NORTH CAROLINA STATE UNIVERSITY Department of Mathematics

MA121-002 Test #3 Form A Thursday, March 28, 2019 Dr. J. Griggs

Please show all work and answers in the stamped blue book provided; one problem per page. Please put your **name, section** (121-002), and **form** (A or B) on the front of the blue book; put your **row letter** and **seat number** on the top right corner of your blue book. Do **not** use a graphing calculator, nor any other calculator that **does** calculus. Simplify completely. (8 questions; 12 points each; 4 points for following directions)

1.) The average cost of a prime-rib dinner was \$15.81 in 1986. In 2010, it was \$27.95. Assuming that the cost is growing exponentially, what will the cost of such a dinner be in 2020?

2.) Find the derivatives: a.) $y=\ln(x^4+5)$ b.) $y=e^{x^2}\ln(4x)$

3.) A typist's speed over a 5-minute interval is given by $W(t)=-6t^2+12t+90$, t in [0, 5], where W(t) is the speed in words per minute at time t. Find the average speed of the typist (average value of the function) over the 5-minute interval.

4.) A metal plate that has been heated cools from 170 degrees to 140 degrees in 20 minutes when surrounded by air at a temperature of 60 degrees. Use Newton's Law of Cooling to approximate its temperature at the end of one hour of cooling. When will the temperature be 90 degrees? $(T=ae^{kt}+M)$

5.) Evaluate the indefinite integrals: a.) $\int(2x^3-4\sqrt{x}+\frac{5}{x^3})dx$ b.) $\int(3e^{5x}+\frac{4}{x})dx$

6.) Find the exact area under the curve $y=x^2+2x+1$ from $x=0$ *to* $x=3$.

7.) Find the area of the region bounded by the two curves:
$y=4x-x^2$ *and* $y=x^2-6x+8$

8.) Find $f(x)$ such that $f'(x)=6x^2-8x+3$ and $f(2)=4$.

Bonus: (5 points) Evaluate $\int_{-2}^{4} f(x)dx$, *where* $f(x)=\begin{cases} x+2, & \text{for } x\le 0 \\ 2-\frac{1}{2}\sqrt{x}, & \text{for } x>0 \end{cases}$

MA 141 TEST #3 FORM A

(8 QUESTIONS – 12 PTS EACH)

1.) 1986 ($t=0$) $y = 15.81$ ($y_0 = 15.81$)

2010 ($t=24$) $y = 27.95$

2020 ($t=34$) $y = ??$

$y = y_0 \cdot e^{kt}$

$y = (15.81)\, e^{kt}$

$27.95 = (15.81)\, e^{k(24)}$

$\frac{27.95}{15.81} = \frac{\cancel{15.81}\, e^{24k}}{\cancel{15.81}}$

$\frac{27.95}{15.81} = e^{24k}$

$\frac{\ln\left(\frac{27.95}{15.81}\right)}{24} = \frac{\cancel{24}k}{\cancel{24}}$ $k \approx .02374$

$y = (15.81)\, e^{(.02374)(34)}$

$y = \$35.44$

price of prime rib in 2020

2.) a.) $y = \ln(x^4+5)$ $y' = \frac{1}{x^4+5} \cdot (4x^3) = \frac{4x^3}{x^4+5}$

b.) $y = e^{x^2} \cdot \ln(4x)$ $y' = e^{x^2} \cdot \left(\frac{1}{4x} \cdot 4\right) + [\ln 4x] \cdot e^{x^2}(2x)$

$y' = \frac{e^{x^2}}{x} + 2x \cdot e^{x^2} \cdot \ln 4x$

3.) AVE Y-VALUE $= \frac{1}{5-0} \int_0^5 (-6t^2 + 12t + 90)\, dt$

12 pts

$= \frac{1}{5}\left[-6\left(\frac{t^3}{3}\right) + 12\left(\frac{t^2}{2}\right) + 90t\right]_0^5$

$= \frac{1}{5}\left[-2t^3 + 6t^2 + 90t\right]_0^5 = \frac{1}{5}\left[\left(-2(5)^3 + 6(5)^2 + 90(5)\right) - 0\right]$

$= \frac{1}{5}\left[-250 + 150 + 450\right] = \frac{1}{5}[350] = 70$ words per minute

(page 2)

4.) $T = a \cdot e^{kt} + m$ $m = 60$

2PTS

① $t=0$ $T=170$

② $t=20$ $T=140$

③ $t=60$ $T=??$

④ $t=??$ $T=90$

$t=0$ $T=170$

$170 = a \cdot e^{k(0)} + 60$ (solve for a)

$170 = a + 60$

$170 - 60 = a = 110$

$T = 110e^{kt} + 60$

$t=20$ $T=140$

$140 = 110e^{k(20)} + 60$ (solve for k)

$\frac{140-60}{110} = e^{20k}$

$\ln\left(\frac{140-60}{110}\right) = 20k$

$\frac{\ln\left(\frac{140-60}{110}\right)}{20} = k \approx -.01592$

$T = 110e^{-.01592t} + 60$

$t=60$ $T=??$

$T = 110e^{-.01592(60)} + 60$

$T = 102.32°$

(page 3)

$t = ??$ $T = 90$

$$90 = 110 \cdot e^{-.01592t} + 60$$

$$\frac{90 - 60}{110} = e^{-.01592t}$$

$$\frac{\ln\left(\frac{90-60}{110}\right)}{-.01592} = \frac{\cancel{-.01592}t}{\cancel{-.01592}} \approx 81.6 \text{ min}$$

5.) a.) $\int \left(2x^3 - 4\sqrt{x} + \frac{5}{x^3}\right) dx$

$$= \int \left(2x^3 - 4x^{1/2} + 5x^{-3}\right) dx$$

$$= 2\left(\frac{x^4}{4}\right) - 4\left(\frac{x^{3/2}}{3/2}\right) + 5\left(\frac{x^{-2}}{-2}\right) + C$$

$$= \frac{1}{2}x^4 - \frac{8}{3}x^{3/2} - \frac{5}{2}x^{-2} + C$$

b.) $\int \left(3 \cdot e^{5x} + \frac{4}{x}\right) dx = 3\int e^{5x} dx + 4\int \frac{1}{x} dx$

$$= 3 \cdot \left(\frac{1}{5}e^{5x}\right) + 4 \cdot \ln|x| + C$$

$$= \frac{3}{5}e^{5x} + 4 \cdot \ln|x| + C$$

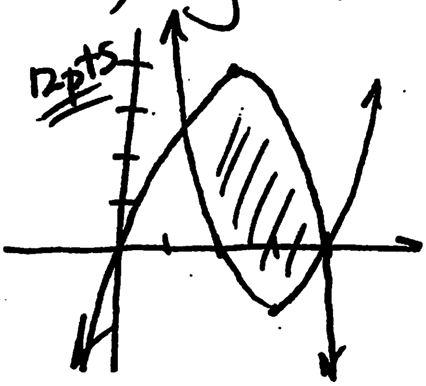
6.) $\int_0^3 (x^2 + 2x + 1) dx = \left.\frac{x^3}{3} + \cancel{2}\left(\frac{x^2}{\cancel{2}}\right) + x\right]_0^3$

$$= \left[\frac{(3)^3}{3} + (3)^2 + 3\right] - \left[\frac{(0)^3}{3} + (0)^2 + (0)\right] = \frac{27}{3} + 9 + 3 - 0$$

$$= 21$$

7.) $y = 4x - x^2$ and $y = x^2 - 6x + 8$

12 pts



graph not required

pts of int: $4x - x^2 = x^2 - 6x + 8$

$-4x + x^2 \quad + x^2 - 4x$

$0 = 2x^2 - 10x + 8$

$0 = 2(x^2 - 5x + 4)$

$0 = 2(x-4)(x-1)$

$x = 1 \quad x = 4$

$$\text{AREA} = \int_1^4 \left[(4x - x^2) - (x^2 - 6x + 8)\right] dx$$

$$A = \int_1^4 (-2x^2 + 10x - 8)\,dx = -2\frac{x^3}{3} + 10 \cdot \frac{x^2}{2} - 8x \Big]_1^4$$

$$= \frac{-2}{3}x^3 + 5x^2 - 8x \Big]_1^4 = \left[\frac{-2}{3}(4)^3 + 5(4)^2 - 8(4)\right] - \left[\frac{-2}{3}(1)^3 + 5(1)^2 - 8(1)\right]$$

$$= \left(\frac{-2}{3}(64) + 5(16) - 32\right) - \left(\frac{-2}{3} + 5 - 8\right)$$

$$= \frac{-128}{3} + 80 - 32 + \frac{2}{3} - 5 + 8 = 9$$

8.) $f'(x) = 6x^2 - 8x + 3 \qquad f(2) = 4$

$$f(x) = \int (6x^2 - 8x + 3)\,dx$$

$$f(x) = 6\cdot\frac{x^3}{3} - 8\cdot\frac{x^2}{2} + 3x + C$$

$$f(x) = 2x^3 - 4x^2 + 3x + C \quad \text{(find C)} \quad x = 2,\ y = 4$$

$$4 = 2(2)^3 - 4(2)^2 + 3(2) + C$$

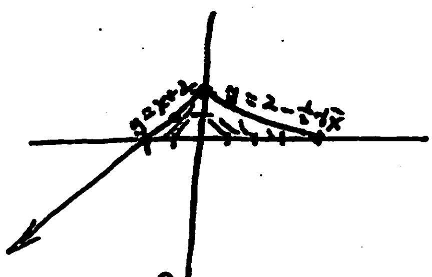
$$4 = 16 - 16 + 6 + C$$

$$4 = 6 + C \qquad C = -2$$

$$f(x) = 2x^3 - 4x^2 + 3x - 2$$

BONUS: (5 pts)

$$f(x) = \begin{cases} x+2, & x \le 0 \\ 2 - \frac{1}{2}\sqrt{x}, & x > 0 \end{cases}$$



$y = x+2$	
x	y
0	2
-1	1
-2	0

$y = 2 - \frac{1}{2}\sqrt{x}$	
x	y
0	2
1	3/2
4	0

$$\int_{-2}^{4} f(x)\,dx = \int_{-2}^{0} f(x)\,dx + \int_{0}^{4} f(x)\,dx$$

$$= \int_{-2}^{0} (x+2)\,dx + \int_{0}^{4} \left(2 - \frac{1}{2}\sqrt{x}\right)dx$$

$$\left[\frac{x^2}{2} + 2x\right]_{-2}^{0} + \left[2x - \frac{1}{2}\frac{x^{3/2}}{3/2}\right]_{0}^{4} = \left[\left(\cancel{\frac{0^2}{2} + 2(0)}\right) - \left(\frac{(-2)^2}{2} + 2(-2)\right)\right]$$

$$+ \left[\left(2(4) - \frac{1}{3}(4)^{3/2}\right) - \left(\cancel{2(0) - \frac{1}{3}(0)^{3/2}}\right)\right] = -(2-4) + \left(8 - \frac{8}{3}\right) = 10 - \frac{8}{3} = \frac{22}{3}$$