Thursday, March 7

4.1: ANTIDERIVATIVES
(integrals)

indefinite integral

with respect to x

$$\int (2x+1)\,dx = x^2+x+C$$

$$d(\;??\;) = 2x+1$$

$$d(x^2+x) \overset{??}{=} 2x+1$$

$$d(x^2+x+7) \overset{??}{=} 2x+1$$

$$d(x^2+x-13) \overset{??}{=} 2x+1$$

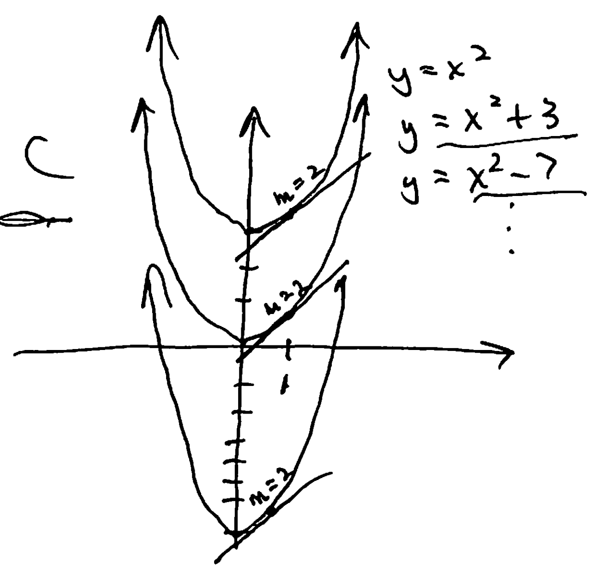
x^2+x ; x^2+x+7 ; x^2+x-13 ; . . . . ; x^2+x+C

family of curves

$$\int (2x)\cdot dx = x^2+C$$

$$d(x^2+c) = 2x$$

M-TAN



$$\int 2 \cdot dx = 2x + C$$

$$\int 2 \cdot dt = 2t + C$$

$$\int 2 \cdot dr = 2r + C$$

$$\int (2x + 1)\, dx = x^2 + x^1 + C$$

DERIV: $x^2 \xrightarrow{\text{DER.}} 2x^1$

$$\int (K)\, dx = Kx + C$$

$$\int (x^3 + x^2 + x)\, dx$$

$$= \frac{x^4}{4} + \frac{x^3}{3} + \frac{x^2}{2} + C$$

$$d(x^4 + x^3 + x^2 + C) \overset{??}{=} 4x^3 + 3x^2 + 2x$$

$$d\left(\frac{1}{4}x^4 + \frac{1}{3}x^3 + \frac{1}{2}x^2 + C\right) = \cancel{\tfrac{1}{4}}(\cancel{4}x^3) + \cancel{\tfrac{1}{3}}(\cancel{3}x^2) + \cancel{\tfrac{1}{2}} \cdot (\cancel{2}x)$$

integrand

$$\int (x^3 + x^2 + x^1)\,dx$$

$$= \frac{x^4}{4} + \frac{x^3}{3} + \frac{x^2}{2} + C$$

$$\int x^r\,dx = \frac{x^{r+1}}{r+1} + C$$

$(r \neq -1)$

check: (using deriv.)

$$d\left(\frac{x^4}{4} + \frac{x^3}{3} + \frac{x^2}{2} + c\right) \stackrel{?}{=} x^3 + x^2 + x$$

* $$\boxed{\int a \cdot x^r \cdot dx = a \cdot \frac{x^{r+1}}{r+1} + C}$$ *

$(r=-1)$ general power rule

* $$\boxed{\int x^{-1}\,dx = \int \left(\frac{1}{x}\right) dx = \ln|x| + C}$$

$$\int \left(13x^3 - 25x^2 + \pi x - \frac{5}{x} + 4x^{3/5} - 7x^{-1/3} + 8x^0\right)$$

$$= 13 \cdot \frac{x^4}{4} - 25\frac{x^3}{3} + \pi \cdot \frac{x^2}{2} - 5 \cdot \ln|x|$$

$$+ 4 \cdot \frac{x^{8/5}}{8/5} - 7\frac{x^{2/3}}{2/3} + 8x$$

$$+ C$$

$$\int \frac{5}{x}\,dx = \int 5 \cdot \frac{1}{x}\,dx$$

check: $d\left(\frac{13}{4}x^4\right) \overset{??}{=} \frac{13}{\cancel{4}} \cdot \cancel{4}x^3$

check: $-7x^{2/3} \cdot \frac{3}{2} = -\frac{21}{2}x^{2/3}$

$$d\left(-\frac{21}{2}x^{2/3}\right) = \boxed{-\frac{21}{\cancel{2}} \cdot \frac{\cancel{2}}{3}} x^{-1/3} = -7x^{-1/3}$$

$$\int e^x\,dx = e^x + C$$

$$\int 5e^x\,dx = 5 \cdot e^x + C$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax} + C$$

$$d\left(e^{8x}\right) = e^{8x} \cdot 8$$

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$$\int e^{5x}\,dx = \frac{1}{5}e^{5x} + C$$

$$\left[\int e^{ax}\,dx = \frac{1}{a}\cdot e^{ax} + C\right]$$

check: $d\left(\frac{1}{5}\cdot e^{5x} + C\right) \stackrel{??}{=}$

$$= \cancel{\frac{1}{5}}\cdot e^{5x}\cdot\cancel{5} + 0$$

$$\int 14{,}805\, e^{-.1057t}\,dt$$

$$= 14{,}805\cdot\frac{1}{-.1057}e^{\boxed{-.1057}t} + C$$