MA121-002

Tuesday, April 2

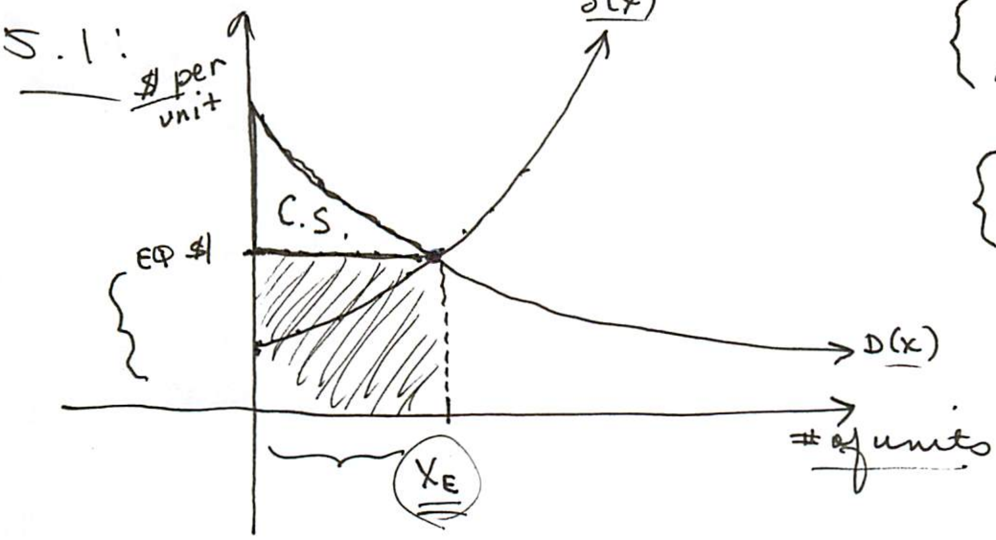
- TEST #3 (RETURNED THURS)
- this week (5.1; 5.2; 5.3)

5.1: (consumer surplus; producer surplus)

5.2: (accumulation models using $P = P_0 \cdot e^{kt}$)

5.3: (improper integrals) $\int_3^{\infty} f(x)\,dx$

5.1:



{ D(x): demand curve

{ S(x): supply curve

1. EQUILIBRIUM POINT

$$D(x) = S(x)$$

⋮

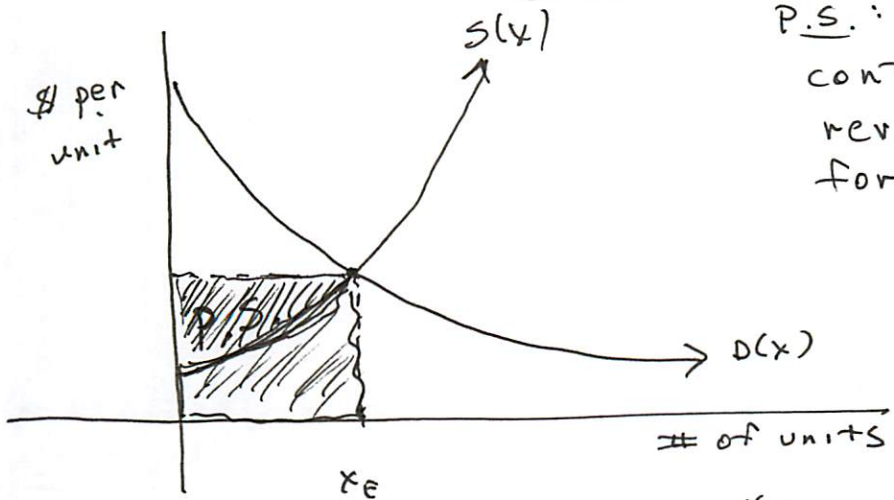
find X_E

2. $ BOX: $(X_E)(\text{EQ }\$) = \$$ ________

3. CONSUMER SURPLUS:

$$C.S. = \int_0^{X_E} D(x)\,dx - [\$ \text{ BOX}]$$

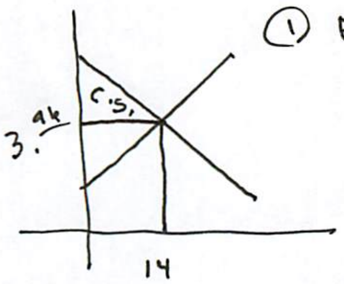
consumer surplus - a feeling that you received more than you paid for.
(not real money)



P.S.: contribution to the revenue (or the profit) for the seller.

$$P.S. = [\$\text{ BOX}] - \int_0^{x_E} S(x)\,dx$$

EX: $D(x) = -.36x + 9$ $\qquad$ $S(x) = .14x + 2$



① EQ. PT:

$$\begin{array}{r} -.36x + 9 = .14x + 2 \\ +.36x - 2 \quad +.36x - 2 \\ \hline 7 = .5x \end{array}$$

$$\frac{7}{.5} = \frac{.5x}{.5} \qquad x = 14$$

$$D(14) = S(14) = .14(14) + 2 = 1.96 + 2 = \$3.96$$

② C.S.: $\displaystyle\int_0^{14} (-.36x + 9)\,dx - [(14)(3.96)]$

$$C.S. = \left[-.36\frac{x^2}{2} + 9x\right]_0^{14} - (14)(3.96)$$

$$C.S. = \left[-.18x^2 + 9x\right]_0^{14} - \underline{(14)(3.\underline{96})}$$

$$C.S. = \left[\left(-.18(14)^2 + 9(14)\right) - \left(\cancel{-.18\,0^2 + 9\,0}\right)\right] - \left(55.\underline{44}\right)$$

$$C.S. = \left[\underline{-.18(196)} + \underline{126}\right] - \underline{55.\underline{44}}$$

$$C.S. = \$35.\underline{28}$$

3 P.S.

$$[\$\ \text{BOX}] - \int_0^{14} S(x)\,dx$$

$$P.S. = \left(55.\underline{44}\right) - \int_0^{14} (\underline{.14x+2})\,dx$$

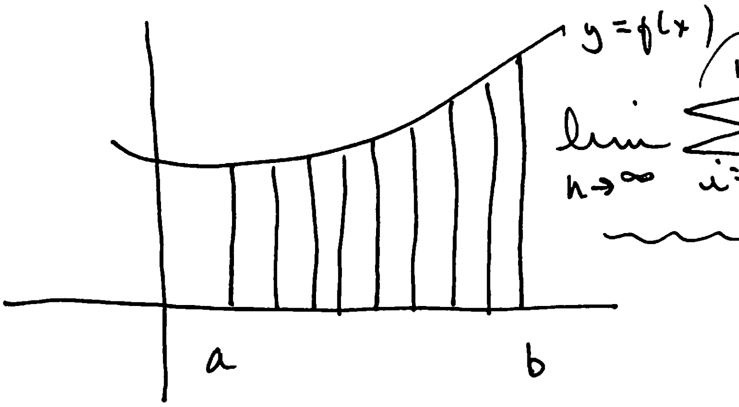
$$= 55.\underline{44} - \left[\frac{.14x^2}{2} + 2x\right]_0^{14}$$

$$= 55.\underline{44} - \left[.\underline{07}x^2 + 2\underline{x}\right]_0^{14}$$

$$= \$55.\underline{44} - \left[\left(\underline{(.07)(14)^2 + 2(14)}\right) - 0\right]$$

$$P.S. = \$13.\underline{72}$$

5.2: ACCUMULATION MODELS



$$\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x = \int_a^b f(x)\cdot dx$$

AREA

FUTURE VALUE:

① ONE-TIME CONTRIBUTION:

$$P = P_0 \cdot e^{kt}$$

future value

present value

$$P = (50{,}000)\, e^{.043(16)}$$

$$P = \underline{\qquad\qquad}$$

② MULTIPLE CONTRIBUTIONS:

10000 (20 yrs)

10000 (19 yrs)

10000 (18 yrs)

...

$$\int_{T_0}^{T_1} P_0 \cdot e^{kt}\, dt$$

22) **Present value of a trust.** In 16 yr, Claire Beasley is to receive \$180,000 under the terms of a trust established by her aunt. Assuming an interest rate of 4.2%, compounded continuously, what is the present value of Claire's trust? \$91,923.51

25.) **Future value of an inheritance.** Upon the death of his uncle, David receives an inheritance of \$50,000, which he invests for 16 yr at 4.3%, compounded continuously. What is the future value of the inheritance? \$99,486.60

31.) **Trust fund.** Bob and Ann MacKenzie have a new grandchild, Brenda, and want to create a trust fund for her that will yield \$250,000 on her 24th birthday.

one time →

a) What lump sum should they deposit now at 5.8%, compounded continuously, to achieve \$250,000? \$62,144.41

multiple

b) The amount in part (a) is more than they can afford, so they decide to invest a constant amount, $R(t)$ dollars per year. Find $R(t)$ such that the accumulated future value of the continuous money stream is \$250,000, assuming an interest rate of 5.8%, compounded continuously. \$4796.74

(multiple. contrib.

a.) $P = P_0 \, e^{Kt}$

$$\frac{250{,}000}{e^{(.058)(24)}} = \frac{P_0 \cdot \cancel{e^{(.058)(24)}}}{\cancel{e^{(.058)(24)}}}$$

$$\boxed{P_0 = \$162{,}144.^{41}}$$

b.) ACCUM FUTURE VALUE $= \int_{t=0}^{t=24} R \cdot e^{Kt} \, dt$

$\int e^{.058t} \, dt$

$$250{,}000 = \int_0^{24} R \cdot e^{(.058)t} \, dt$$

$\int e^{3x} dx = \frac{1}{3} e^{3x} + C$

$$\boxed{\int e^{ax} dx = \frac{1}{a} \cdot e^{ax} + C}$$

$$250{,}000 = R \left[\frac{1}{.058} e^{.058t} \right]_0^{24}$$

$$250{,}000 = \frac{R}{.058} \left[e^{.058(24)} - e^{.058(0)} \right]$$

$$250{,}000 = \frac{R}{.058} \left[4.02288 - 1 \right]$$

$$\frac{(.058)(250{,}000)}{3.02288} = \frac{\frac{R}{\cancel{.058}} \cancel{(3.02288)} \cancel{(.058)}}{\cancel{3.02288}}$$

$$\boxed{\$4796.^{74} = R}$$

each year for 24 years

$(4796.^{74})(24) = 115{,}121$