MA 121 - 002

Thursday, April 4

- finish 5.2: one-time present value; accumulated present value.
- 5.3: improper integrals

(Q3 given out TUES, 4/9 – due THURS, 4/11)

(Test #3 returned today)

5.2:

future value:

(1) one-time FV:

$$P = P_0 \cdot e^{kt}$$

(2) multiple contrib. (ACCUM. FV)

$$\int_{t_1}^{t_2} P_0 \, e^{kt} \, dt$$

45.)

$$\int_0^7 (33.3) e^{.015t} \, dt$$

2013 (t=0)
2020 (t=7)

$$\int e^{at} dt = \frac{1}{a} e^{at}$$

$$= (33.3) \frac{1}{.015} e^{.015t} \Big]_0^7$$

$$= \frac{33.3}{.015} \left[e^{.015(7)} - e^{.015(0)} \right]$$

cumulative use during 2013–2020 (Accum) $= \frac{33.3}{.015}\left[1.1107 - 1\right]$

$\approx$ 245.78 billion barrels

b.) $1635 = \int_0^T (33.3)e^{.015t}\,dt$ (2013) $t=0$, $T=??$

$= (33.3)\left(\frac{1}{.015}e^{.015t}\right)\Big]_0^T$

$1635 = \frac{33.3}{.015}\left[e^{.015t}\right]_0^T$ (solve for T)

$1635 = \frac{33.3}{.015}\left[e^{.015T} - e^{.015(0)}\right]$

$(.015)\frac{(1635)}{33.3} = \frac{33.3}{.015}\left[e^{.015T} - 1\right]\cdot\frac{.015}{33.3}$

$\frac{(.015)(1635)}{33.3} + 1 = e^{.015T} - 1 + 1$

$\frac{(.015)(1635)}{33.3} + 1 = e^{.015T}$

$\frac{\ln\left(\frac{(.015)(1635)}{33.3}+1\right)}{.015} = \frac{.015T}{.015} \approx 36.8$ years

PRESENT VALUE:

① $P = P_0 \cdot e^{kt}$

↑ solve for P_0

$$\frac{P}{e^{kt}} = \frac{P_0 \cdot e^{kt}}{e^{kt}}$$

$$P_0 = P \cdot e^{-kt}$$

$$P_0 = (180{,}000) \cdot e^{-.042(16)}$$

$$P_0 = \frac{(180{,}000)}{e^{(.042)(16)}} \approx 91{,}923.51$$

② ACCUM. PRESENT VALUE:

$$\int_{t_1}^{t_2} P \cdot e^{-kt}\, dt$$

24.) a.) $$\int_0^{40} (125{,}000) \cdot e^{-.04t}\, dt$$

$$= 125{,}000 \left(\frac{1}{-.04} e^{-.04t} \right]_0^{40}$$

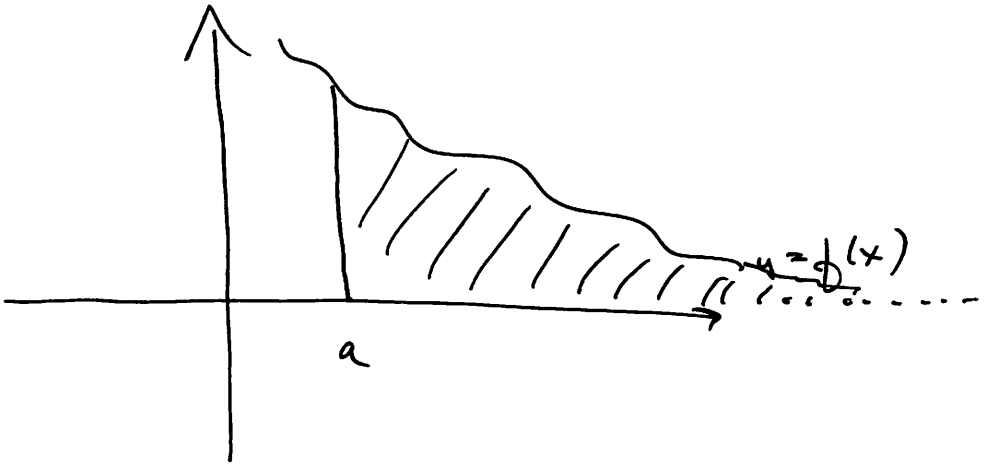
$$= \frac{125{,}000}{-.04} \left[e^{-.04(40)} - e^{-.04(0)} \right]$$

$$= \$2{,}494{,}073.38$$

$e^{-.04(0)} = 1$

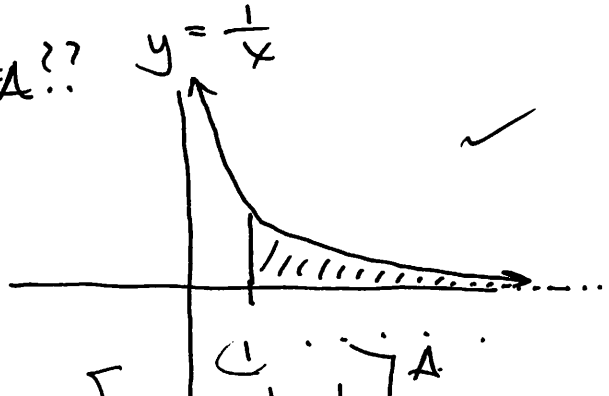
5.3: improper integrals

$$\int_a^{\infty} f(x)\,dx$$



$$\int_1^{\infty} \frac{1}{x}\,dx$$

AREA?? $y = \frac{1}{x}$



$$\lim_{A\to\infty} \int_1^{A} \frac{1}{x}\,dx = \lim_{A\to\infty} \Big[\ln|x| \Big]_1^{A}$$

$$= \lim_{A\to\infty} \left[\ln A - \underbrace{\ln 1}_{0} \right]$$

$$= \lim_{A\to\infty} \left[\ln A \right]^{\infty}$$

no sol.

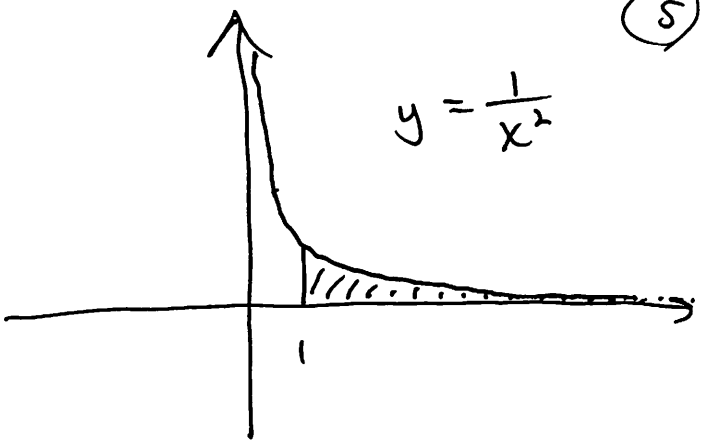
integral DIVERGES

A = 100
ln A = ___

A = 10,000
ln 10,000

A = 1,000,000
ln 1,000,000

$$\int_1^\infty \frac{1}{x^2}\,dx$$



AREA??

$$\lim_{B\to\infty} \int_1^B x^{-2}\,dx$$

$$= \lim_{B\to\infty} \left[\frac{x^{-1}}{-1}\right]_1^B$$

$$= \lim_{B\to\infty} \left[\frac{-1}{x}\right]_1^B$$

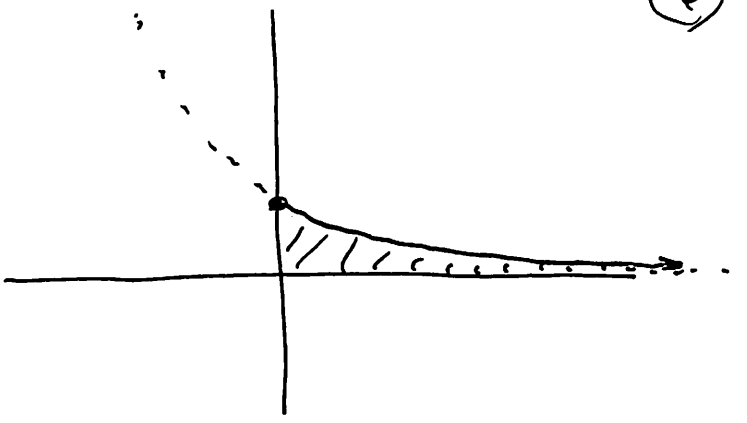
$$= \lim_{B\to\infty} \left[\left(\frac{-1}{B}\right) - \left(\frac{-1}{1}\right)\right]$$

$$= \lim_{B\to\infty} \left(1 - \frac{1}{B}\right) = 1$$

integral CONVERGES

$$\int_1^{1{,}732{,}596} \frac{1}{x^2}\,dx = .99999874$$

$$\int_0^{\infty} e^{-.32t}\,dt$$



$$\lim_{A\to\infty} \int_0^{A} e^{-.32t}\,dt$$

$$\lim_{A\to\infty} \left[\frac{1}{-.32} e^{-.32t}\right]_0^{A}$$

$$\lim_{A\to\infty} \frac{-1}{.32}\left[e^{-.32(A)} - e^{-.32(0)}\right]$$

$$\lim_{A\to\infty} \frac{-1}{.32}\left[\frac{1}{e^{.32A}} - 1\right]$$

$\frac{1}{e^{.32A}} \to 0$

$$\frac{-1}{.32}(-1) = \frac{1}{.32}$$

integral converges

5.2 Exercise Set

For all exercises in this exercise set, use a graphing calculator.

Find the future value P of each amount P_0 invested for time period t at interest rate k, compounded continuously.

1. $P_0 = \$100{,}000$, $t = 6$ yr, $k = 3\%$ \$119,721.74
2. $P_0 = \$55{,}000$, $t = 8$ yr, $k = 4\%$ \$75,742.03
3. $P_0 = \$140{,}000$, $t = 9$ yr, $k = 5.8\%$ \$235,955.31
4. $P_0 = \$88{,}000$, $t = 13$ yr, $k = 4.7\%$ \$162,120.00

Find the present value P_0 of each amount P due t years in the future and invested at interest rate k, compounded continuously.

5. $P = \$100{,}000$, $t = 6$ yr, $k = 3\%$ \$83,527.02
6. $P = \$100{,}000$, $t = 8$ yr, $k = 4\%$ \$72,614.90
7. $P = \$1{,}000{,}000$, $t = 25$ yr, $k = 6\%$ \$223,130.16
8. $P = \$2{,}000{,}000$, $t = 20$ yr, $k = 3.5\%$ \$993,170.61

Find the accumulated future value of each continuous income stream at rate $R(t)$, for the given time T and interest rate k, compounded continuously.

9. $R(t) = \$50{,}000$, $T = 22$ yr, $k = 5\%$ \$2,004,166.02
10. $R(t) = \$125{,}000$, $T = 20$ yr, $k = 6\%$ \$4,833,576.92
11. $R(t) = \$400{,}000$, $T = 20$ yr, $k = 4\%$ \$12,255,409.28
12. $R(t) = \$50{,}000$, $T = 22$ yr, $k = 2.75\%$ \$1,511,367.65

Find the accumulated present value of each continuous income stream at rate $R(t)$, for the given time T and interest rate k, compounded continuously.

13. $R(t) = \$250{,}000$, $T = 18$ yr, $k = 4\%$ \$3,207,798.40
14. $R(t) = \$425{,}000$, $T = 15$ yr, $k = 3.5\%$ \$4,959,684.86
15. $R(t) = \$800{,}000$, $T = 20$ yr, $k = 2.3\%$ \$12,824,916.68
16. $R(t) = \$520{,}000$, $T = 25$ yr, $k = 6\%$ \$6,732,871.95
17. $R(t) = \$5200t$, $T = 18$ yr, $k = 3.1\%$ \$585,877.18
18. $R(t) = \$6400t$, $T = 20$ yr, $k = 3.8\%$ \$784,075.74
19. $R(t) = \$(2000t + 7)$, $T = 30$ yr, $k = 4.5\%$ \$386,076.11
20. $R(t) = \$t^2$, $T = 40$ yr, $k = 4.25\%$ \$6325.14

APPLICATIONS

Business and Economics

21. **Present value of a trust.** In 18 yr, Maggie Oaks is to receive \$200,000 under the terms of a trust established by her grandparents. Assuming an interest rate of 3.8%, compounded continuously, what is the present value of Maggie's trust? \$100,918.91

22. **Present value of a trust.** In 16 yr, Claire Beasley is to receive \$180,000 under the terms of a trust established by her aunt. Assuming an interest rate of 4.2%, compounded continuously, what is the present value of Claire's trust? \$91,923.51

23. **Salary value.** At age 35, Rochelle earns her MBA and accepts a position as vice president of an asphalt company. Assume that she will retire at the age of 65, having received an annual salary of \$95,000, and that the interest rate is 5%, compounded continuously.
    a) What is the accumulated present value of her position? \$1,476,052.70
    b) What is the accumulated future value of her position? \$6,615,209.23

24. **Salary value.** At age 25, Del earns his CPA and accepts a position in an accounting firm. Del plans to retire at the age of 65, having received an annual salary of \$125,000. Assume an interest rate of 4%, compounded continuously.
    a) What is the accumulated present value of his position? \$2,494,073.38
    b) What is the accumulated future value of his position? \$12,353,226.33

25. **Future value of an inheritance.** Upon the death of his uncle, David receives an inheritance of \$50,000, which he invests for 16 yr at 4.3%, compounded continuously. What is the future value of the inheritance? \$99,486.60

26. **Future value of an inheritance.** Upon the death of his aunt, Burt receives an inheritance of \$80,000, which he invests for 20 yr at 3.9%, compounded continuously. What is the future value of the inheritance? \$174,517.78

27. **Decision making.** A group of entrepreneurs is considering the purchase of a fast-food franchise. Franchise A predicts that it will bring in a constant revenue stream of \$80,000 per year for 10 yr. Franchise B predicts that it will bring in a constant revenue stream of \$95,000 per year for 8 yr. Based on a comparison of accumulated present values, which franchise is the better buy, assuming the interest rate is 4.1%, compounded continuously, and both franchises have the same purchase price? A: \$656,292.19, B: \$647,939.34; B is the better buy.

28. **Decision making.** A group of entrepreneurs is considering the purchase of a fast-food franchise. Franchise A predicts that it will bring in a constant revenue stream of \$120,000 per year for 8 yr. Franchise B predicts that it will bring in a constant revenue stream of \$112,000 per year for 10 yr. Based on a comparison of accumulated present values, which franchise is the better buy, assuming the interest rate is 5.4%, compounded continuously, and both franchises have the same purchase price? A: \$779,534.72, B: \$865,411.03; B is the better buy.

29. **Decision making.** An athlete attains free agency and is looking for a new team. The Bronco Crunchers offer a

38. **Negotiating a sports contract.** Gusto Stick is a professional baseball player who has just become a free agent. His attorney begins negotiations with an interested team by asking for a contract that provides Gusto with an income stream given by $R_1(t) = 800{,}000 + 340{,}000t$, over 10 yr, where t is in years. (Round all answers to the nearest $100.)
   a) What is the accumulated future value of the offer, assuming an interest rate of 5%, compounded continuously? $30,605,600
   b) What is the accumulated present value of the offer, assuming an interest rate of 5%, compounded continuously? $18,563,300
   c) The team counters by offering an income stream given by $R_2(t) = 600{,}000 + 210{,}000t$. What is the accumulated present value of this counteroffer? $12,298,800
   d) Gusto comes back with a demand for an income stream given by $R_3(t) = 1{,}000{,}000 + 250{,}000t$. What is the accumulated present value of this income stream? $16,889,800
   e) Gusto signs a contract for the income stream in part (d) but decides to live on $500,000 each year, investing the rest at 5%, compounded continuously. What is the accumulated future value of the remaining income, assuming an interest rate of 5%, compounded continuously? $21,359,300

39. **Accumulated present value.** The Wilkinsons want to have $100,000 in 10 yr for a down payment on a retirement home. Find the continuous money stream, $R(t)$ dollars per year, that they need to invest at 4.33%, compounded continuously, to generate $100,000. $7990.75/yr

40. **Accumulated present value.** Tania wants to have $20,000 in 5 yr for her dream vacation. Find the continuous money stream, $R(t)$ dollars per year, that she needs to invest at 5.125%, compounded continuously, to generate $20,000. $3509.36/yr

Life and Physical Sciences

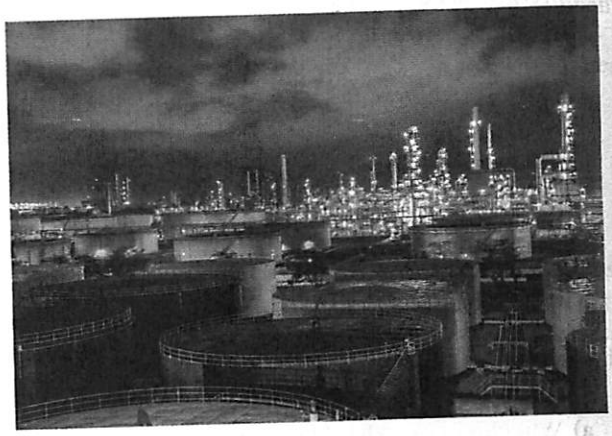
41. **Demand for natural gas.** In 2013 ($t = 0$), world consumption of natural gas was approximately 117.2 trillion cubic feet and was growing exponentially at about 1.24% per year. (*Source:* U.S. Energy Information Administration.) If the demand continues to grow at this rate, how many cubic feet of natural gas will the world use from 2015 to 2025? 1279.1 trillion cubic feet

42. **Demand for aluminum ore (bauxite).** In 2013 ($t = 0$), bauxite production was approximately 232 million metric tons, and the demand was growing exponentially at a rate of 2.6% per year. (*Source:* minerals.usgs.gov.) If the demand continues to grow at this rate, how many metric tons of bauxite will the world use from 2015 to 2030? 4483.3 million metric tons

43. **Depletion of natural gas.** The world reserves of natural gas were approximately 6597 trillion cubic feet in 2013. (*Source:* www.cia.com.) Assuming that the growth described in Exercise 41 continues and that no new reserves are found, when will the world reserves of natural gas be depleted? In 42.7 yr, or in 2055

44. **Depletion of aluminum ore (bauxite).** In 2013, the world reserves of bauxite were about 65 billion metric tons. (*Source:* U.S. Geological Survey summaries, Jan. 2014.) Assuming that the growth described in Exercise 42 continues and that no new reserves are discovered, when will the world reserves of bauxite be depleted? In 81.32 yr, or about 2094

45. **Demand for and depletion of oil.** In 2013, annual world demand for crude oil was approximately 33.3 billion barrels, and it was projected to increase by 1.5% per year. (*Sources*: Based on information from www.iea.gov and www.cia.gov.)



   a) Assuming an exponential growth model, predict the demand in 2020. Approximately 36.99 billion barrels
   b) World reserves of crude oil in 2013 were approximately 1635 billion barrels. Assuming that no new oil reserves are found, when will the reserves be depleted? Approximately 36.8 yr after 2013

The model

$$\int_0^T Pe^{-kt}\,dt = \frac{P}{k}(1 - e^{-kT})$$

can be applied to calculate the buildup of a radioactive material that is being released into the atmosphere at a constant annual rate. Some of the material decays, but more continues to be released. The amount present at time T is given by the integral above, where P is the amount released per year and k is the half-life.

46. **Radioactive buildup.** Plutonium-239 has a decay rate of approximately 0.003% per year. Suppose plutonium-239 is released into the atmosphere for 20 yr at a constant rate of 1 lb per year. How much plutonium-239 will be present in the atmosphere after 20 yr? Approximately 19.994 lb

47. **Radioactive buildup.** Cesium-137 has a decay rate of 2.3% per year. Suppose cesium-137 is released into the atmosphere for 20 yr at a rate of 1 lb per year. How much cesium-137 will be present in the atmosphere after 20 yr? 16.031 lb

SYNTHESIS

Capitalized cost. *The capitalized cost, c, of an asset over its lifetime is the total of the initial cost and the present value of*

MA121-002 Test #3

A's: 39 (20.6 % of the class)
B's: 63 (33.3 % of the class) } 53.9%

C's: 58 (30.6 % of the class)

D's: 16 (8.5 % of the class)
F's: 13 (6.9 % of the class) } 15.4%

Class average: 79.7