MA121-002

Thursday, april 11

- today 5.7: SEPARABLE DIFFERENTIAL EQUATIONS

TUES: 4/16: 6.1; test review

THURS: 4/18: TEST #4 (4.5, 5.1, 5.2, 5.3, 5.6, 5.7)

5.7: DIFFERENTIAL EQUATIONS

(has a deriv. in it)

1. $y' = 11x$, find y:

$$y = 11\frac{x^2}{2} + C$$

2. $f'(x) = x^{2/3} - x$ $\boxed{f(1) = 6}$ $x=1$, $y=6$

find $f(x)$:

$$f(x) = \frac{x^{5/3}}{5/3} - \frac{x^2}{2} + C$$

$$f(x) = \frac{3}{5}x^{5/3} - \frac{1}{2}x^2 + C$$

$$6 = \frac{3}{5}(1)^{5/3} - \frac{1}{2}(1)^2 + C$$

$$6 = \frac{3}{5} - \frac{1}{2} + C$$

$$\boxed{6 - 3/5 + 1/2 = C}$$

SEPARABLE D.E. :

(1) $$\boxed{\frac{dy}{dx} = \frac{2x}{y}}$$

$$y\,\cancel{dx}\,\frac{\boxed{dy}}{\cancel{dx}} = \frac{2x}{\cancel{y}}\cdot\boxed{dx}\cdot\cancel{y}$$

goal : (x's & dx's on one side,
y's & dy's on the other side).

$$\int y \cdot dy = \int 2x\,dx$$

$$\frac{y^2}{2} + C_1 = \cancel{2}\cdot\frac{x^2}{\cancel{2}} + C_2$$

$$2\left(\frac{y^2}{2}\right) = \left(x^2 + C\right)2$$

(solve for y ....)

$$y^2 = 2x^2 + K$$

$$y = \pm\sqrt{2x^2 + K}$$

(2) $\dfrac{dy}{dx} = 5x^4 \cdot y$

$e^{\ln 8} = 8$

$e^{\ln u} = u$

$\dfrac{1}{y} \quad \cancel{dx} \cdot \dfrac{dy}{\cancel{dx}} = 5x^4 \cdot \cancel{y}\, dx \cdot \dfrac{1}{\cancel{y}}$

$\int \dfrac{1}{y} \cdot dy = \int 5x^4\, dx$

$\rightarrow \boxed{\ln|y|} = \boxed{x^5 + C}$

(solve for y ...)

exponentiate both sides

$e^{\ln|y|} = e^{x^5 + C}$

$|y| = e^{x^5} \cdot e^{c}$ $\quad (e^c = K)$

$y = e^{x^5} \cdot K$

3 $\boxed{\frac{dP}{dt} = k \cdot P}$

is

pop.

directly proportional to

rate of change of pop.

$P = P_0 e^{kt}$

$$\frac{1}{P} \cdot \cancel{dt} \cdot \frac{dP}{\cancel{dt}} = k \cdot \cancel{P} \cdot dt \, \frac{1}{\cancel{P}}$$

$$\int \frac{1}{P}\, dP = \int k \cdot dt$$

$$\ln|P| = kt + C$$

$$e^{\ln|P|} = e^{kt+C}$$

$$|P| = e^{kt} \cdot e^{C} \qquad e^{C} = A$$

$$|P| = A \cdot e^{kt}$$

$$\boxed{P = A \cdot e^{kt}}$$

$t = 0$
$P = P_0$

$$P_0 = A \cdot e^{k(0)}$$

$$P_0 = A \cdot 1$$

$$A = P_0$$

$$P = P_0 \cdot e^{kt}$$

(4) $y' = 2x - xy$

$$\frac{1}{2-y} \cancel{dx} \frac{dy}{\cancel{dx}} = x(2-y) \cdot dx \, \frac{1}{2-y}$$

$$-1 \int \frac{1}{\boxed{2-y}} \boxed{dy}^{-1} = \int x \, dx$$

$u = 2-y$

$du = -1 \cdot dy$

$$= \frac{x^2}{2} + C$$

$$-1 \int \frac{1}{u} du = \frac{x^2}{2} + C$$

$\ln a^{r} = \boxed{r} \ln a$

$$\boxed{-1} \cdot \ln|u| = \frac{x^2}{2} + C$$

$$\boxed{-1} \cdot \ln|2-y| = \frac{x^2}{2} + C$$

$$\ln|2-y|^{-1} = \frac{x^2}{2} + C$$

$$e^{\ln\left|\frac{1}{2-y}\right|} = e^{\frac{x^2}{2} + C}$$

$e^c = B$

$$\left|\frac{1}{2-y}\right| = e^{\frac{x^2}{2}} \cdot e^c$$

$$\boxed{\frac{1}{2-y} = B \cdot e^{\frac{x^2}{2}}} \ *$$