

Thursday, March 21

TODAY:

• 4.3 (continued)

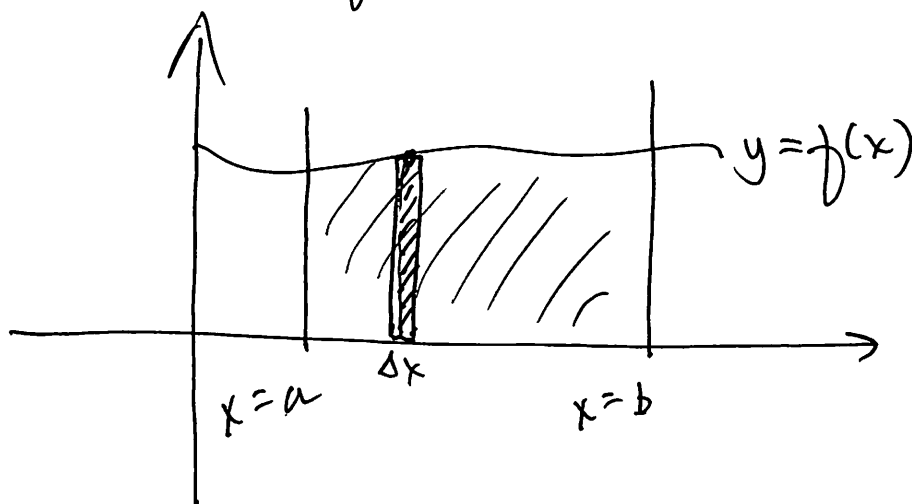
• 4.4 (AREA BETWEEN TWO CURVES) (AVE VALUE OF A FUNCTION)

TEST #3 (THURS, MARCH 28)

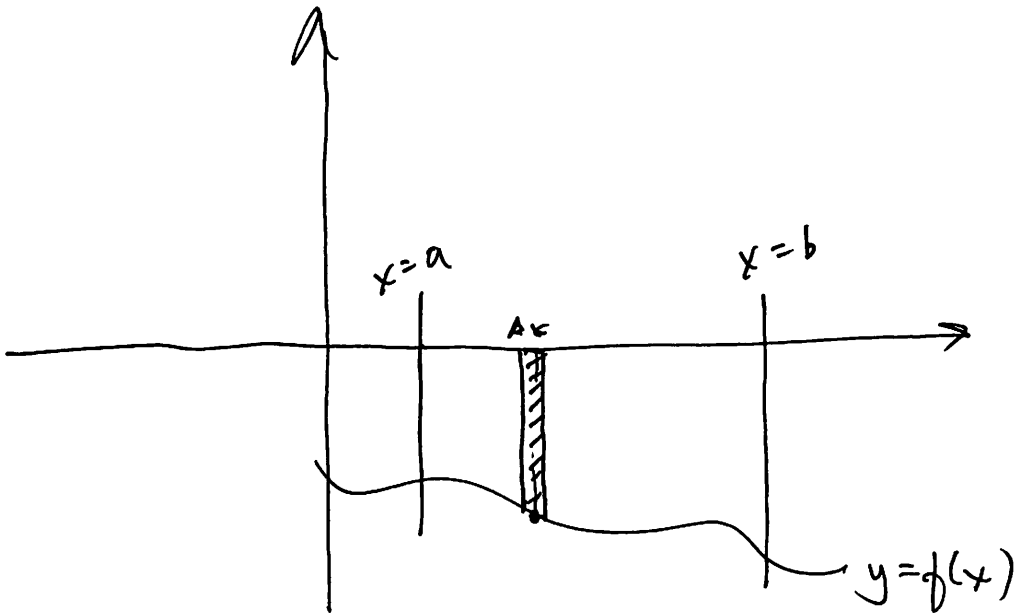
4.3:

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n \underbrace{f(x_i)}_{\text{HT.}} \cdot \underbrace{\Delta x}_{\text{WIDTH.}} = \text{AREA} = \int_a^b f(x) dx$$

$$\left[\begin{array}{c} \text{where} \\ F'(x) = f(x) \end{array} \right] = F(x) \Big|_a^b = F(b) - F(a)$$



$$\int_a^b f(x) dx = +$$



$$\int_a^b f(x) dx$$

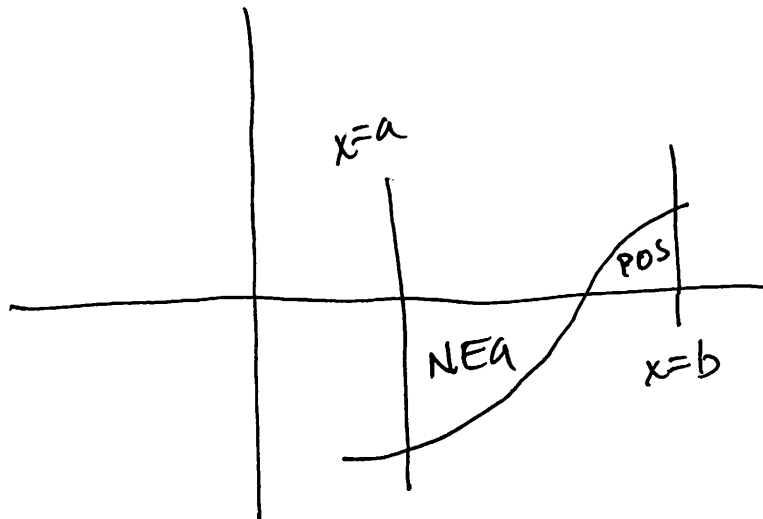
- +

$$= \underline{\underline{NEG}}$$



$$Area = \underline{\underline{NET}}$$

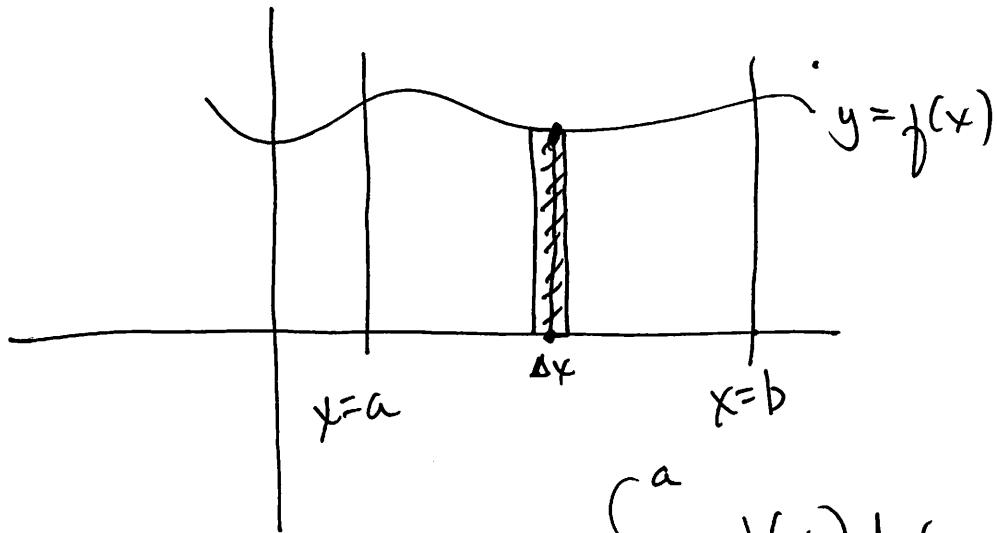
$Area$



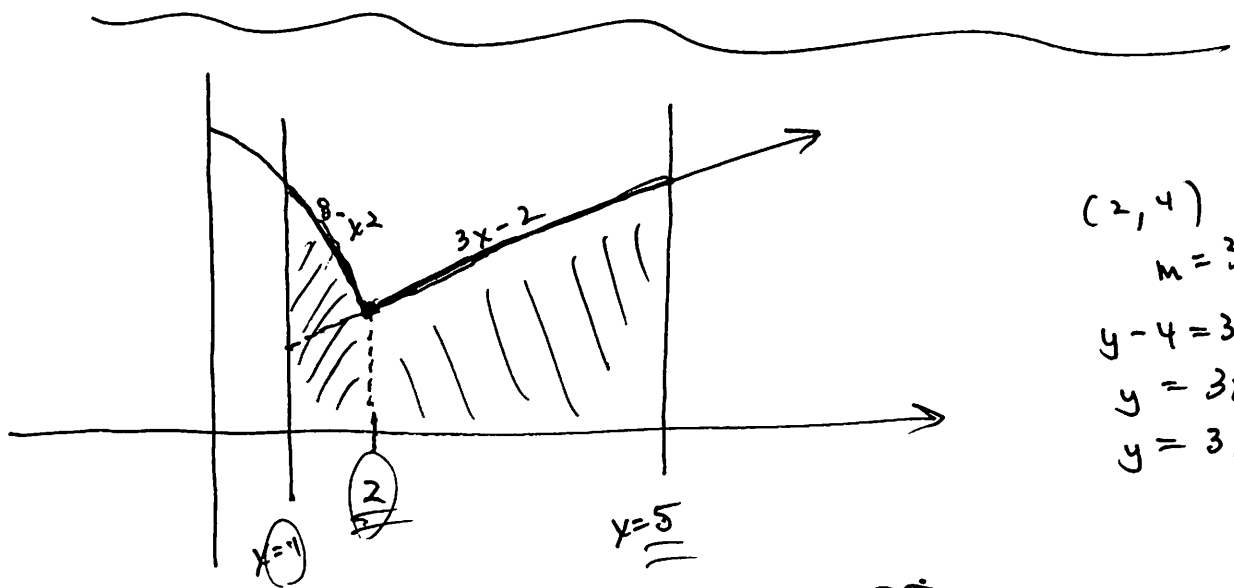
$$\underline{\underline{NET}}$$

$Area$

(3)



$$\int_a^b f(x) dx = \text{NEGA}$$



$$(2, 4)$$

$$m = 3$$

$$y - 4 = 3(x - 2)$$

$$y = 3x - 6 + 4$$

$$y = 3x - 2$$

$$\int_1^5 f(x) dx = A = \int_1^2 f(x) dx + \int_2^5 f(x) dx$$

$$\text{ex: } f(x) = \begin{cases} 8 - x^2, & 1 \leq x \leq 2 \\ 3x - 2, & 2 \leq x \leq 5 \end{cases}$$

$$A = \int_1^2 (8 - x^2) dx + \int_2^5 (3x - 2) dx$$

(4)

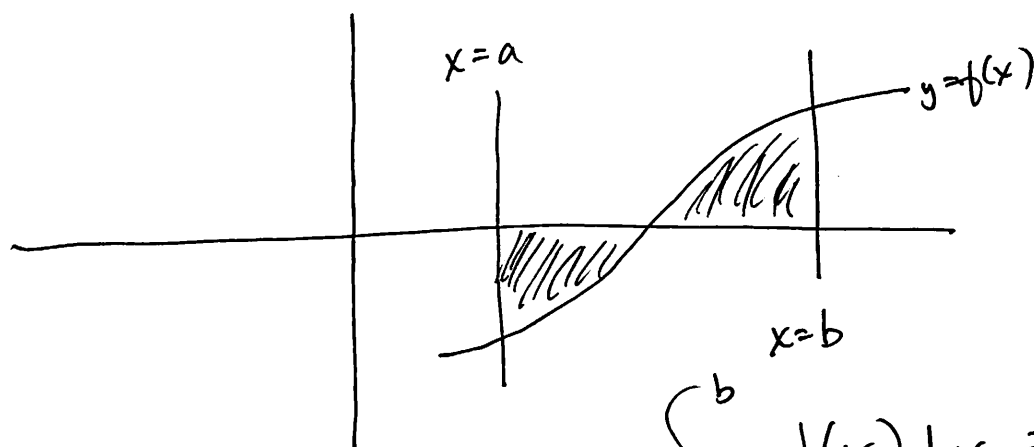
$$A = \int_1^2 (\underline{8} - \underline{x^2}) \underline{dx} + \int_2^5 (\underline{3x} - \underline{2}) \underline{dx}$$

$$A = \left[8x - \frac{x^3}{3} \right]_1^2 + \left[3 \cdot \frac{x^2}{2} - 2x \right]_2^5$$

$$A = \left[\left(8 \cdot 2 - \frac{2^3}{3} \right) - \left(8 \cdot 1 - \frac{1^3}{3} \right) \right] + \left[\left(\frac{3}{2}(5)^2 - 2(5) \right) - \left(\frac{3}{2}(2)^2 - 2(2) \right) \right]$$

$$A = (16) - \frac{8}{3} (-8) + \frac{1}{3} + \frac{75}{2} (10) - (6) (+4)$$

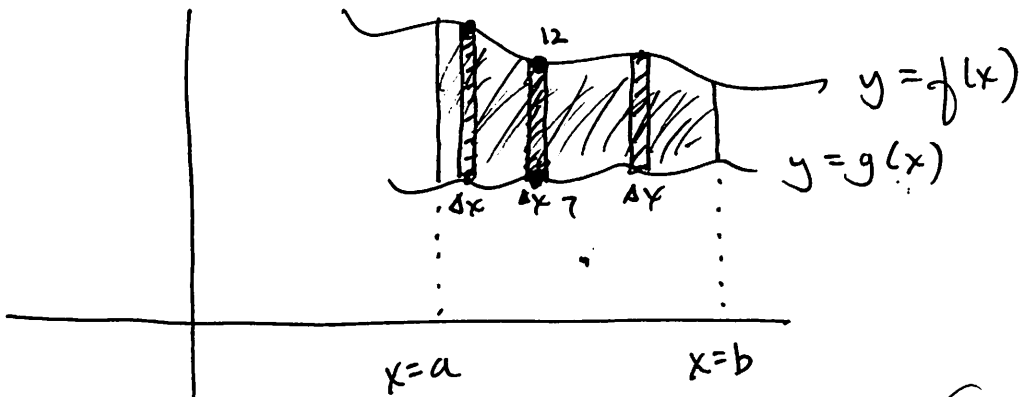
$$A = \text{_____ sq. units}$$



$$\int_a^b f(x) dx = 0$$

4.4:

AREA BETWEEN TWO CURVES



(1) $A = \int_a^b f(x) dx - \int_a^b g(x) dx$

Below the equations are two small diagrams illustrating the area elements. The first diagram shows a shaded rectangle with width Δx and height $f(x) - g(x)$. The second diagram shows a shaded rectangle with width Δx and height $f(x)$.

The final formula is boxed:

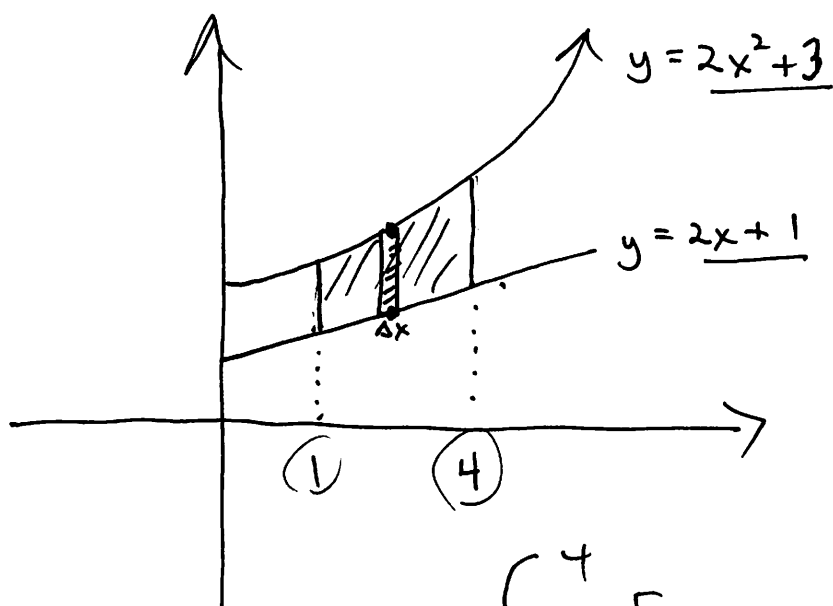
$$A = \int_a^b [f(x) - g(x)] dx$$

(2) $A = \int_a^b \underbrace{(f(x) - g(x))}_{\text{H.T.}} \underbrace{dx}_{\text{WIDTH.}}$

Annotations for the formula:

- $f(x)$: y-value on UPPER CURVE
- $g(x)$: y-value on LOWER CURVE
- $H.T.$: Height (Height of Trapezium)
- $WIDTH.$: Width

(6)



$$A = \int_1^4 \left[(2x^2 + 3) - (2x + 1) \right] dx$$

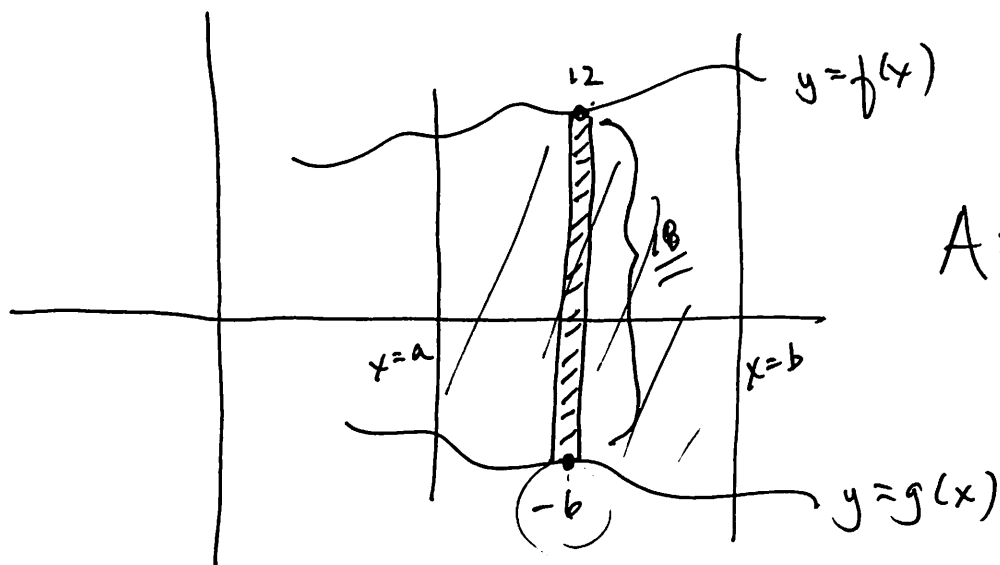
$$A = \int_1^4 (2x^2 - 2x + 2) dx$$

$$A = \left[\frac{2x^3}{3} - \cancel{2} \frac{x^2}{\cancel{x}} + 2x \right]_1^4$$

$$A = \left[\frac{2}{3}(4)^3 - (4)^2 + 2(4) \right] - \left[\frac{2}{3}(1)^3 - (1)^2 + 2(1) \right]$$

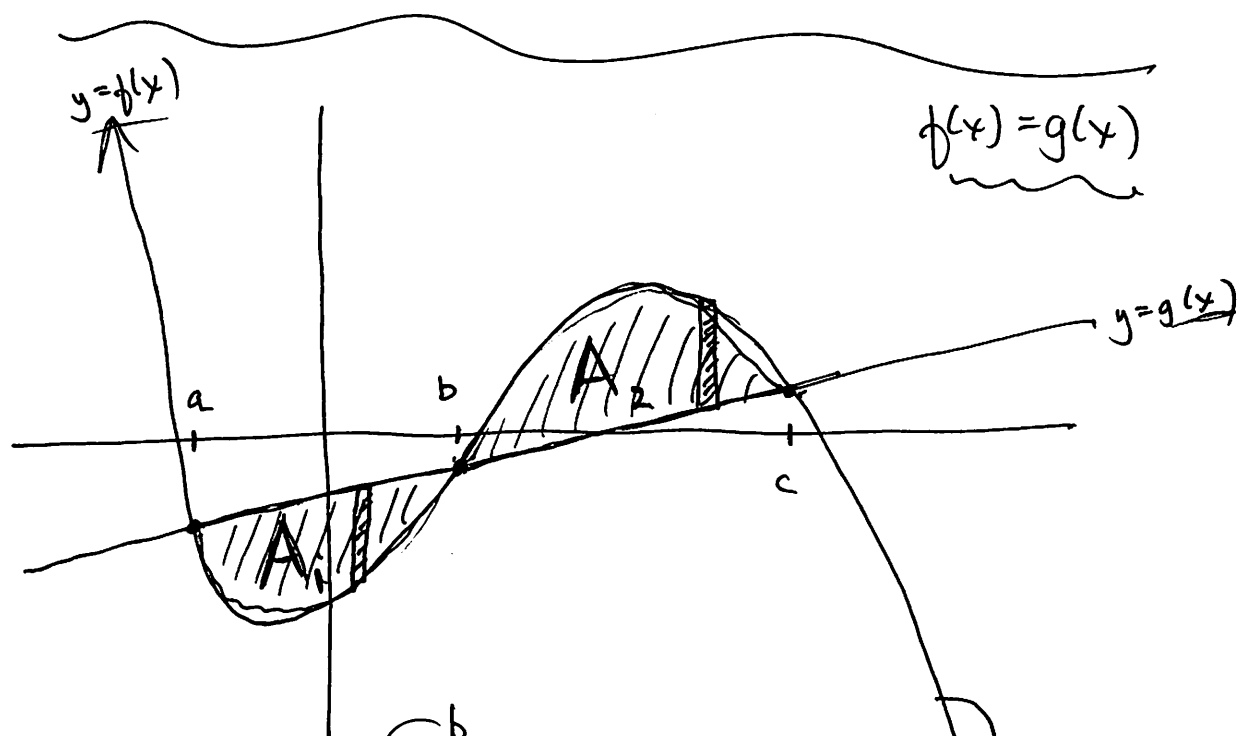
$$A = \underline{\hspace{2cm}}$$

7



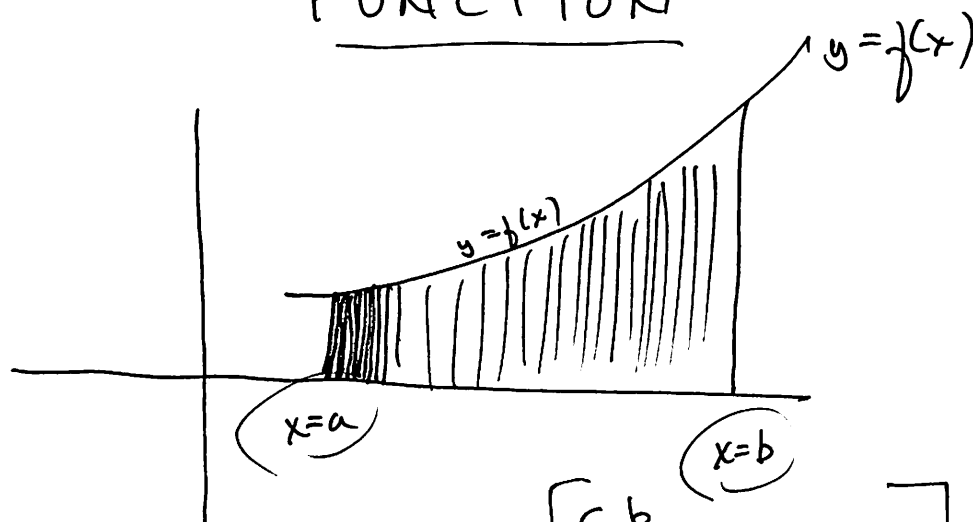
$$A = \int_a^b [f(x) - g(x)] dx$$

$\uparrow$
 $12 - (-6)$



$$\left. \begin{aligned} A_1 &= \int_a^b [g(x) - f(x)] dx \\ A_2 &= \int_b^c [f(x) - g(x)] dx \end{aligned} \right\} A = A_1 + A_2$$

4.4: AVERAGE VALUE OF A FUNCTION



(AVE. Y-VALUE)
(AVE. HEIGHT)

WIDTH: $b-a$

$$\frac{\text{AREA}}{\text{WIDTH}} = \text{AVE HT}$$

$$\frac{\left[\int_a^b f(x) dx \right]}{b-a}$$

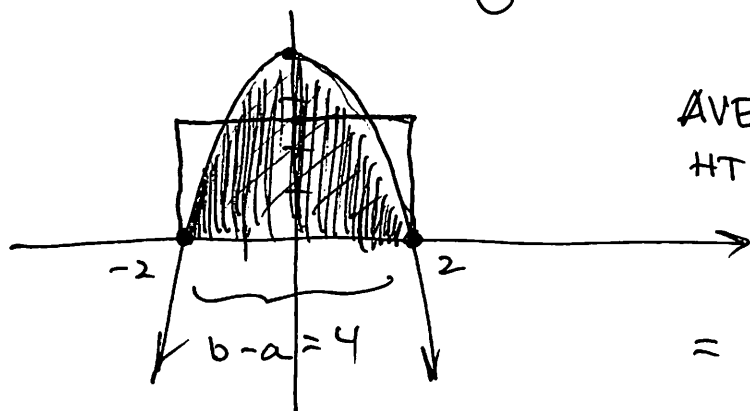
$$= \text{AVE HT}$$

(9)

ex

$$\text{AVE.} = \frac{1}{b-a} \int_a^b f(x) dx$$

$y = 4 - x^2$ find the average value of the function on $[-2, 2]$



$$\text{AVE} = \frac{1}{2 - (-2)} \int_{-2}^2 (4 - x^2) dx$$

$$= \frac{1}{4} \left[4x - \frac{x^3}{3} \right]_{-2}^2$$

$$= \frac{1}{4} \left[\left(4 \cdot 2 - \frac{2^3}{3} \right) - \left(4 \cdot (-2) - \frac{(-2)^3}{3} \right) \right]$$

$$= \frac{1}{4} \left[\left(8 - \frac{8}{3} + 8 - \frac{8}{3} \right) \right]$$

$$= \frac{1}{4} \left[16 - \frac{16}{3} \right] = \frac{1}{4} \left[\frac{48}{3} - \frac{16}{3} \right]$$

$$= \frac{1}{4} \left[\frac{32}{3} \right] = \frac{8}{3}$$