NORTH CAROLINA STATE UNIVERSITY Department of Mathematics

MA121-002 Test #2 Form A Tuesday, February 26, 2019 Dr. J. Griggs

Please put all work and answers in the stamped blue book provided; one problem per page (the back of a page constitutes a new page). Graphing calculators cannot be used because they do calculus; however, scientific calculators are allowed. Please put your **name, row letter** and **seat number** on the blue book. Seven questions – 14 points each; 5 point bonus question . Simplify all derivatives completely. Turn in the test copy with the blue book.

1.) Find all intercepts and asymptotes; use $f'(x)$ to determine where the function is increasing and decreasing; graph it: $f(x)=\dfrac{4x-1}{x+3}$

2.) Find and use $f'(x)$ *and* $f''(x)$ in order to find all critical points, points of inflection, areas where the function is increasing, decreasing, concave up and concave down, and all relative maximum and minimum values of the function. Graph the function: $f(x)=x^3-3x+4$

3.) A stone is projected upward from a platform 45 feet above the ground with an initial velocity of 120 feet per second. The height *s(t)* above the ground (in feet) at time *t* (in seconds) is given by $s(t)=-16t^2+120t+45$. Find the height of the stone at t=2 sec, the velocity of the stone at t=2 sec, and the acceleration of the stone at t=2 sec. (appropriate units are important)

4.) Find the critical point(s); determine where the function is increasing and decreasing; and graph: $y=1+(x-3)^{2/3}$

5.) Find the absolute maximum and absolute minimum values of the function on the given closed interval using calculus techniques: $f(x)=x^3-x^2-x+4 \text{ on } [0,3]$

6.) Find the equation of the tangent line to $y=\sqrt{2x^2+7x}$ at the point (1, 3).

7.) A real estate company owns 120 apartments which are fully occupied when the rent is $800 per month. Analytics indicate that for each $25 increase in rent, 3 apartments will become unoccupied. What rent should be charged in order to obtain the largest gross income? What is the largest gross income?

Bonus: (5 points) For the function $f(x)=\dfrac{x^3-6x^2+5x+12}{x^2-2x-8}=\dfrac{(x-3)(x^2-3x-4)}{x^2-2x-8}$ find all intercepts and all asymptotes (including oblique asymptotes). Does this function have a "hole" in it? If so, where is it? Use all of the above information to sketch a graph of this function. ($f'(x)$ and $f''(x)$ are not necessary for this sketch)

MA121 TEST #2 FORM A

(14 points each)

1.) $f(x) = \dfrac{4x-1}{x+3}$

$f'(x) = \dfrac{(x+3)(4)-(4x-1)(1)}{(x+3)^2}$

$f'(x) = \dfrac{4\cancel{x}+12-4\cancel{x}+1}{(x+3)^2}$

$f'(x) = \dfrac{13}{(x+3)^2}$

① $f'(x) \neq 0$

② $f'(x)$ undef. at $x=-3$

$f(-3) = ?$ no! (VERT ASYMP)

intercepts:

① y-int: $(x=0)$

$y = \dfrac{4(0)-1}{0+3} = \dfrac{-1}{3}$

$(0, -1/3)$

2.) x-int: $(y=0)$

$0 = \dfrac{4x-1}{x+3}$

$4x-1=0$

$x = 1/4$

$(1/4, 0)$

$f'(x)$:

$f'(-4) = +$ | -3 | $f'(0) = +$

$f(x)$ INCR | | $f(x)$ INCR

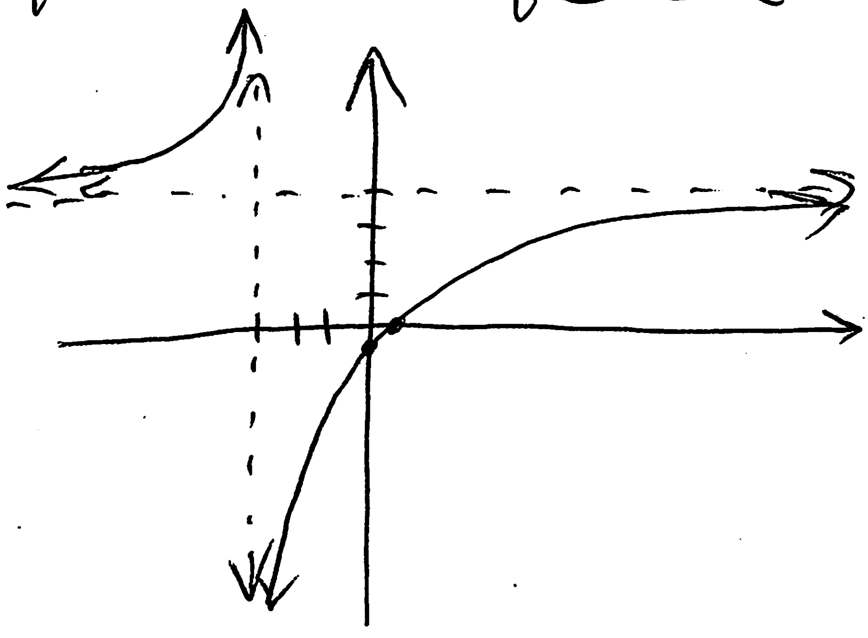
ASYMPTOTES:

① VERT: $\boxed{x=-3}$

② HORIZ:

$\lim_{x \to \infty} \dfrac{4x-1}{x+3} = 4$

$\boxed{y=4}$



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2.) $f(x) = x^3 - 3x + 4$

$f'(x) = 3x^2 - 3 = 3(x^2-1) = 3(x-1)(x+1)$

① $f'(x) = 3(x-1)(x+1) = 0$

$x = 1$ & $x = -1$

$f(1) = 1^3 - 3(1) + 4$	$f(-1) = (-1)^3 - 3(-1) + 4$
$f(1) = 1 - 3 + 4$	$f(-1) = -1 + 3 + 4$
$f(1) = 2$	$f(-1) = 6$
$(1, 2)$ CRIT. PT.	$(-1, 6)$ CRIT PT

(horizontal tangent lines there)

$f'(x)$:

$f'(-2) = +$ | -1 | $f'(0) = -$ | $+1$ | $f'(2) = +$

$f(x)$ INCR | -1 | $f(x)$ DECR | $+1$ | $f(x)$ INCR

$f''(x) = 6x$

① $f''(x) = 6x = 0$

$x = 0$

$f(0) = 4$ $\Rightarrow$ $(0, 4)$ point of inflection

$f''(-1) = -$ | 0 | $f''(1) = +$

$f(x)$ CONC D... | 0 | $f(x)$ CONC. UP

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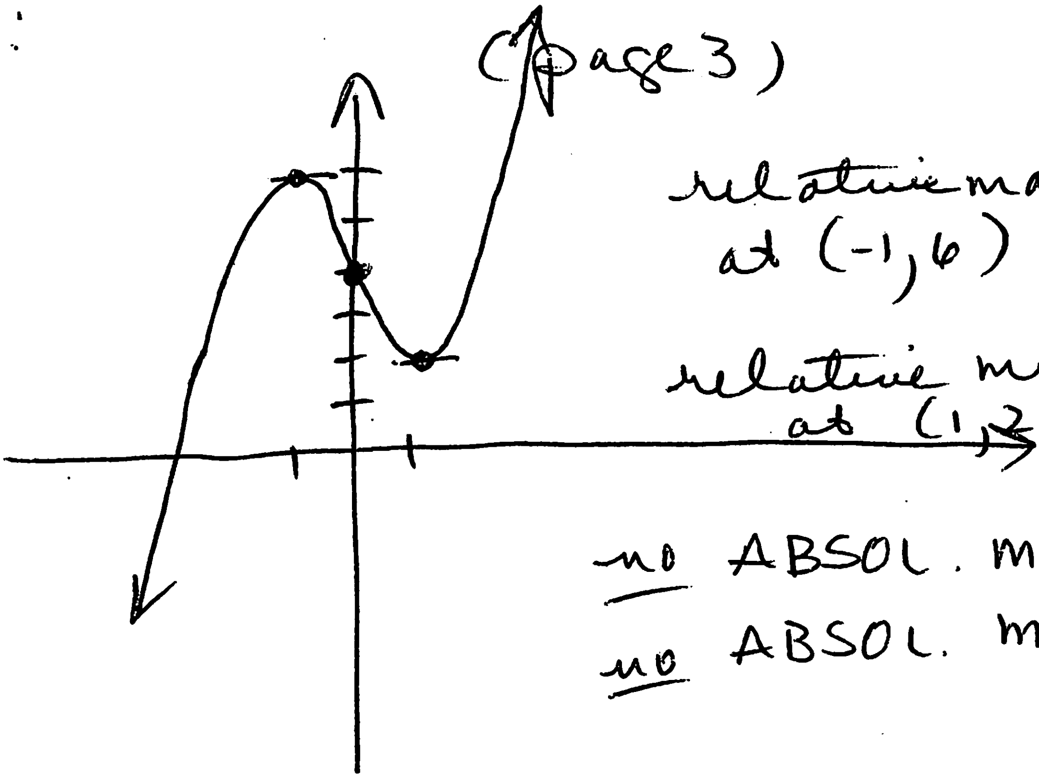

relative max at $(-1, 6)$

relative min at $(1, 2)$

no ABSOL. MAX

no ABSOL. MIN

3.) $s(t) = -16t^2 + 120t + 45$

$s(2) = -16(2)^2 + 120(2) + 45$

$s(2) = -64 + 240 + 45 = 221 \text{ feet}$

$v(t) = s'(t) = -32t + 120$

$v(2) = -32(2) + 120$

$v(2) = -64 + 120 = 56 \frac{ft}{sec}$

$a(t) = v'(t) = s''(t) = -32$

$a(2) = -32 \frac{ft}{sec} / sec$ or $-32 \frac{ft}{sec^2}$

4.) $y = 1 + (x-3)^{2/3}$

$y' = \frac{2}{3}(x-3)^{-1/3}(1) = \frac{2}{3\sqrt[3]{x-3}} = y'$

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① $y' = 0$ no!

② y' undef. when $x = 3$

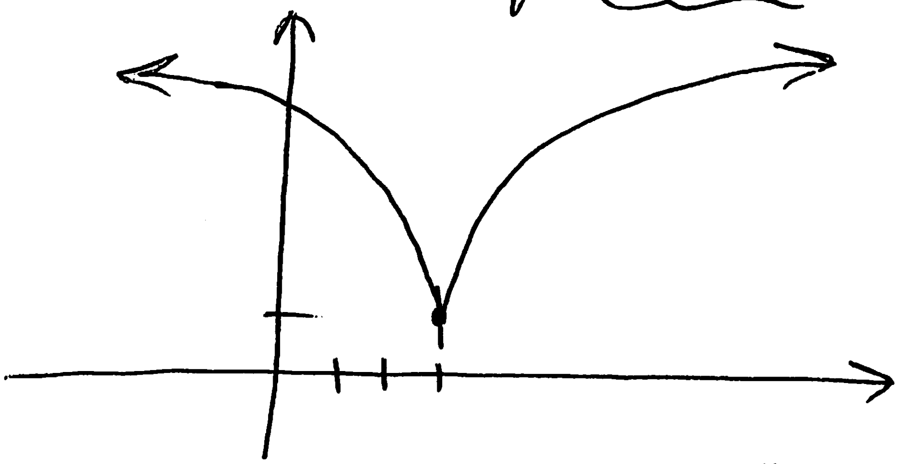
$f(3) = 1 + (3-3)^{2/3} = 1 + 0 = 1$

$(3, 1)$ vertical tangent line there

$f'(x)$:

$f'(0) = -$ | $f'(4) = +$

$f(x)$ DECR. ③ $f(x)$ INCR



5.) $f(x) = x^3 - x^2 - x + 4$ on $[0, 3]$

endpoints:

$x=0$: $f(0) = 0^3 - 0^2 - 0 + 4 \Rightarrow (0, 4)$

$x=3$: $f(3) = 3^3 - 3^2 - 3 + 4 = 27 - 9 - 3 + 4 = 19$

$\Rightarrow (3, 19)$

$f'(x) = 3x^2 - 2x - 1$

$0 = 3x^2 - 2x - 1 = (3x+1)(x-1)$

$\cancel{x = -1/3}$ outside of interval | $x = 1$

$(1, f(1)) = (1, 3)$

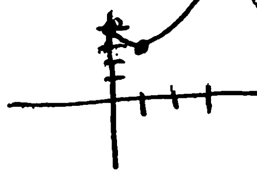
$f(1) = 1^3 - 1^2 - 1 + 4$

$f(1) = 3$

absolute max: 19 at $(3, 19)$

absolute min 3 at $(1, 3)$

(graph not necessary)



6.) $y = (2x^2 + 7x)^{1/2}$ at $(1, 3)$

$y' = \frac{1}{2}(2x^2 + 7x)^{-1/2}(4x + 7)$

$y' = \dfrac{(4x+7)}{2 \cdot \sqrt{2x^2 + 7x}}$

$m_{TAN} = y'$ when $x = 1$: $\dfrac{4(1)+7}{2\sqrt{2 \cdot 1^2 + 7(1)}} = \dfrac{11}{2 \cdot 3} = \dfrac{11}{6}$

eq: $y - 3 = \frac{11}{6}(x - 1)$

7.) 120 apartments; \$800 per month

(let x = # of increases in rent)

INCOME = $(800 + 25x) \cdot (120 - 3x)$

rent ↑ # of apartments

$I(x) = (800 + 25x)(120 - 3x)$

$I(x) = 96{,}000 + 3000x - 2400x - 75x^2$

$I(x) = 96{,}000 + 600x - 75x^2$

$I'(x) = 600 - 150x = 0$

$150x = 600$

$x = \frac{600}{150} = 4$ (increases in rent)

[max or min?
$I''(x) = -150$
$\therefore$ c. down
$\therefore$ max]

rent: $(800 + 25(4)) = 800 + 100 = \$900$

# of apts: $(120 - 3(4)) = 120 - 12 = 108$

income: $(900)(108) = \$97{,}200^{00}$

BONUS: (5 PTS)

$$f(x) = \frac{(x-3)(x^2-3x-4)}{x^2-2x-8} = \frac{(x-3)\cancel{(x-4)}(x+1)}{\cancel{(x-4)}(x+2)}$$

$(x \neq 4)$

$$f(x) = \frac{(x-3)(x+1)}{(x+2)}$$

"hole" in the graph at $x=4$

$\left(4, \frac{5}{6}\right)$

$$f(x) = \frac{x^2-2x-3}{x+2}$$

$$\frac{(x-3)(x+1)}{(x+2)} = \frac{(4-3)(4+1)}{(4+2)} = \frac{(1)(5)}{(6)} = \frac{5}{6}$$

1) VERT ASYMP:

$x = -2$

2) HORIZ ASYMP:

$$\lim_{x\to\infty} \frac{x^2-2x-3}{x+2} = \text{D.N.E.}$$

no horiz asymp

3) OBLIQUE (SLANT) ASYMP:

$$\begin{array}{r} x-4+\frac{5}{x+2} \\ x+2 \overline{\smash{)}\, x^2-2x-3} \\ -(x^2+2x) \\ \hline -4x-3 \\ -(-4x-8) \\ \hline 5 \end{array}$$

$y = x-4$

4) int:

at $x=0$

$$\frac{0^2-2(0)-3}{0+2}$$

$\left(0, -\frac{3}{2}\right)$

at $y=0$

$$0 = \frac{(x-3)(x+1)}{x+2}$$

$x = 3$ or $x = -1$

$(-1, 0)$ & $(3, 0)$