MA 141-004

Thursday, March 21

- TEST #3 (first hour)
- 4.1: (AREAS & RIEMANN SUMS) (second hour)

4.1:

$$\sum_{i=1}^{6} 4^i = 4^1+4^2+4^3+4^4+4^5+4^6$$

SIGMA (summation)

$= \underline{\qquad\qquad}$

$$\sum_{i=1}^{n} i = 1+2+3+\cdots+n$$

sum of "n" consecutive integers

$$\boxed{\sum_{i=1}^{n} i = \frac{n(n+1)}{2}}$$

① $\sum_{i=1}^{4} i = 1+2+3+4 = \frac{4(4+1)}{2}$

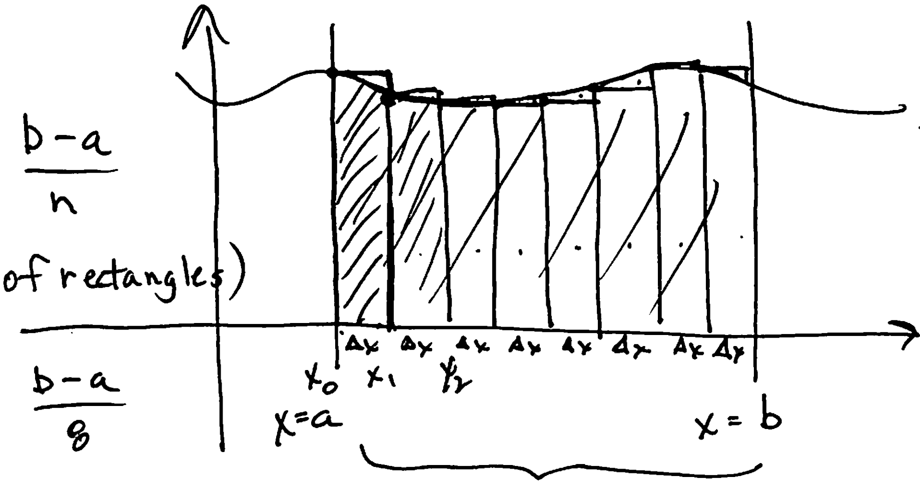
② $\sum_{i=1}^{100} i = 1+2+3+\cdots+98+99+100$

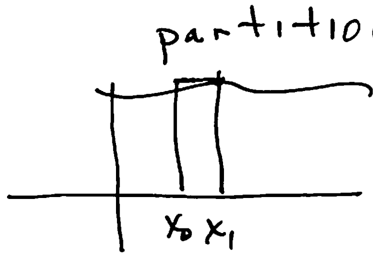
$= \frac{100(101)}{2} = 5050$

$$\Delta x = \frac{b-a}{n}$$

(n = # of rectangles)

$$\Delta x = \frac{b-a}{8}$$





Approx: using rectangles

$$\text{AREA} \approx \underbrace{\left[f(x_0)\right]\cdot \Delta x}_{A_1} + \underbrace{\left[f(x_1)\right]\cdot \Delta x}_{A_2} + \underbrace{\left[f(x_2)\right]\cdot \Delta x}_{A_3}$$

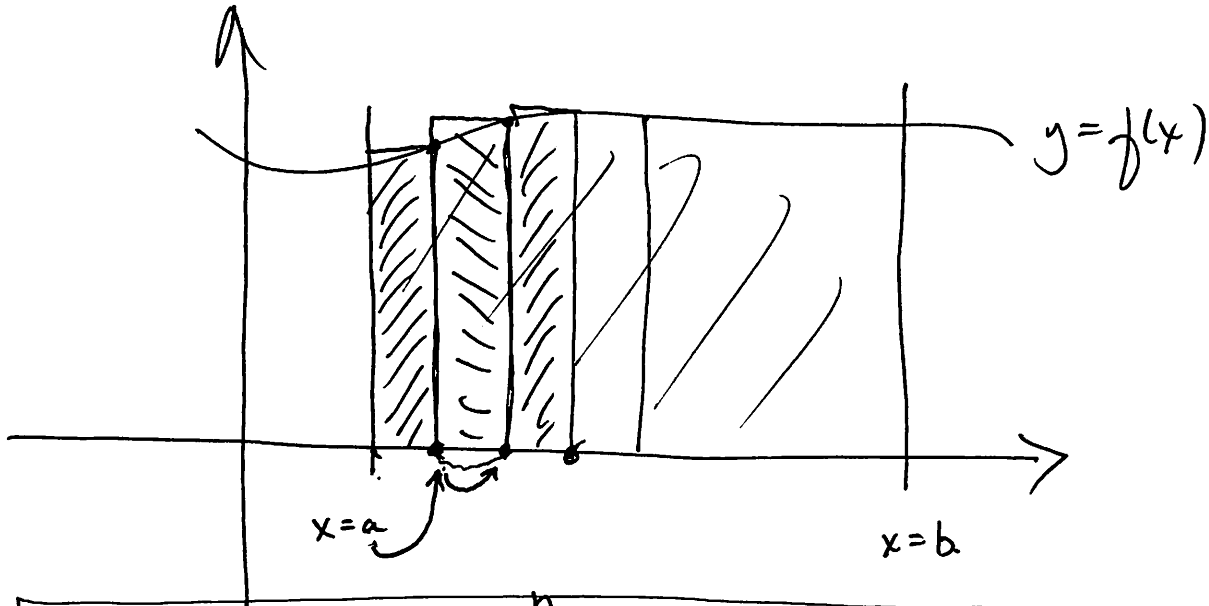
$$+ \cdots$$

as $\left(\begin{matrix} n \to \infty \\ \Delta x \to 0 \end{matrix}\right)$, then the accur. of our AREA calculation INCR.

$$\text{AREA} \approx \sum_{i=1}^{n} f(x_i)\cdot \Delta x$$

$$\text{AREA} \approx f(x_1)\cdot \Delta x + f(x_2)\cdot \Delta x + f(x_3)\,\Delta x + \cdots f(x_{n-1})\cdot \Delta x + f(x_n)\,\Delta x$$

$$\boxed{A = \lim_{n\to\infty} \sum_{i=1}^{n} \underbrace{f(x_i)}_{\text{HT}}\, \underbrace{\Delta x}_{\text{WDTH}}}$$



$$\boxed{A = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\cdot\Delta x}$$

$$x_i = \underline{????} = \underline{a + i\cdot\Delta x}$$

$$\Delta x = \frac{b-a}{n}$$

$$f(x_1)\cdot\Delta x = f(a+1\cdot\Delta x)\cdot\Delta x$$

$$f(x_2)\cdot\Delta x = f(a+2\Delta x)\cdot\Delta x$$

$$f(x_3)\cdot\Delta x = f(a+3\cdot\Delta x)\cdot\Delta x$$

$$\vdots$$

$$\boxed{f(x_i)\,\Delta x = f(a+i\cdot\Delta x)\cdot\Delta x}$$

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i) \cdot \Delta x$$

$$\boxed{A = \lim_{n\to\infty} \sum_{i=1}^{n} f(a + i\cdot\Delta x)\cdot \Delta x}$$

$$x_i = a + i \cdot \Delta x$$

$$\Delta x = \frac{b-a}{n}$$

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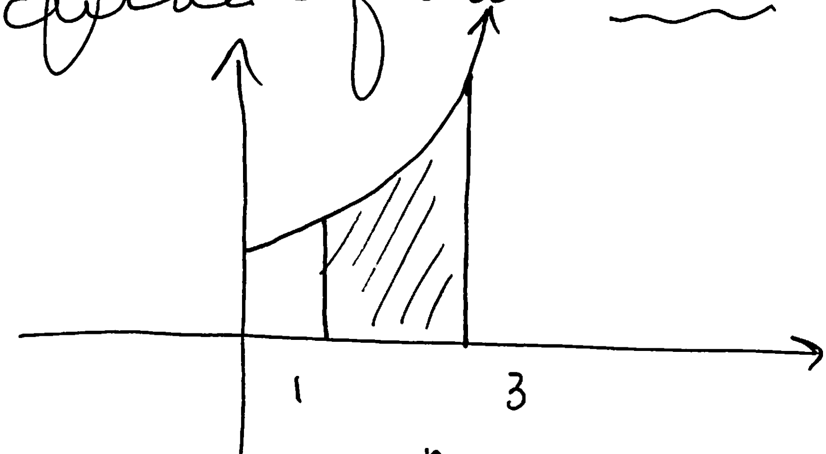
ex: $f(x) = x^2 + 1$

find the AREA under this curve from $x=1$ to $x=3$.



$$1 + \frac{2i}{n}$$

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} f(a + i\cdot\Delta x)\cdot \Delta x$$

$a = 1$

$b = 3$

$\Delta x = \frac{b-a}{n}$

$\Delta x = \frac{2}{n}$

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} f\left(1 + i \cdot \frac{2}{n}\right) \cdot \left(\frac{2}{n}\right)$$

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} \left[\left(1+\frac{2i}{n}\right)^2 + 1\right] \cdot \left(\frac{2}{n}\right)$$

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} \left[\left(1+\frac{2i}{n}\right)^2 + 1\right] \cdot \left(\frac{2}{n}\right)$$

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} \left(2 + \frac{4i}{n} + \frac{4i^2}{n^2}\right) \cdot \left(\frac{2}{n}\right)$$

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{4}{n} + \frac{8i}{n^2} + \frac{8i^2}{n^3}\right)$$

$$A = \lim_{n\to\infty} \left[\sum_{i=1}^{n} \frac{4}{n} + \sum_{i=1}^{n} \frac{8i}{n^2} + \sum_{i=1}^{n} \frac{8i^2}{n^3}\right]$$

$$A = \lim_{n\to\infty} \left[\frac{4}{n} \sum_{i=1}^{n} 1 + \frac{8}{n^2} \sum_{i=1}^{n} i + \frac{8}{n^3} \sum_{i=1}^{n} i^2\right]$$

$$\frac{4}{n}(n) \qquad \frac{8}{n^2}\left(\frac{n(n+1)}{2}\right)$$

ADD: $\lim_{x \to 1^+} (1-x)$ will email.

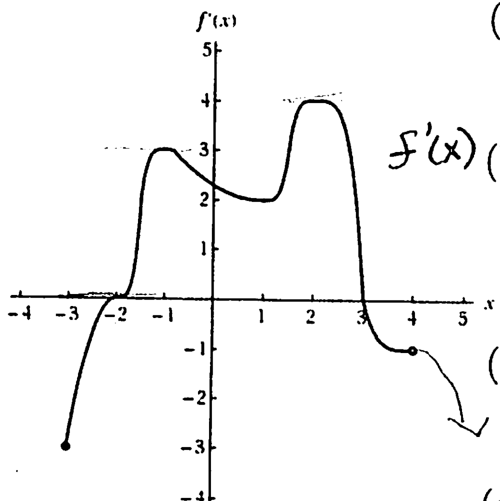
MA 141-004 S19 Recitation 10, Mar 20

Recall that the indefinite integral $\int f(x)dx$ is the family of antiderivatives $F(x)+C$ such that $\frac{d}{dx}[F(x)+C] = f(x)$.

1. Find the following indefinite integrals:

$$a) \int \left[x^{\frac{5}{2}} + \cos(x)\right] dx \qquad b) \int 4e^{12x}dx \qquad c) \int \frac{10}{1+x^2}dx$$

2. Suppose **the derivative** of some function $f(x)$ is graphed below. The domain of the function $f(x)$ is the interval $[-3, 4]$.



(a) Find the intervals on which $f(x)$ is increasing and the intervals on which it is decreasing.

(b) Find the critical numbers of $f(x)$ and determine, for each, if it is the location of a local maximum or minimum (or neither).

(c) Find the intervals on which $f(x)$ is concave up and the intervals on which it is concave down.

(d) Find the points of inflection of $f(x)$.

3. The end of a house has the shape of a square surmounted by an equilateral triangle. Suppose the length of the base is measured to be 39 feet, with a maximum error in measurement of 1 inch. Calculate the area of the end. Then use differentials to estimate the maximum error in the calculation of the area. (Hint: If the length of the base is x then the area is $A(x) = (1 + \frac{\sqrt{3}}{4})x^2$.)

4. Waste Management Company wants to construct a dumpster in the shape of a rectangular solid with no top whose length is twice its width. Its volume is fixed at 36 yd^3. Find the **dimensions** of the dumpster that will **minimize its surface area.**

1 Regarding Differentials and Indefinite Integrals

Given a variable x we associate to it dx, the *differential of* x, and consider it another independent variable. If $y = f(x)$ is a differentiable function of x, then we call df the *differential of* f which is defined to be $df = f'(x)dx$. When dx is small we use $\delta y = df$ as an approximation.

Theorem 4. Power Rule for Indefinite Integrals
The indefinite integral of the function $f(x) = x^p$ is the family of antiderivatives given by

$$\int x^p\, dx = \frac{x^{p+1}}{p+1} + C \qquad C, p \in \mathbb{R} \quad p \neq -1$$

Derivative and Indefinite Integral Formulas for the Trigonometric Functions

$\boxed{\frac{d}{dx}F(x) = f(x)}$ $\qquad$ $\boxed{\int f(x)\,dx = F(x) + C}$

1. $\frac{d}{dx}\sin ax = a\cos ax$ $\qquad$ $\int \cos ax\,dx = \frac{1}{a}\sin ax + C \qquad a \neq 0$
2. $\frac{d}{dx}\cos ax = -a\sin ax$ $\qquad$ $\int \sin ax\,dx = -\frac{1}{a}\cos ax + C \qquad a \neq 0$
3. $\frac{d}{dx}\tan ax = a\sec^2 ax$ $\qquad$ $\int \sec^2 ax\,dx = \frac{1}{a}\tan ax + C \qquad a \neq 0$

Derivative and Indefinite Integral Formulas for e^x and $\ln x$.

$\boxed{\frac{d}{dx}F(x) = f(x)}$ $\qquad$ $\boxed{\int f(x)\,dx = F(x) + C}$

1. $\frac{d}{dx}e^x = e^x$ $\qquad$ $\int e^x\,dx = e^x + C \Rightarrow \int e^{kx}\,dx = \frac{1}{k}e^{kx}$
2. $\frac{d}{dx}\ln|x| = \frac{1}{x}\,,\ |x| \neq 0$ $\qquad$ $\int \frac{1}{x}\,dx = \ln|x| + C$

Derivative and Indefinite Integral Formulas for the Inverse Trigonometric Functions

$\boxed{\frac{d}{dx}F(x) = f(x)}$ $\qquad$ $\boxed{\int f(x)\,dx = F(x) + C}$

1. $\frac{d}{dx}\arcsin x = \frac{1}{\sqrt{1-x^2}}\,, \quad |x| < 1$ $\qquad$ $\int \frac{1}{\sqrt{1-x^2}}\,dx = \arcsin x + C$
2. $\frac{d}{dx}\arccos x = -\frac{1}{\sqrt{1-x^2}}\,, \quad |x| < 1$ $\qquad$ $\int -\frac{1}{\sqrt{1-x^2}}\,dx = \arccos x + C$
3. $\frac{d}{dx}\arctan x = \frac{1}{1+x^2}\,, \quad x \in \mathbb{R}$ $\qquad$ $\int \frac{1}{1+x^2}\,dx = \arctan x + C$