NORTH CAROLINA STATE UNIVERSITY Department of Mathematics

MA121-002 Test #3 Form B Thursday, March 28, 2019 Dr. J. Griggs

Please show all work and answers in the stamped blue book provided; one problem per page. Please put your **name, section** (121-002), and **form** (A or B) on the front of the blue book; put your **row letter** and **seat number** on the top right corner of your blue book. Do **not** use a graphing calculator, nor any other calculator that **does** calculus. Simplify completely. (8 questions; 12 points each; 4 points for following directions)

1.) The average cost of a prime-rib dinner was \$14.81 in 1986. In 2010, it was \$26.95. Assuming that the cost is growing exponentially, what will the cost of such a dinner be in 2020?

2.) Find the derivatives: a.) $y=\ln(x^5+4)$ b.) $y=e^{x^2}\ln(7x)$

3.) A typist's speed over a 5-minute interval is given by $W(t)=-6t^2+12t+90$, t in [0, 5], where W(t) is the speed in words per minute at time t. Find the average speed of the typist (average value of the function) over the 5-minute interval.

4.) A metal plate that has been heated cools from 180 degrees to 150 degrees in 20 minutes when surrounded by air at a temperature of 60 degrees. Use Newton's Law of Cooling to approximate its temperature at the end of one hour of cooling. When will the temperature be 100 degrees? $(T=ae^{kt}+M)$

5.) Evaluate the indefinite integrals: a.) $\int(5x^3-6\sqrt{x}+\frac{7}{x^3})dx$ b.) $\int(5e^{3x}+\frac{6}{x})dx$

6.) Find the exact area under the curve $y=x^2+3x+1$ from $x=0$ *to* $x=3$.

7.) Find the area of the region bounded by the two curves:
$y=4x-x^2$ *and* $y=x^2-6x+8$

8.) Find $f(x)$ such that $f'(x)=9x^2-10x+4$ and $f(1)=5$ .

Bonus: (5 points) Evaluate $\int_{-2}^{4} f(x)dx$, *where* $f(x)=\begin{cases} x+2, & \text{for } x\le 0 \\ 2-\frac{1}{2}\sqrt{x}, & \text{for } x>0 \end{cases}$

MA 121 TEST #3 FORM B

(8 QUESTIONS – 12 POINTS EACH)

1.) 1986 $(t=0)$ $y = 14.81$ $(y_0 = 14.81)$

2010 $(t=24)$ $y = 26.95$

2020 $(t=34)$ $y = ??$

$y = y_0 \cdot e^{kt}$

$y = (14.81)e^{kt}$

$26.95 = (14.81)e^{k(24)}$

$\frac{26.95}{14.81} = e^{24k}$

$\frac{\ln\left(\frac{26.95}{14.81}\right)}{24} = \frac{\cancel{24}k}{\cancel{24}}$ $k \approx .024945$

$y = (14.81)e^{(.024945)(34)}$

$y = \$34.59$

price of prime rib in 2020

2.) a.) $y = \ln(x^5+4)$ $y' = \frac{1}{x^5+4} \cdot 5x^4 = \frac{5x^4}{x^5+4}$

b.) $y = e^{x^2} \cdot \ln(7x)$ $y' = e^{x^2}\left(\frac{1}{\cancel{7}x} \cdot \cancel{7}\right) + [\ln(7x)] \cdot e^{x^2} \cdot 2x$

$y' = \frac{e^{x^2}}{x} + 2x \cdot e^{x^2} \cdot \ln(7x)$

3.) AVE Y-VALUE $= \frac{1}{5-0} \int_0^5 (-6t^2+12t+90)\,dt$

12 pts

$= \frac{1}{5}\left[-6\left(\frac{t^3}{3}\right) + 12\left(\frac{t^2}{2}\right) + 90t\right]_0^5$

$= \frac{1}{5}\left[-2t^3 + 6t^2 + 90t\right]_0^5 = \frac{1}{5}\left[\left(-2(5)^3 + 6(5)^2 + 90(5)\right) - 0\right]$

$= \frac{1}{5}\left[-250 + 150 + 450\right] = \frac{1}{5}[350] = 70$ words per minute

(4) $T = a \cdot e^{kt} + M \qquad m = 60$

① $t = 0 \qquad T = 180$

② $t = 20 \qquad T = 150$

③ $t = 60 \qquad T = ??$

④ $t = ?? \qquad T = 100$

$t = 0 \qquad T = 180$

$180 = a \cdot e^{k(0)} + 60$ (solve for a)

$180 = a + 60$

$180 - 60 = a = 120$

$T = 120\, e^{kt} + 60$

$t = 20 \qquad T = 150$

$150 = 120\, e^{k(20)} + 60$ (solve for k)

$\frac{150-60}{120} = e^{20k}$

$\ln\left(\frac{150-60}{120}\right) = 20k$

$\frac{\ln\left(\frac{150-60}{120}\right)}{20} = k \approx -.014384$

$T = 120\, e^{-.014384t} + 60$

$t = 60 \qquad T = ??$

$T = 120\, e^{-.014384(60)} + 60$

$T = 110.625^{\circ}$

$t = ??\quad T = 100$

$100 = 120 \cdot e^{-.014384t} + 60$

$\frac{100-60}{120} = e^{-.014384t}$

$\frac{\ln\left(\frac{100-60}{120}\right)}{-.014384} = \frac{\cancel{-.014384}\,t}{\cancel{-.014384}}$ $t \approx 76.4$ min

5.) a.) $\int \left(5x^3 - 6\sqrt{x} + \frac{7}{x^3}\right)dx = \int \left(5x^3 - 6x^{1/2} + 7x^{-3}\right)dx$

$= 5\left(\frac{x^4}{4}\right) - 6\left(\frac{x^{3/2}}{3/2}\right) + 7\left(\frac{x^{-2}}{-2}\right) + C$

$= \frac{5}{4}x^4 - 4x^{3/2} - \frac{7}{2}x^{-2} + C$

b.) $\int \left(5 \cdot e^{3x} + \frac{6}{x}\right)dx = 5\int e^{3x}dx + 6\int \frac{1}{x}dx$

$= 5\left(\frac{1}{3}e^{3x}\right) + 6 \cdot \ln|x| + C = \frac{5}{3}e^{3x} + 6 \cdot \ln|x| + C$

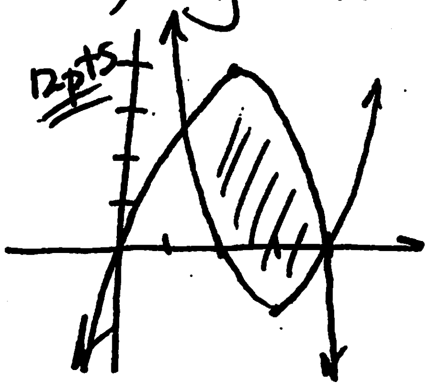
6.) $\int_0^3 (x^2 + 3x + 1)\,dx = \left.\frac{x^3}{3} + \frac{3x^2}{2} + x\right]_0^3$

$= \left[\frac{(3)^3}{3} + \frac{3(3)^2}{2} + (3)\right] - \left[\frac{0^3}{3} + \frac{3(0)^2}{2} + 0\right]$

$= \frac{27}{3} + \frac{27}{2} + 3 - 0 = 12 + \frac{27}{2} = \frac{24}{2} + \frac{27}{2} = \frac{51}{2}$

7.) $y = 4x - x^2$ and $y = x^2 - 6x + 8$

12 pts



graph not required

pts of int: $4x - x^2 = x^2 - 6x + 8$

$$-4x + x^2 \quad +x^2 - 4x$$

$$0 = 2x^2 - 10x + 8$$

$$0 = 2(x^2 - 5x + 4)$$

$$0 = 2(x-4)(x-1)$$

$$x = 1 \quad x = 4$$

$$\text{AREA} = \int_1^4 \left[(4x - x^2) - (x^2 - 6x + 8)\right] dx$$

$$A = \int_1^4 (-2x^2 + 10x - 8)\,dx = -2\frac{x^3}{3} + 10 \cdot \frac{x^2}{2} - 8x \Big]_1^4$$

$$= \frac{-2}{3}x^3 + 5x^2 - 8x \Big]_1^4 = \left[\frac{-2}{3}(4)^3 + 5(4)^2 - 8(4)\right] - \left[\frac{-2}{3}(1)^3 + 5(1)^2 - 8(1)\right]$$

$$= \left(\frac{-2}{3}(64) + 5(16) - 32\right) - \left(\frac{-2}{3} + 5 - 8\right)$$

$$= \frac{-128}{3} + 80 - 32 + \frac{2}{3} - 5 + 8 = 9$$

8.) $f'(x) = 9x^2 - 10x + 4 \qquad f(1) = 5$

$$f(x) = \int (9x^2 - 10x + 4)\,dx$$

$$f(x) = \frac{9x^3}{3} - 10\frac{x^2}{2} + 4x + C$$

$$f(x) = 3x^3 - 5x^2 + 4x + C \quad (\text{find } C)$$

$x = 1$
$y = 5$

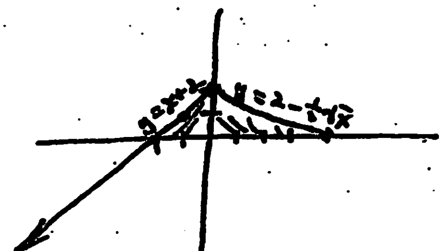
$$5 = 3(1)^3 - 5(1)^2 + 4(1) + C$$

$$5 = 2 + C \qquad C = 3$$

$$f(x) = 3x^3 - 5x^2 + 4x + 3$$

BONUS: (5 pts)

$$f(x) = \begin{cases} x + 2, & x \le 0 \\ 2 - \frac{1}{2}\sqrt{x}, & x > 0 \end{cases}$$




$y = x+2$

| $x$ | $y$ |
|---|---|
| 0 | 2 |
| -1 | 1 |
| -2 | 0 |

$y = 2 - \frac{1}{2}\sqrt{x}$

| $x$ | $y$ |
|---|---|
| 0 | 2 |
| 1 | 3/2 |
| 4 | 0 |

$$\int_{-2}^{4} f(x)\,dx = \int_{-2}^{0} f(x)\,dx + \int_{0}^{4} f(x)\,dx$$

$$= \int_{-2}^{0} (x+2)\,dx + \int_{0}^{4} \left(2 - \frac{1}{2}\sqrt{x}\right)dx$$

$$\left[\frac{x^2}{2} + 2x\right]_{-2}^{0} + \left[2x - \frac{1}{2}\frac{x^{3/2}}{3/2}\right]_{0}^{4} = \left[\left(\cancel{\frac{0^2}{2} + 2(0)}\right) - \left(\frac{(-2)^2}{2} + 2(-2)\right)\right]$$

$$+ \left[\left(2(4) - \frac{1}{3}(4)^{3/2}\right) - \left(\cancel{2(0) - \frac{1}{3}(0)^{3/2}}\right)\right] = -(2-4) + \left(8 - \frac{8}{3}\right) = 10 - \frac{8}{3} = \frac{22}{3}$$