MA 121 - 002

Tuesday, March 25

- today 4.5 (integr by subst); TEST REVIEW
- Thurs, March 27: TEST #3
  (3.1; 3.2; 3.3; 3.4; 3.5; 4.1; 4.2, 4.3; 4.4)

4.5: INTEGRATION USING SUBSTITUTION

(change of variable method)

① $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$

$(n \neq -1)$

$$\int u^n\,du = \frac{u^{n+1}}{n+1} + C$$

$(n \neq -1)$

② $\int x^{-1}\,dx = \int \frac{1}{x}\,dx = \ln|x| + C$

$$\int u^{-1}\,du = \int \frac{1}{u}\,du = \ln|u| + C$$

→ ③ $\int e^x\,dx = e^x + C$

$$\rightarrow \int e^u\,du = e^u + C$$

ex:

$$\frac{1}{12}\int (3t^4+2)^5 \cdot \boxed{t^3 \cdot dt\,(12)}$$

let $u = 3t^4+2$

$$\cancel{dt}\cdot\frac{du}{\cancel{dt}} = 12t^3 \cdot dt$$

$$du = 12t^3 \cdot dt$$

$$\frac{1}{12}\int (u^5) \cdot du = \frac{1}{12}\,\frac{u^6}{6} + C$$

$$= \frac{1}{72}(3t^4+2)^6 + C$$

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ex:

$$\frac{1}{8}\int \frac{1}{2+8x}\,\boxed{dx \cdot 8}$$

$\int \frac{1}{x}\,dx$

$\int \frac{1}{u}\cdot du$

let $u = 2+8x$

$$\cancel{dx}\,\frac{du}{\cancel{dx}} = 8 \cdot dx$$

$$du = 8 \cdot dx$$

$$\frac{1}{8}\int \frac{1}{u}\,du = \frac{1}{8}\ln|u| + C$$

$$= \frac{1}{8}\ln|2+8x| + C$$

ex:

$$\int x \sqrt[5]{1-x^2}\, dx$$

$$\int u^n\,du = \frac{u^{n+1}}{n+1} + C$$

$$-\frac{1}{2}\int \boxed{(1-x^2)^{1/5}} \cdot \boxed{x \cdot dx\,(-2)}$$

let $u = 1-x^2$

$du = -2x\,dx$

$$-\frac{1}{2}\int u^{1/5} \cdot du = -\frac{1}{2}\,\frac{u^{6/5}}{6/5} + C$$

$$= -\frac{1}{2}\cdot\frac{5}{6}(1-x^2)^{6/5} + C$$

$$= \frac{-5}{12}(1-x^2)^{6/5} + C$$

ex:

$$\frac{1}{2}\int x \cdot \boxed{e^{x^2}}\, dx\,(2)$$

let $u = x^2$

$du = 2x\,dx$

$$= \frac{1}{2}\int e^u \cdot du = \frac{1}{2}e^u + C$$

$$= \frac{1}{2}e^{x^2} + C$$

ex:

$$\int (\ln x)^7 \cdot \frac{1}{x} \cdot dx$$

let $u = \ln x$

$du = \frac{1}{x}\, dx$

$$\int u^7\, du = \frac{u^8}{8} + C$$

$$= \frac{(\ln x)^8}{8} + C$$

ex: DEF. INTEGRAL (w/SUBST.)

$$\int_{x=1}^{x=4} \frac{2x+1}{x^2+x+1}\, dx$$

① if $x=1$, $u = x^2+x+1$

$u = 3$

② if $x=4$, $u = x^2+x+1$

$u=21$

let $\boxed{u = x^2+x+1}$

$du = \boxed{(2x+1)}\, dx$

$$\int_{\cancel{1}}^{\cancel{4}} \frac{du}{u} = \int_{\cancel{1}\,\boxed{3}}^{\cancel{4}\,\boxed{21}} \frac{1}{u}\, du$$

$$\int_3^{21} \frac{1}{u}\,du = \ln|u| \Big]_3^{21}$$

$$= \ln|21| - \ln|3|$$

$$= \ln 21 - \ln 3$$

$$= \ln\frac{21}{3} = \ln 7 \approx$$

<u>or</u>

$$\int \frac{1}{u}\,du = \ln|u| \Big]$$

$$= \ln|x^2+x+1| \Big]_1^4$$

$$= \ln|4^2+4+1| - \ln|1^2+1+1|$$

$$= \ln(21) - \ln(3)$$

$$= \ln\frac{21}{3} = \ln 7$$