MA121-002

Thursday, March 21

TODAY:

- 4.3 (continued)
- 4.4 (AREA BETWEEN TWO CURVES)(AVE VALUE OF A FUNCTION)

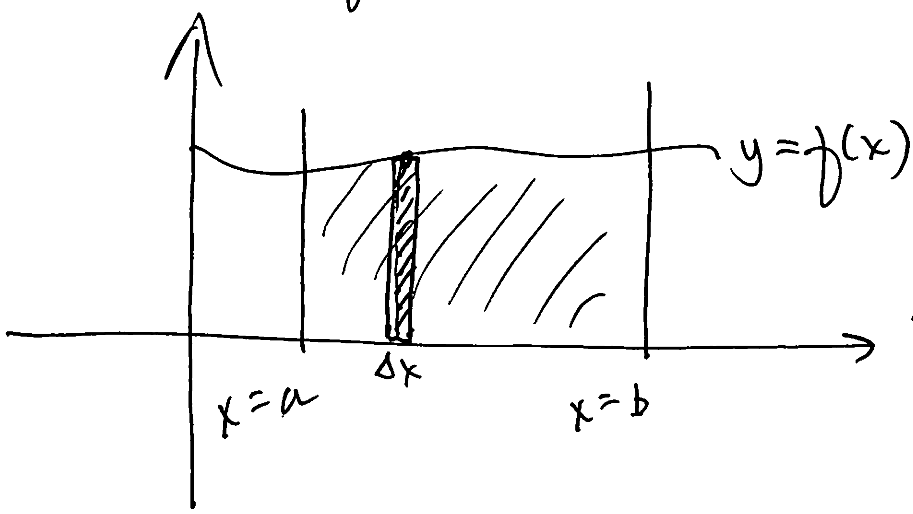
TEST #3 (THURS, MARCH 28)

4.3:

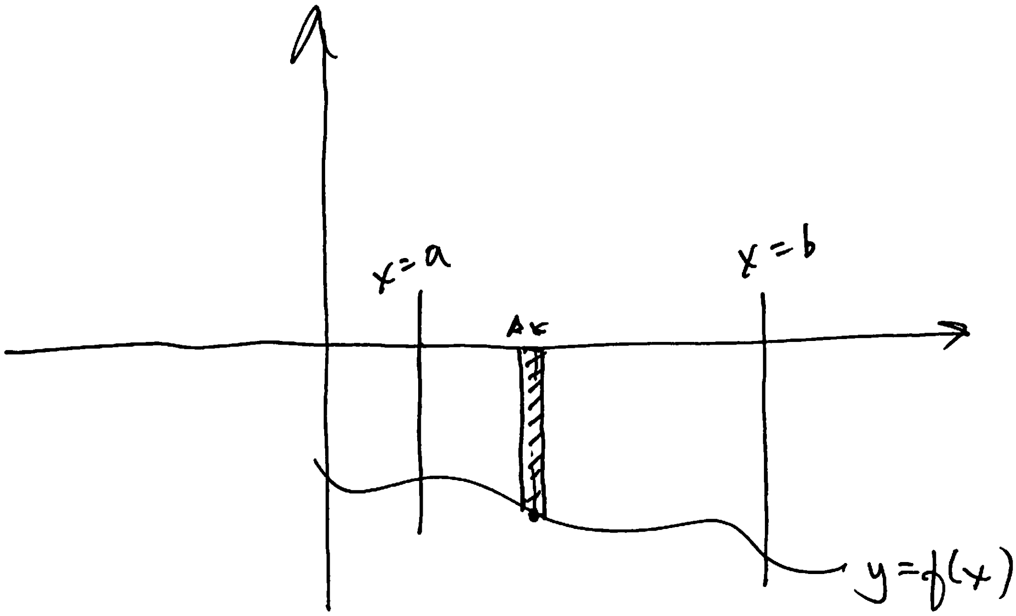
$$\lim_{n\to\infty}\sum_{i=1}^{n} \underbrace{f(x_i)}_{\text{HT.}} \cdot \underbrace{\Delta x}_{\text{WDTH.}} = \text{AREA} = \int_a^b f(x)\,dx$$

$$= F(x)\Big]_a^b = F(b) - F(a)$$

[where $F'(x) = f(x)$]



$$\int_a^b \underbrace{f(x)}_{+}\,\underbrace{dx}_{+} = +$$

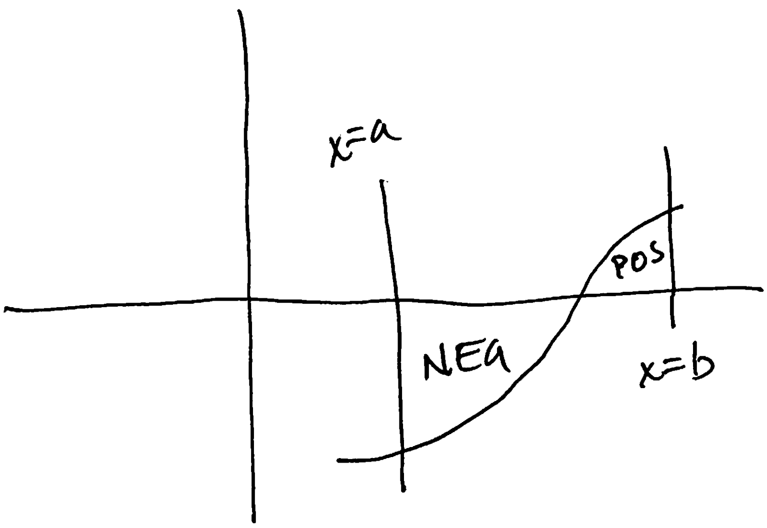


$$\int_a^b \underbrace{f(x)}_{-}\,\underbrace{dx}_{+}$$

$$= \underline{\underline{\text{NEG}}}$$

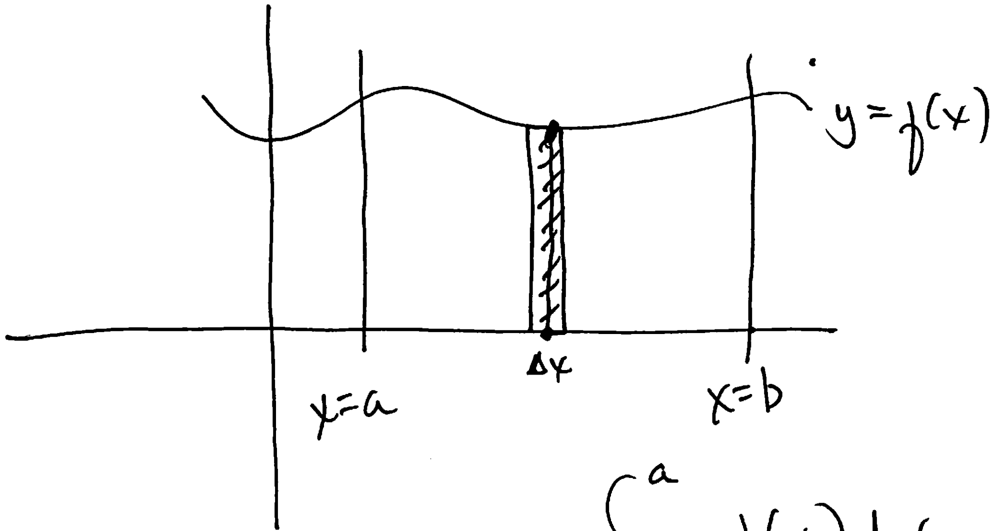
AREA = <u>NET</u>

~~AREA~~

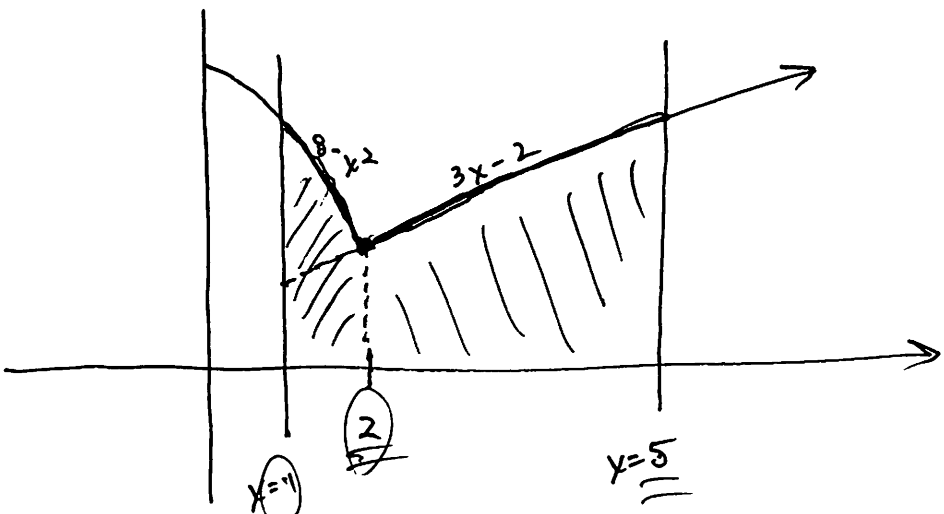


<u>NET</u>

~~AREA~~



$$\int_b^a \underbrace{f(x)}_{+}\,\underbrace{dx}_{-} = \underline{NEG}$$



(2,4)
$m = 3$
$y - 4 = 3(x-2)$
$y = 3x - 6 + 4$
$y = 3x - 2$

$$\int_1^5 f(x)\,dx = A = \int_1^2 f(x)\,dx + \int_2^5 f(x)\,dx$$

ex: $$f(x) = \begin{cases} 8 - x^2, & 1 \le x \le 2 \\ 3x - 2, & 2 \le x \le 5 \end{cases}$$

$$A = \int_1^2 (8 - x^2)\,dx + \int_2^5 (3x - 2)\,dx$$

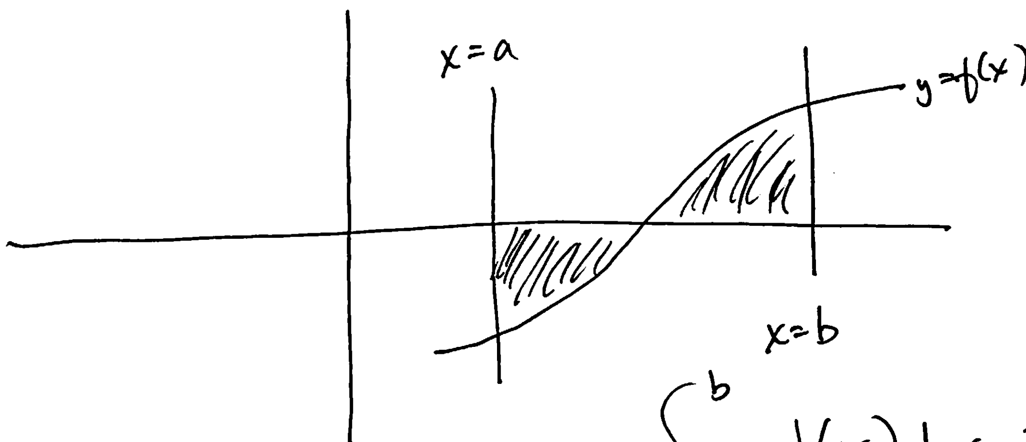
$$A = \int_1^2 (8 - x^2)\,dx + \int_2^5 (3x - 2)\,dx$$

$$A = \left[8x - \frac{x^3}{3}\right]_1^2 + \left[3 \cdot \frac{x^2}{2} - 2x\right]_2^5$$

$$A = \left[\left(8 \cdot 2 - \frac{2^3}{3}\right) - \left(8 \cdot 1 - \frac{1^3}{3}\right)\right] + \left[\left(\frac{3}{2}(5)^2 - 2(5)\right) - \left(\frac{3}{2}(2)^2 - 2(2)\right)\right]$$

$$A = 16 - \frac{8}{3} - 8 + \frac{1}{3} + \frac{75}{2} - 10 - 6 + 4$$

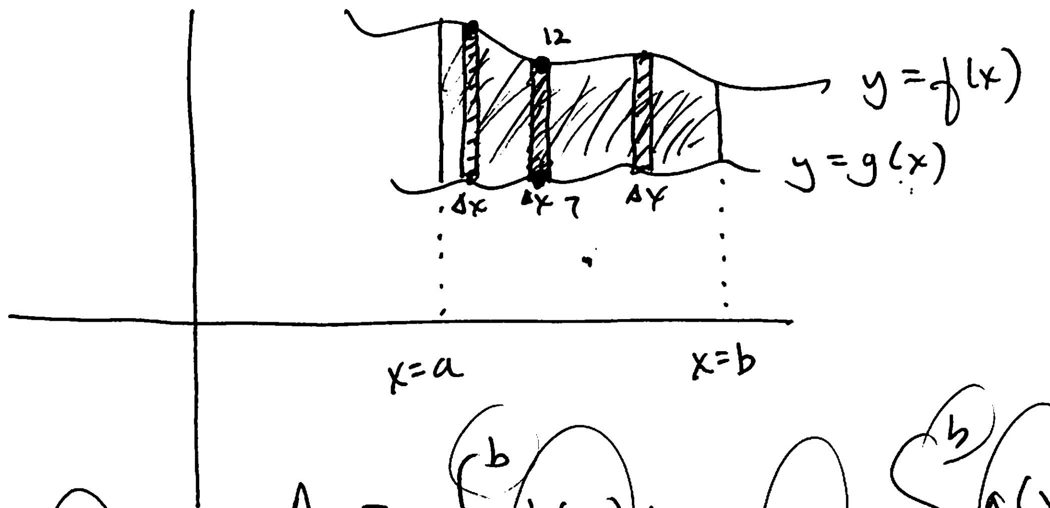
$$A = \underline{\qquad\qquad} \text{ sq. units}$$



$$\int_a^b f(x)\,dx = 0$$

4.4:

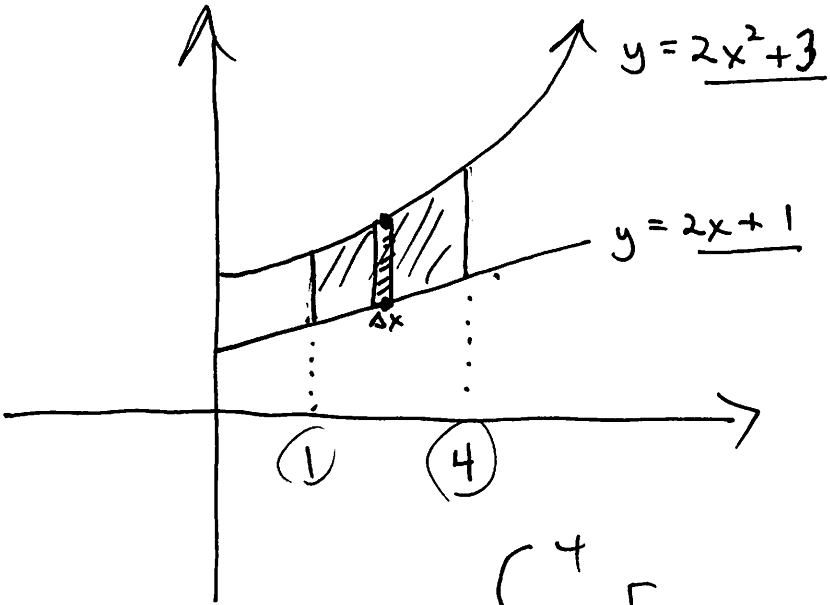
AREA BETWEEN TWO CURVES



(1) $$A = \int_a^b f(x)\,dx - \int_a^b g(x)\,dx$$

$$\boxed{A = \int_a^b \left[f(x) - g(x)\right] dx}$$

(2) $$\boxed{A = \int_a^b \underbrace{(\overset{\text{y-value on UPPER CURVE}}{f(x)} - \overset{\text{y-value on LOWER CURVE}}{g(x)})}_{\text{HT.}} \underbrace{dx}_{\text{WDTH.}}}$$



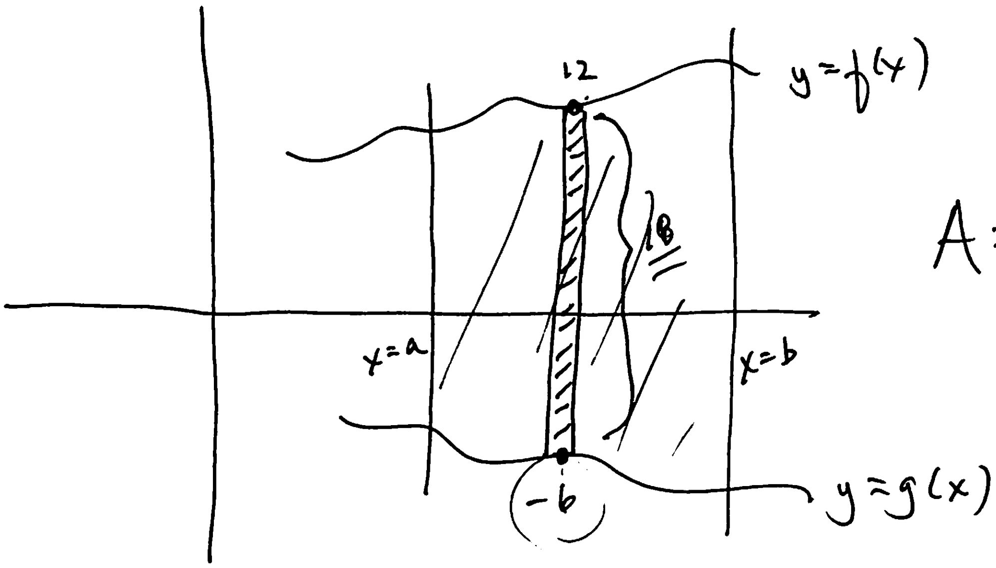
$$A = \int_1^4 \left[(2x^2+3) - (2x+1) \right] dx$$

$$A = \int_1^4 (2x^2 - 2x + 2)\, dx$$

$$A = \left[2\frac{x^3}{3} - \cancel{2}\frac{x^2}{\cancel{2}} + 2x \right]_1^4$$

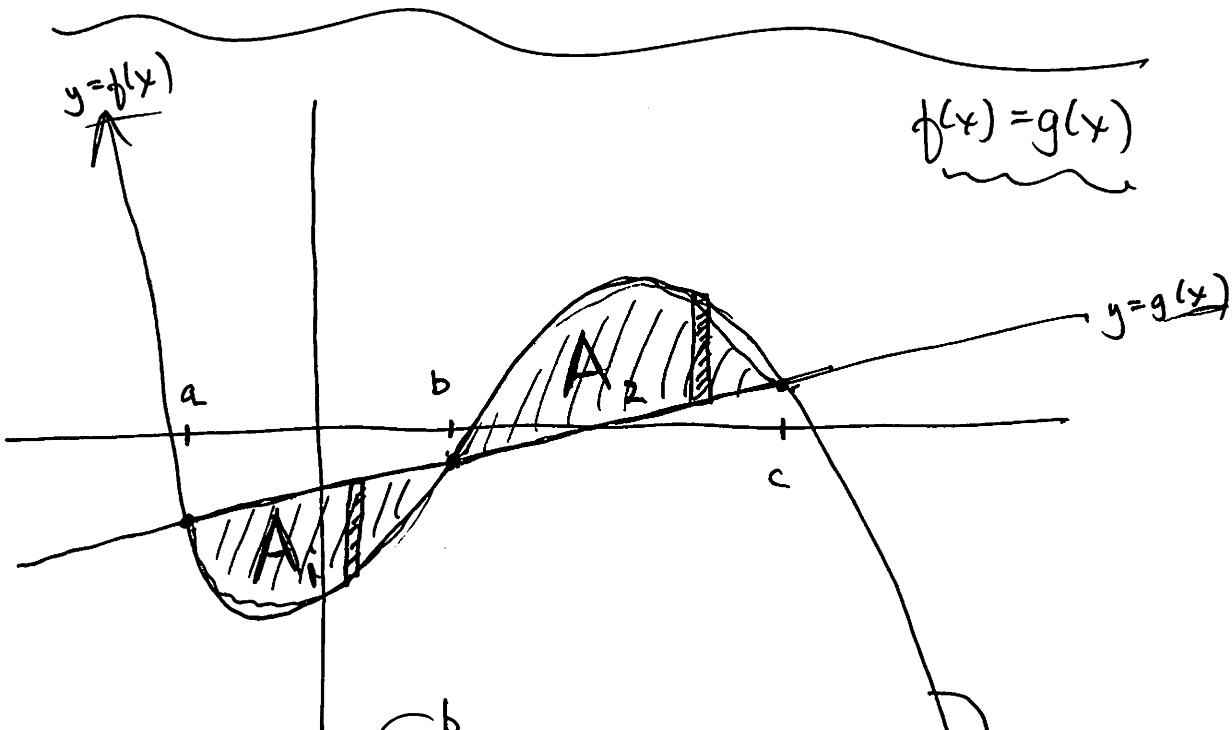
$$A = \left[\frac{2}{3}(4)^3 - (4)^2 + 2(4) \right] - \left[\frac{2}{3}(1)^3 - (1)^2 + 2(1) \right]$$

$$A = \underline{\qquad\qquad}$$



$$A = \int_a^b \left[f(x) - g(x) \right] dx$$

$12 - (-6)$



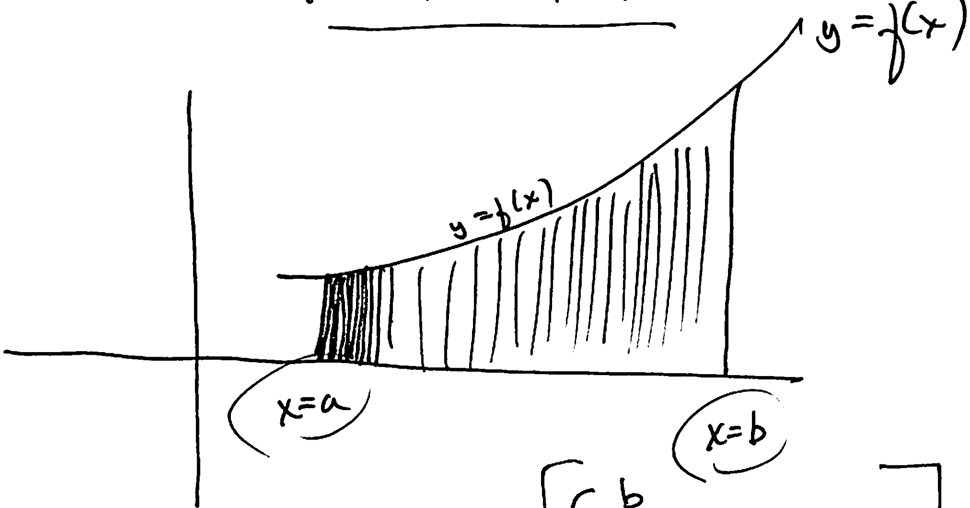
$f(x) = g(x)$

$$A_1 = \int_a^b \left[g(x) - f(x) \right] dx$$

$$A_2 = \int_b^c \left[f(x) - g(x) \right] dx$$

$A = A_1 + A_2$

4.4: AVERAGE VALUE OF A FUNCTION



(AVE. Y-VALUE)

(AVE. HEIGHT)

WIDTH: b−a

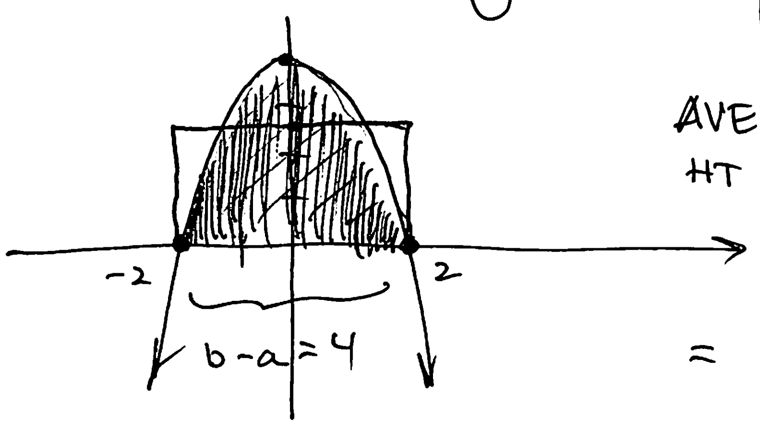
$$\frac{\text{AREA}}{\text{WIDTH}} = \text{AVE HT}$$

$$\frac{\left[\displaystyle\int_a^b f(x)\,dx\right]}{\underbrace{b-a}} = \text{AVE HT}$$

ex

$$\text{AVE. HT} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

$y = 4 - x^2$ find the average value of the function on $[-2, 2]$



$$\text{AVE HT} = \frac{1}{2-(-2)}\int_{-2}^{2}(4-x^2)\,dx$$

$$= \frac{1}{4}\left[4x - \frac{x^3}{3}\right]_{-2}^{2}$$

$$= \frac{1}{4}\left[\left(4\cdot 2 - \frac{2^3}{3}\right) - \left(4\cdot(-2) - \frac{(-2)^3}{3}\right)\right]$$

$$= \frac{1}{4}\left[\left(8 - \frac{8}{3} + 8 - \frac{8}{3}\right)\right]$$

$$= \frac{1}{4}\left[16 - \frac{16}{3}\right] = \frac{1}{4}\left[\frac{48}{3} - \frac{16}{3}\right]$$

$$= \frac{1}{4}\left[\frac{32}{3}\right] = \frac{8}{3}$$