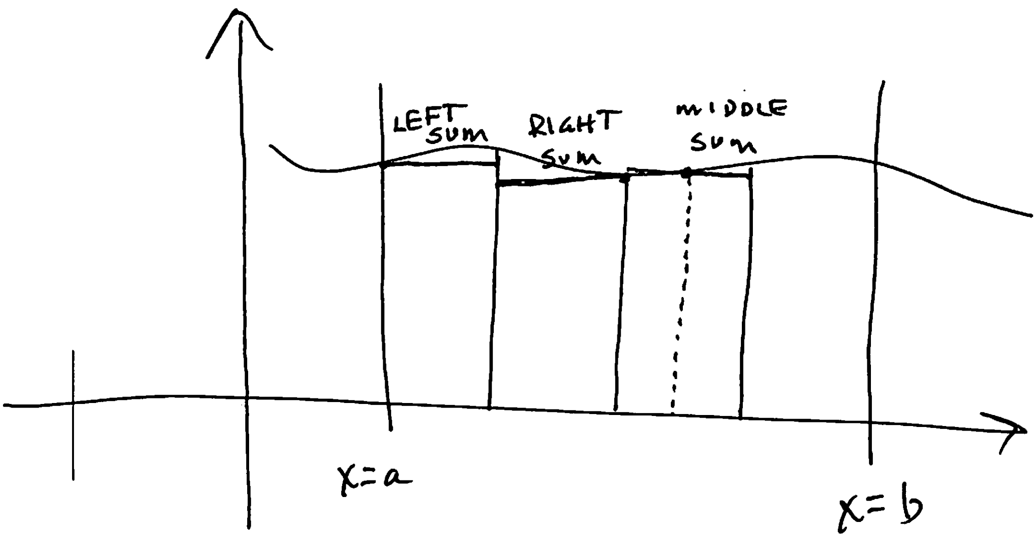
MA 141 – 004

Tuesday, March 25

4.1: AREAS & RIEMANN SUM

4.2: DEFINITE INTEGRAL

4.3: FUNDAMENTAL THEOREM OF CALCULUS



$$\sum_{i=1}^{n} 1 = n$$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{4} i = \frac{1+2+3+4+5+6}{\frac{6(6+1)}{2} = \frac{6(7)}{2} = 21}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^{4} i^2 = 1^2+2^2+3^2+4^2 = \underline{30}$$

$1+4+9+16$

$$= \frac{4(4+1)(2\cdot4+1)}{6} = \frac{4(5)(9)}{6} = \frac{180}{6} = \underline{30}$$

$$\sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$$

$$A = \lim_{n\to\infty}\left[\frac{4}{n}\sum_{i=1}^{n} 1 + \frac{8}{n^2}\sum_{i=1}^{n} i + \frac{8}{n^3}\sum_{i=1}^{n} i^2\right]$$

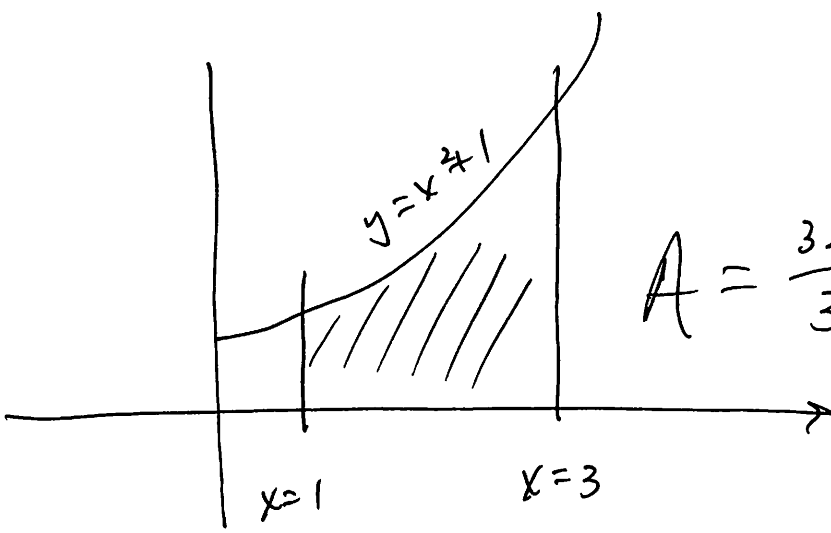
$$A = \lim_{n\to\infty}\left[\frac{4}{n}(n) + \frac{8}{n^2}\left(\frac{n(n+1)}{2}\right) + \frac{8}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right)\right]$$

$$A = \lim_{n\to\infty}\left[4 + \frac{8}{2}\left(\frac{n^2+n}{n^2}\right) + \frac{8}{6}\left(\frac{2n^3+\cdots}{1\cdot n^3}\right)\right]$$

$$\lim_{n\to\infty}\frac{1\cdot n^2+n}{1\cdot n^2} = 1 \qquad \lim_{n\to\infty}\frac{2n^3+\cdots}{1\cdot n^3} = 2$$

$$A = \left[4 + \frac{8}{2}\cdot 1 + \frac{8}{6}\cdot 2\right]$$

$$A = 4 + 4 + \frac{8}{3} = 10\tfrac{2}{3} = \frac{32}{3}$$



$$A = \frac{32}{3}$$

Definition 1. **Definite Integral of a Continuous Function on $[a, b]$**

Let $f(x)$ be defined and continuous on the interval $[a, b]$. For each partition

$$a = x_0 < x_1 < x_2 < \cdots < x_{n-1} < x_n = b$$

of the interval $[a, b]$ into n equal parts, the length of each subinterval is $\boxed{\Delta x = \frac{b-a}{n}}$ *and each* $\boxed{x_i = a + i\Delta x}$ *$i = 1, 2, \cdots, n$. The* **definite integral** *of f on $[a, b]$, denoted $\int_a^b f(x)\,dx$, is defined by*

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

provided this limit exists.

Properties of the Definite Integral

Let a, b and c be real numbers with $a < b$ and f and g be continuous functions. Then

1. $\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$
2. $\int_a^a f(x)\,dx = 0$
3. $\int_a^b c\,dx = c(b-a)$
4. $\int_a^b (f(x) \pm g(x))\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$
5. $\int_a^b cf(x)\,dx = c\int_a^b f(x)\,dx$
6. $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$
7. If $f(x) \geq 0$ for all $x \in [a, b]$, then $\int_a^b f(x)\,dx \geq 0$
8. If $f(x) \geq g(x)$ for all $x \in [a, b]$, then $\int_a^b f(x)\,dx \geq \int_a^b g(x)\,dx$
9. If $m \leq f(x) \leq M$ for all $x \in [a, b]$, then $m(b-a) \leq \int_a^b f(x)\,dx \leq M(b-a)$

Theorem 4. The Fundamental Theorem of Calculus

Let f be continuous on $[a, b]$.

Part 1: The function G defined by

$$G(x) = \int_a^x f(t)\, dt \qquad \text{for all } x \text{ in } [a, b]$$

is an antiderivative of f on $[a, b]$.

Part 2: If F is any antiderivative of f on $[a, b]$, then

$$\int_a^b f(x)\, dx = F(b) - F(a)$$