)}}} - 1\right]$$

$$= \frac{2}{(-.023)}(-1) = \boxed{86.96 \text{ lbs}}$$