Tuesday, April 23

- Q4 turned in:
- finish Ch 6 material
  (6.2: partial derivatives)
  - will be included on final exam
- begin final exam review

6.2: partial derivatives (1st order partials)

$$f(x,y) = x^3 - 8x^2y^5 + \frac{11x^2}{y} = z$$

$$(x, y, f(x,y))$$

$$(x, y, z)$$

$$\frac{\partial z}{\partial x} = \lim_{h \to 0} \frac{f(x+h, y) - f(x, y)}{h} = f_x$$

y is constant (fixed)

(treat "y" as if it were constant)

$$\frac{\partial z}{\partial y} = \lim_{k \to 0} \frac{f(x, y+k) - f(x, y)}{k} = f_y$$

x is constant (fixed)

(treat "x" as if it were constant)

$$f(x,y) = 8x^2 - 3xy + 11y^3 + 5$$

find f_x & f_y : (1st order partial deriv)

① $f(x,y) = \underline{8x^2} - 3x\boxed{y} + \underline{11}\boxed{y^3} + \underline{5}$ (y is fixed)

$f_x = 8(2x) - 3y(1) + 0 + 0$

$$\boxed{f_x = 16x - 3y}$$

② $f(x,y) = 8\boxed{x^2} - 3\boxed{x}y + \underline{11y^3} + \underline{5}$ (x is fixed)

$f_y = 0 - 3x(1) + 11(3y^2) + 0$

$$\boxed{f_y = -3x + 33y^2}$$

$$f(x,y) = 8x^2 - 3xy + 11y^3 + 2$$

find f_x & f_y : (1st partial derivatives)

1) $f(x,y) = 8x^2 - 3xy + 11y^3 + 2$ (y is fixed)

$$f_x = 8(2x) - 3y(1) + 0 + 0$$

$$\boxed{f_x = 16x - 3y}$$

2) $f(x,y) = 8x^2 - 3xy + 11y^3 + 2$ (x is fixed)

$$f_y = 0 - 3x(1) + 11(3y^2) + 0$$

$$\boxed{f_y = -3x + 33y^2}$$

$$f(x,y) = x^3 - 8x^2y^5 + \frac{11x^2}{y}$$

① f_x:

$$f(x,y) = x^3 - 8x^2\boxed{y^5} + \frac{11x^2}{\boxed{y}}$$

$$f_x = 3x^2 - 8y^5(2x) + \frac{11}{y}(2x)$$

$$\boxed{f_x = 3x^2 - 16xy^5 + \frac{22x}{y}}$$

② f_y:

$$f(x,y) = \boxed{x^3} - 8\boxed{x^2}y^5 + \frac{11\boxed{x^2}}{\cancel{y}} \cdot y^{-1}$$

$$= 0 - 8x^2(5y^4) + 11x^2(-1 \cdot y^{-2})$$

$$\boxed{f_y = -40x^2y^4 - \frac{11x^2}{y^2}}$$

$$f(x,y) = e^{5xy}$$

$$\boxed{\begin{array}{l} f(x) = e^{8x} \\ f'(x) = e^{8x} \cdot (8) \end{array}}$$

$$f(x,y) = e^{5y(x)}$$

$$f(x,y) = e^{5x(y)}$$

① f_x:

$$\boxed{f_x = e^{5y(x)} \cdot (5y)}$$

② f_y:

$$\boxed{f_y = e^{5xy} \cdot (5x)}$$

2^{nd} order partial deriv:

1^{st} order partials:

f_x ; f_y

2^{nd} order partials

f_{xx} ; f_{xy} ; f_{yy} ; f_{yx}

$$f(x,y) = \underline{8x^2} + \underline{11x^3y^4} - \underline{5y^4}$$

$$f_x = 16x + 11y^4(3x^2) - 0$$

$$\boxed{f_x = 16x + 33x^2y^4} \ \ast$$

$$f_{xx} = 16 + 33y^4(2x)$$

$$\boxed{f_{xx} = 16 + 66xy^4}$$

$$f_{xy} = 0 + 33x^2(4y^3)$$

$$\boxed{f_{xy} = 132x^2y^3}$$

$$f(x,y) = \underline{8x^2} + \underline{11x^3y^4} - \underline{5y^4}$$

$$f_y = 0 + 11x^3(4y^3) - 5(4y^3)$$

$$\boxed{f_y = 44x^3y^3 - 20y^3} \ \ast$$

$$f_{yy} = 44x^3(3y^2) - 20(3y^2)$$

$$\boxed{f_{yy} = 132x^3y^2 - 60y^2}$$

2nd order partial derivs:

2nd order mixed partials: f_{xx} ; f_{xy} ; f_{yy} ; f_{yx}

1st order partials: f_x ; f_y

$$f(x,y) = 8x^2 + 11x^3y^4 - 5y^4$$

$$f_x = 16x + 11y^4(3x^2) - 0$$

$$\boxed{f_x = 16x + 33x^2y^4}$$

$$f_{xx} = 16 + 33y^4(2x)$$

$$\boxed{f_{xx} = 16 + 66xy^4}$$

$$f_{xy} = 0 + 33x^2(4y^3)$$

$$\boxed{f_{xy} = 132x^2y^3}$$

$$f(x,y) = 8x^2 + 11x^3y^4 - 5y^4$$

$$f_y = 0 + 11x^3(4y^3) - 5(4y^3)$$

$$\boxed{f_y = 44x^3y^3 - 20y^3}$$

$$f_{yy} = 44x^3(3y^2) - 20(3y^2)$$

$$\boxed{f_{yy} = 132x^3y^2 - 60y^2}$$

$$f_{yx} = 44y^3(3x^2) - 0$$

$$\boxed{f_{yx} = 132x^2y^3}$$

$$f_{xx} = 16 + 66xy^4 \qquad f_{yy} = 132x^3y^2 - 60y^2$$

$$f_{xy} = 132x^2y^3 \quad = \quad f_{yx} = 132x^2y^3$$

MAX / MIN / SADDLE POINT FOR $f(x,y)$:

1.) find f_x, f_y, f_{xx}, f_{yy}, f_{xy}:

2.) set $f_x = 0$ & $f_y = 0$ $\Rightarrow (a,b)$

3.) D TEST: (2nd DERIV TEST)

$$D = f_{xx}(a,b) \cdot f_{yy}(a,b) - \left[f_{xy}(a,b)\right]^2$$

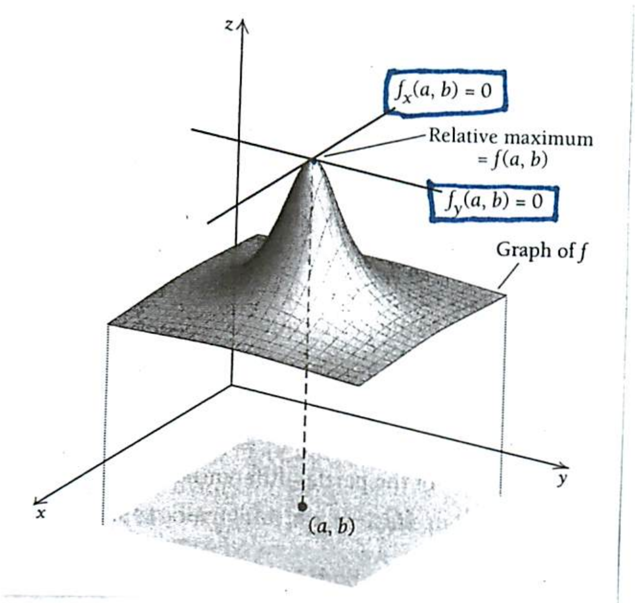
a.) if $D > 0$ & $f_{xx}(a,b) < 0$
$\longrightarrow (a,b,f(a,b))$ is a MAX

b.) if $D > 0$ & $f_{xx}(a,b) > 0$
$\longrightarrow (a,b,f(a,b))$ is a MIN

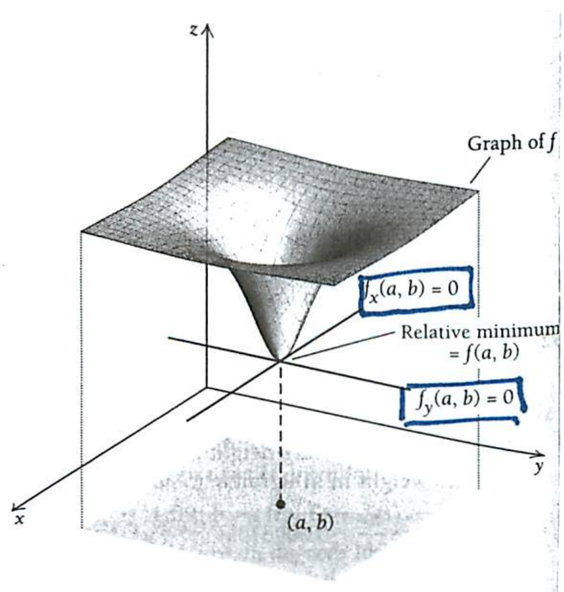
c.) if $D < 0 \longrightarrow (a,b,f(a,b))$ is a SADDLE POINT

d.) if $D = 0 \longrightarrow$ TEST FAILS

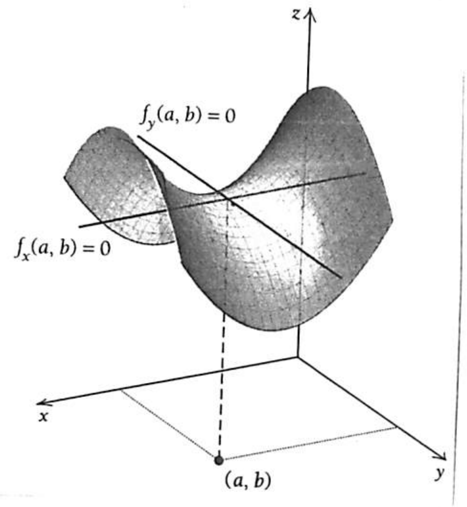
RELATIVE MAX:

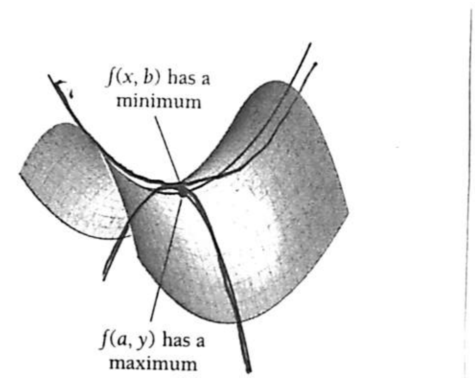


RELATIVE MIN:



SADDLE POINT:





NORTH CAROLINA STATE UNIVERSITY Department of Mathematics

MA121-002 Quiz #4 Due Tuesday, April 23, 2019 (at the beginning of class) J. Griggs

Three points per question; one point for following directions. You are to work **individually** on this quiz; it is permissible to use your book and/or notes from the class. Show **all** work and any graphs/diagrams on <u>**this**</u> sheet – use the back of this sheet, if necessary.

1.) Evaluate the improper integral $\int_2^\infty 7x^{-2}\,dx$. Does this integral converge or diverge?

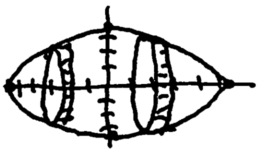
$$\lim_{A\to\infty}\int_2^A x^{-2}\,dx = 7\cdot\lim_{A\to\infty}\left[\frac{x^{-1}}{-1}\right]_2^A = 7\cdot\lim_{A\to\infty}\left[\frac{-1}{x}\right]_2^A$$

$$= 7\cdot\lim_{A\to\infty}\left[\left(\frac{-1}{A}\right)-\left(-\frac{1}{2}\right)\right] = 7\cdot\lim_{A\to\infty}\left[\frac{1}{2}-\frac{1}{A}\right] = \frac{7}{2}$$

∴ integral converges

2.) A regulation football used in the NFL is 11 inches from tip to tip and 7 inches in diameter at its thickest (the regulations allow for slight variations in these dimensions). The shape of a football can be modeled by the function $f(x) = -0.116x^2 + 3.5$ *for* $-5.5 \le x \le 5.5$ where x is in inches. Find the volume of the football by rotating the area bounded by the graph of f around the x-axis.

$$\text{VOL} = \int_{-5.5}^{5.5} \pi (f(x))^2\,dx = \pi\int_{-5.5}^{5.5}(-.116x^2+3.5)^2\,dx$$



$$= \pi\int_{-5.5}^{5.5}(.013456x^4 - .812x^2 + 12.25)\,dx$$

$$= \pi\left[.013456\cdot\frac{x^5}{5} - .812\cdot\frac{x^3}{3} + 12.25x\right]_{-5.5}^{5.5} = \pi\left[\left(\frac{.013456}{5}(5.5)^5 - \frac{.812}{3}(5.5)^3 + (12.25)(5.5)\right) - \left(\frac{.013456}{5}(-5.5)^5 - \frac{.812}{3}(-5.5)^3 + (12.25)(-5.5)\right)\right]$$

$$\approx 71.774\pi \approx 225.48 \text{ in}^3$$

3.) A hockey goalie's *goals against average* (A) is a function of the number of goals allowed (g) and the number of minutes played (m) and is given by $A(g,m) = \frac{60g}{m}$. Find the goals against average of a goalie who allows 35 goals while playing 820 minutes.

$$A(\underset{\text{goals}}{35}, \underset{\text{min}}{820}) = \frac{60(35)}{820} \approx \boxed{2.56}$$