

Please put all work and answers in the blue book provided. Nothing written on the test itself will be graded. Scientific calculators OK; do **not** use a graphing calculator. Validate all graphs with appropriate calculus techniques. Put your name, row number and seat number on the front of the blue book.

1.) Evaluate: a.) $\lim_{x \rightarrow 0} \frac{e^x - 1 - x}{x^2}$ b.) $\lim_{x \rightarrow 0^+} x^{x^3}$

2.) For the function $f(x) = x^3 - x^2 - x + 6$ on $[-3, 2]$, using the first and second derivative - find the critical points and points of inflection; where the function is increasing, decreasing; where it is concave up, concave down; all relative and absolute maximum and minimum values; graph the function.

3.) Use Newton's Method to find the positive root of the equation $f(x) = 2x - e^{-x} = 0$ using $x_1 = .3$, find x_2 and x_3 .

4.) A closed wooden box (top included) is to be constructed to have a volume of 45 cubic feet. The length of the box must be 5 times the height. Once it is built, it will be coated with a very expensive acrylic - so its surface area needs to be minimized. What dimensions should the box have in order to minimize the surface area?

5.) Use differentials to approximate the increase in volume of a cube if the length of each edge of the cube is changed from 11 cm to 11.1 cm. What is the exact change in volume?

6.) Integrate: a.) $\int (x^4 - 4 \sin x + \frac{5}{\sqrt[3]{x}}) dx$ b.) $\int (5e^{2x} - \frac{3}{x} + \frac{2}{1+x^2}) dx$

7.) Using $f(x) = x^2 - 4x + 2$ on the interval $[1, 4]$, verify the hypotheses of the Mean Value Theorem and find c in $[1, 4]$ guaranteed by the conclusion. Construct a graph of this scenario.

MA141 TEST #3B SOLUTIONS:

(7 questions; 14 points each)

1.) a.) $\lim_{x \rightarrow 0} \frac{e^x - 1 - x}{x^2}$ $\xrightarrow{\text{L'HOP}}$ $\lim_{x \rightarrow 0} \frac{e^x - 1}{2x}$ $\xrightarrow{\text{L'HOP}}$

7pts

$= \lim_{x \rightarrow 0} \frac{e^x}{2} = \frac{1}{2}$

b.) $\lim_{x \rightarrow 0^+} x^{x^3}$ (rewrite) $\rightarrow \lim_{x \rightarrow 0^+} e^{\ln x^{x^3}}$

7pts

$= \lim_{x \rightarrow 0^+} e^{(x^3)(\ln x)} = e^{\lim_{x \rightarrow 0^+} (x^3)(\ln x)}$

exponent only $\dots \lim_{x \rightarrow 0^+} (x^3)(\ln x)$

$= \lim_{x \rightarrow 0^+} \frac{(\ln x)}{\frac{1}{x^3}}$ $\xrightarrow{\text{L'HOP}}$ $\lim_{x \rightarrow 0^+} \frac{\frac{1}{x}}{\frac{-3}{x^4}}$

$= \lim_{x \rightarrow 0^+} \left(\frac{1}{x} \right) \cdot \left(\frac{x^4}{-3} \right) = \lim_{x \rightarrow 0^+} \frac{x^3}{-3} = 0$

$\therefore e^0 = 1$

2.) $f(x) = x^3 - x^2 - x + 6$ on $[-3, 2]$

14pts endpoints:

$f(-3) = (-3)^3 - (-3)^2 - (-3) + 6 = -27 \rightarrow (-3, -27)$

$f(2) = (2)^3 - (2)^2 - (2) + 6 = 8 \rightarrow (2, 8)$

(page 2)

$$f'(x) = 3x^2 - 2x - 1 = 0$$
$$(3x+1)(x-1) = 0$$

~~($f'(x)$ never undef)~~

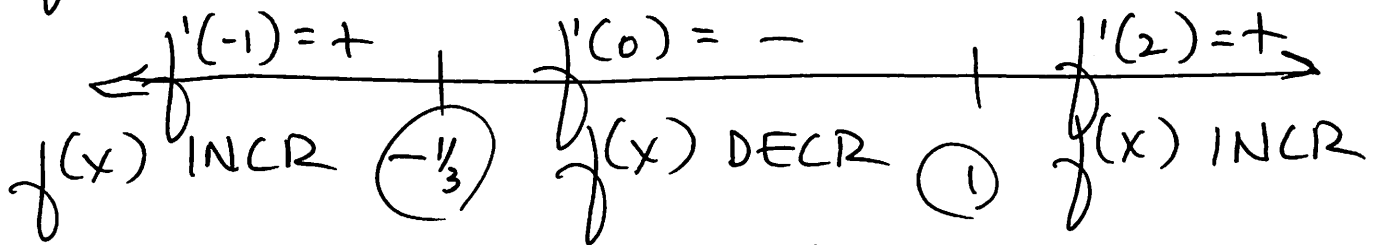
$$x = -\frac{1}{3} \quad x = 1$$

$$f(-\frac{1}{3}) = (-\frac{1}{3})^3 - (-\frac{1}{3})^2 - (-\frac{1}{3}) + 6 = \frac{167}{27} \approx 6.185$$

$$f(1) = 1^3 - 1^2 - 1 + 6 = 5$$

critical points: $(-\frac{1}{3}, 6.185)$ & $(1, 5)$
"FLAT"

$f'(x)$:



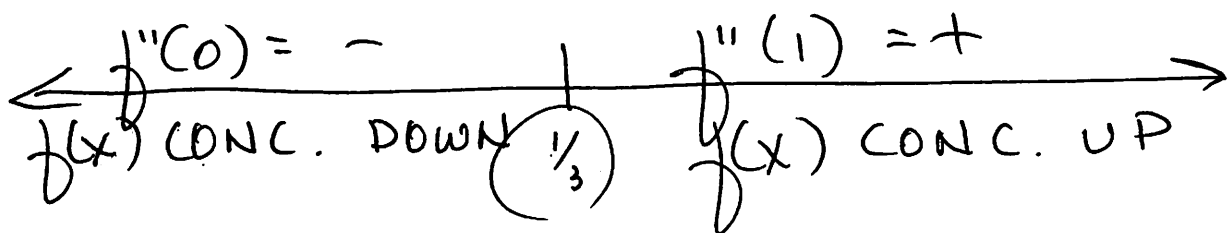
$$f''(x) = 6x - 2 = 0$$
$$6x = 2 \quad x = \frac{1}{3}$$

~~($f''(x)$ never undef)~~

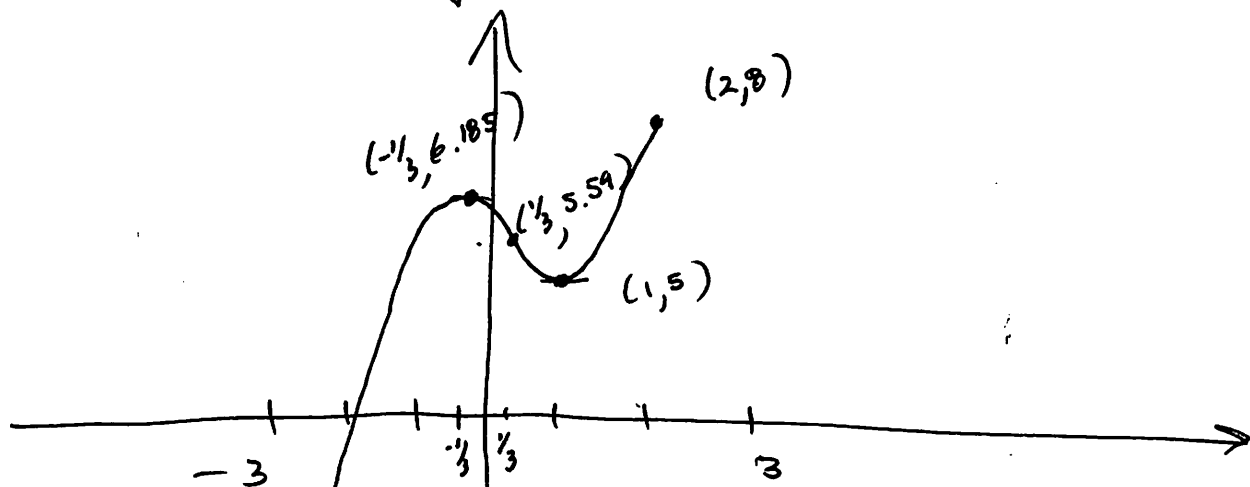
$$f(\frac{1}{3}) = (\frac{1}{3})^3 - (\frac{1}{3})^2 - (\frac{1}{3}) + 6 = \frac{151}{27} \approx 5.59$$

$(\frac{1}{3}, 5.59)$ pt. of inflection

$f''(x)$:



(Page 3)



ABS MAX:
8 (2, 8)

ABS MIN:
-27 (-3, -27)

rel max:
(2, 8) & (-1/3, 6.1845)

rel min:
(1, 5) & (-3, -27)

3.) $f(x) = 2x - e^{-x}$ $x_1 = .3$

14pts

$$f'(x) = 2 + e^{-x}$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$\begin{aligned} x_2 &= x_1 - \frac{f(x_1)}{f'(x_1)} = .3 - \frac{f(.3)}{f'(.3)} \\ &= .3 - \frac{[.6 - .7408]}{[2 + .7408]} = .3 + .0514 \\ &= \underline{.3514} \end{aligned}$$

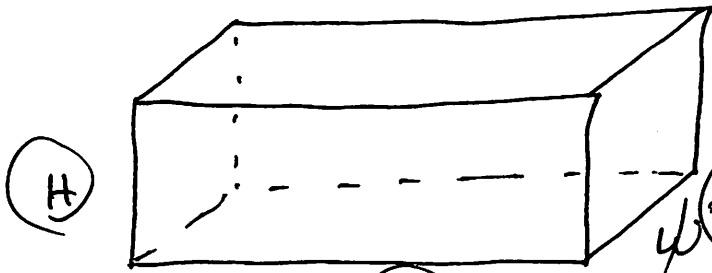
(page 4)

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = .3514 - \frac{[.7028 - .7037]}{[2 + .7037]}$$

$$x_3 = .3514 + .00033$$

$$x_3 = .35173$$

4.)
14 pts



$$V = 45$$

$$(H)(w)(5H) = 45$$

$$5H^2w = 45$$

$$w = \frac{45}{5H^2} = \left(\frac{9}{H^2}\right)$$

surface area
↑

$$S = 2[(H)(5H)] + 2[(w)(H)] + 2[(5H)(w)]$$

$$S = 10H^2 + 2wH + 10wH = 10H^2 + 12wH$$

$$S = 10H^2 + 12\left(\frac{9}{H^2}\right)(H) = 10H^2 + \frac{108}{H} = S$$

$$S' = 20H - \frac{108}{H^2} = 0$$

$$20H = \frac{108}{H^2}$$

$$\frac{20H}{1} = \frac{108}{H^2}$$

$$20H^3 = 108 \quad H^3 = \frac{108}{20} = \frac{54}{10} = 5.4$$

$$H = \sqrt[3]{5.4} \approx 1.7544 \approx 1.75 \text{ ft}$$

(page 5)

$$5H \approx 5(1.7544) \approx \boxed{8.77 \text{ ft}}$$

$$W \approx \frac{9}{H^2} \approx \boxed{2.94 \text{ ft}}$$

max or min?

$$S'' = 20 + \frac{216}{H^3} = +$$

$\therefore$ CONCL. UP $\therefore$ MIN

5.) CUBE $V = X^3$ $X = 11$ $\Delta X = .1$

14pts $dV = V'(X) \cdot \Delta X$

$$= (3X^2) \cdot \Delta X = (3 \cdot 11^2)(.1) = \boxed{36.3 \text{ cm}^3}$$

APPROX

$$\Delta V = (11.1)^3 - (11)^3 = 1367.631 - 1331$$

$$= \boxed{36.631 \text{ cm}^3}$$

EXACT

6.) a.) $\int (X^4 - 4 \sin X + 5 \cdot X^{-1/3}) dX$

7pts

$$= \frac{X^5}{5} - 4(-\cos X) + 5\left(\frac{X^{2/3}}{2/3}\right) + C$$

$$= \boxed{\frac{X^5}{5} + 4 \cos X + \frac{15}{2} X^{2/3} + C}$$

(page 6)

b.) $\int (5e^{2x} - \frac{3}{x} + \frac{2}{1+x^2}) dx$

7pts

$$= 5(\frac{1}{2}e^{2x}) - 3(\ln|x|) + 2\tan^{-1}x + C$$
$$= \frac{5}{2}e^{2x} - 3\ln|x| + 2\tan^{-1}x + C$$

7.) $f(x) = x^2 - 4x + 2$ on $[1, 4]$

14pts hypotheses: ① $f(x)$ contin on $[1, 4]$??
yes, it is a polynomial

② $f(x)$ differentiable on $(1, 4)$
yes, deriv. exists for all values in $(1, 4)$

$$f'(x) = 2x - 4 = 0$$

$$2x = 4$$
$$x = 2$$

Vertex: $(2, -2)$

$$f(1) = -1 \quad f(4) = 2$$

find c such that...

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$2c - 4 = \frac{f(4) - f(1)}{4 - 1}$$

$$2c - 4 = \frac{2 - (-1)}{4 - 1} = \frac{3}{3} = 1$$

$$2c - 4 = 1 \quad 2c = 5 \quad \boxed{c = \frac{5}{2}}$$
$$\frac{5}{2} \in (1, 4)$$

