

Please put all work and answers in the stamped blue/green book provided. No graphing calculators; do not use a calculator that does calculus. Validate all work with appropriate calculus techniques. Simplify completely – unless directed otherwise in the problem. Fold the test copy and turn it in along with the blue/green book. (Name; row/seat on the front)

1.) Find $\frac{dy}{dx}$:

a.) $y^3 + 4xy = 2x^5 + xy^4$ b.) $y = (\ln x)^{\sin x}$ (b: use logarithmic differentiation)

2.) Find y' :

a.) $y = \sqrt{1-x^2} \cdot \sin^{-1} x$ b.) $y = \frac{1 - \sec x}{\tan x}$

3.) Compute the first and second derivatives of $f(x) = \frac{8x+1}{5x-3}$

4.) A ball is projected upward from a platform 40 feet above the ground with an initial velocity of 62 feet per second. The height of the ball at time t (in seconds) is given by $s(t) = -16t^2 + 62t + 40$. Find the velocity and acceleration of the ball as functions of time. How long is the ball in the air? What is the velocity of the ball when it hits the ground?

5.) Find $f'(x)$: a.) $f(x) = e^{x^3} \cdot \ln(3x+1)$ b.) $f(x) = \arctan(\sin(x^3 + 2x))$

6.) Find the equation of the line tangent to the curve $f(x) = \sin x \cdot \cos x$ at the point $\left(\frac{\pi}{6}, \frac{\sqrt{3}}{4}\right)$.

7.) A balloon is rising at a constant speed of 6 ft/sec. A boy is biking along a straight road at a speed of 18 ft/sec. When he passes under the balloon, it is 48 feet above him. How fast is the distance between the boy and the balloon changing 2 seconds later?

SOLUTIONS

(7 QUESTIONS — 14 POINTS EACH)

1.) a.) $y^3 + 4xy = 2x^5 + xy^4$

7pts

DERIV:

$$3y^2 \cdot \frac{dy}{dx} + [(4y) \cdot \frac{dy}{dx} + (y) \cdot 4] = 10x^4 + [(x) \cdot 4y^3 \frac{dy}{dx} + (y^4) \cdot (1)]$$

$$3y^2 \cdot \frac{dy}{dx} + 4x \cdot \frac{dy}{dx} - 4xy^3 \cdot \frac{dy}{dx} = 10x^4 + y^4 - 4y$$

$$\frac{dy}{dx} [3y^2 + 4x - 4xy^3] = 10x^4 + y^4 - 4y$$

$$\frac{dy}{dx} = \frac{10x^4 + y^4 - 4y}{3y^2 + 4x - 4xy^3}$$

b.) $y = (\ln x)^{\sin x}$ (use log. diff.)

$$\ln y = \ln (\ln x)^{\sin x} \quad \ln y = (\sin x) \cdot [\ln (\ln x)]$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = (\sin x) \cdot \left[\frac{1}{\ln x} \cdot \frac{1}{x} \right] + (\ln (\ln x)) \cdot [\cos x]$$

$$\frac{dy}{dx} = y \left[\frac{\sin x}{x \cdot \ln x} + \cos x \cdot \ln (\ln x) \right]$$

$$\frac{dy}{dx} = (\ln x)^{\sin x} \left[\frac{\sin x}{x \cdot \ln x} + \cos x \cdot \ln (\ln x) \right]$$

2.) a.) $y = (1-x^2)^{1/2} \cdot \sin^{-1} x$

7pts

$$y' = (1-x^2)^{1/2} \cdot \left(\frac{1}{\sqrt{1-x^2}} \right) + (\sin^{-1} x) \cdot \frac{1}{2} (1-x^2)^{-1/2} (-2x)$$

$$y' = 1 - \frac{x \cdot \sin^{-1} x}{\sqrt{1-x^2}}$$

(page 2)

2.) b.) $y = \frac{1 - \sec x}{\tan x}$
7pts

$$y' = \frac{(\tan x)(-1 \cdot \sec x \cdot \tan x) - (1 - \sec x)(\sec^2 x)}{(\tan x)^2}$$

$$y' = \frac{-\sec x \cdot \tan^2 x - \sec^2 x + \sec^3 x}{(\tan x)^2}$$

OK to STOP HERE

$$y' = \frac{\sec x [-\tan^2 x - \sec x + \sec^2 x]}{(\tan x)^2}$$

$$y' = \frac{\sec x [-\tan^2 x + \sec^2 x - \sec x]}{(\tan x)^2}$$

$$y' = \frac{\sec x [1 - \sec x]}{(\tan x)^2}$$

3.) $f(x) = \frac{8x+1}{5x-3}$ find $f'(x)$ & $f''(x)$:
14pts

$$f'(x) = \frac{(5x-3)(8) - (8x+1)(5)}{(5x-3)^2} = \frac{40x - 24 - 40x - 5}{(5x-3)^2}$$

$$f'(x) = \frac{-29}{(5x-3)^2} = -29(5x-3)^{-2}$$

$$f''(x) = -29[-2(5x-3)^{-3}(5)] = \frac{290}{(5x-3)^3} = f''(x)$$

(page 3)

4.) $s(t) = -16t^2 + 62t + 40$

1+pts

$s(t)$: FEET

t : SECONDS

$$s'(t) = \boxed{v(t) = -32t + 62}$$

$$s''(t) = v'(t) = \boxed{a(t) = -32}$$

in the air for how long
(when does it hit the ground?)

$$0 = s(t) = -16t^2 + 62t + 40$$

$$t = \frac{-62 \pm \sqrt{(62)^2 - 4(-16)(40)}}{2(-16)}$$

$$t = \frac{-62 \pm \sqrt{3844 + 2560}}{-32} = \frac{-62 \pm 80.025}{-32}$$

$$\left\{ \begin{array}{l} t_1 = \frac{-62 - 80.025}{-32} \approx \boxed{4.44 \text{ sec}} \end{array} \right.$$

$$\left\{ \begin{array}{l} t_2 = \frac{-62 + 80.025}{-32} \approx \text{NEG (time} \geq 0) \end{array} \right.$$

$$v(4.44) = -32(4.44) + 62 = \boxed{-80.08 \frac{\text{ft}}{\text{sec}}}$$

when it hits
the ground

5) a.) $f(x) = e^x \cdot \ln(3x+1)$ (page 4)

$$f'(x) = e^{x^3} \left[\frac{1}{3x+1} (3) \right] + \ln(3x+1) \cdot \left[e^{x^3} \cdot (3x^2) \right]$$

$$f'(x) = 3e^{x^3} \left[\frac{1}{3x+1} + x^2 \cdot \ln(3x+1) \right]$$

b.) $f(x) = \arctan(\sin(x^3+2x))$
7pts
 $f'(x) = \frac{1}{1 + [\sin(x^3+2x)]^2} \cdot \cos(x^3+2x) (3x^2+2)$

$$f'(x) = \frac{(3x^2+2) \cdot \cos(x^3+2x)}{1 + [\sin(x^3+2x)]^2}$$

6.) $f(x) = \sin x \cdot \cos x$ $(\pi/6, \sqrt{3}/4)$

$$y - \frac{\sqrt{3}}{4} = m(x - \pi/6)$$

$$m = m_{\text{TAN}} = f'(x) = (\sin x)(-\sin x) + (\cos x)(\cos x)$$

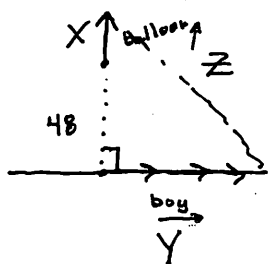
$$f'(x) = -\sin^2 x + \cos^2 x$$

$$f'(\pi/6) = -(\sin \pi/6)^2 + (\cos \pi/6)^2$$

$$f'(\pi/6) = -(\frac{1}{2})^2 + (\frac{\sqrt{3}}{2})^2 = -\frac{1}{4} + \frac{3}{4} = \frac{1}{2}$$

$$y - \frac{\sqrt{3}}{4} = \frac{1}{2}(x - \pi/6)$$

7.)



$$\frac{dx}{dt} = 6 \frac{\text{ft}}{\text{sec}}$$

$$\frac{dy}{dt} = 18 \frac{\text{ft}}{\text{sec}}$$

$$2(48+y) \cdot \frac{dx}{dt} + 2y \frac{dy}{dt} = 2z \frac{dz}{dt}$$

$$2(48+12) \cdot (6) + 2(36)(18) = 2(69.97) \frac{dz}{dt}$$

$$360 + 648 = 69.97 \frac{dz}{dt}$$

$$1008 = 69.97 \frac{dz}{dt}$$

$$14.41 \text{ ft/sec} = \frac{dz}{dt}$$

AT THIS INSTANT:

$$t = 2 \text{ sec}$$



$$60^2 + 36^2 = z^2$$

$$z = \sqrt{60^2 + 36^2} \approx 69.97$$