Tuesday, March 5

- this week (3.4; 3.5; 4.1)
- interview quiz (Q2) due March 19

today: 3.4: exponential decay
(standard decay; Newton's law of cooling)

BEGIN: 3.5 deriv of $y = a^x$ and $y = \log_a x$

3.4: exponential decay

$$y = y_0 \cdot e^{kt} \qquad (A = P \cdot e^{rt})$$

(k is NEG)

ex: half-life; carbon dating

C_{14} 5730 years

$$y = y_0 \cdot e^{kt}$$

$$\frac{\frac{1}{2}y_0}{y_0} = \frac{y_0 \cdot e^{k(5730)}}{y_0}$$

$$\ln e^{5730k} = 5730k$$

$$\frac{1}{2} = e^{5730k} \quad \text{(solve for k)}$$

$$\frac{\ln(1/2)}{5730} = \frac{5730k}{5730} \qquad k \approx -.0001209$$

40. **Cooling.** After warming the water in a hot tub to 100°, the heating element fails. The surrounding air temperature is 40°, and in 5 min the water temperature drops to 95°.

a) Find the value of the constant a in Newton's Law of Cooling. 60

b) Find the value of the constant k. Round to five decimal places. 0.01740

c) What is the water temperature after 10 min? 90°

d) How long does it take the water to cool to 41°? 235 min

e) Find the rate of change of the water temperature, and interpret its meaning.

EXAMPLE 3 **Life Science: Carbon Dating.** The radioactive element carbon-14 has a half-life of 5730 yr. The percentage of carbon-14 present in the remains of plants and animals is used to determine age. Archaeologists found that the linen wrapping from one of the Dead Sea Scrolls had lost 22.3% of its carbon-14. How old was the linen wrapping?

$$y = y_0 \cdot e^{(-.000120968)t}$$

$$77.7\% = 100\% \cdot e^{-.000120968t}$$

(lost 22.3% of C_{14})

$$.777 = e^{-.000120968t} \quad \text{(solve for t)}$$

$$\frac{\ln(.777)}{-.000120968} = \frac{-.000120968\,t}{-.000120968}$$

$$2085 \text{ yrs} \approx t$$

NEWTON'S LAW OF COOLING:

rate at which the object cools is directly proportional to the difference in temp. between the object and the surrounding medium.

(T = temp of object)

$$\frac{dT}{dt} = k \cdot (T - M) \quad \star$$

$$\frac{dP}{dt} = k \cdot P \qquad \frac{dT}{dt} = k \cdot T$$

$$T = a \cdot e^{kt} + m$$ ✓

T = temp. of object

k = rate at which it cools

t = time

m = surrounding med.

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① $t=0$ $T=100^\circ$ $m=40^\circ$

$t=5$ min $T=95^\circ$

$t=10$ min $T=??$

$$T = a \cdot e^{kt} + m$$

$$100 = a \cdot e^{k(0)} + 40$$

$$100 = a + 40$$

$$-40 \quad -40$$

$$60 = a$$

$$T = 60 \cdot e^{kt} + 40$$

② $$95 = 60 \cdot e^{k(5)} + 40$$

$$-40 \quad -40$$

$$\frac{55}{60} = \frac{60 \cdot e^{5k}}{60} \qquad \frac{55}{60} = e^{5k}$$

$$\frac{55}{60} = e^{5k}$$

$$\frac{\ln\left(\frac{55}{60}\right)}{5} = \frac{5k}{5} \qquad k \approx -.0174$$

$$T = 60 \cdot e^{-.0174t} + 40$$

$T = ??$ $\quad t = 10$

$$T = 60 \cdot e^{-.0174(10)} + 40$$

$$T \approx 90.42^{\circ}$$

$T = 41$ $\quad t = ??$

$$41 = 60 \cdot e^{-.0174(t)} + 40$$

$$-40 \qquad\qquad -40$$

$$\frac{1}{60} = \frac{60 \cdot e^{-.0174t}}{60}$$

$$\frac{1}{60} = e^{-.0174t}$$

$$\frac{\ln(1/60)}{-.0174} = \frac{-.0174t}{-.0174}$$

$$t \approx 235 \text{ min}$$

3.5:

$y = a^x$ & $y = \log_a x$

ex: $y = 2^x$, $y = 5^x$, $y = (17.1)^x$

ex: $y = \log_8 x$; $y = \log_2 x$, . . .

$\ln 8 = 2.08$ ← the power you would raise "e" to get 8

$(e^{2.08} = 8)$

$e^{\ln 8} = 8$

$e^{\ln 5} = 5$

$e^{\ln 103} = 103$

$e^{\ln a^x} = a^x$

$\ln 2^{(3)} = 3 \cdot \ln 2$

$y = a^x$

(rewrite ...)

$y = e^{\ln a^x} = e^{x \cdot \ln a}$

find $y' = ??$

$y = e^{x \cdot \ln a}$

$y' = e^{x \cdot \ln a} \cdot d(x \cdot \ln a)$

$y' = a^x \cdot \ln a$

$y = a^x$

$y' = a^x \cdot \ln a$

ex: $y = 2^x$

$y' = 2^x \cdot \ln 2$

$a = 5, 10$

$x \cdot \ln 5$

$x = \ln 10$

$y = 5^{8x^2 - x}$ $\quad y' = 5^{8x^2-x} \cdot (\ln 5)(16x - 1)$

↑ chain rule

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yes $\quad f(x) = \log_a x$ (rewrite...) $\quad f'(x) = ??$

$$a^{f(x)} = x$$

$$\frac{a^{f(x)} \cdot \ln a \cdot f'(x)}{a^{f(x)} \cdot \ln a} = \frac{1}{a^{f(x)} \cdot \ln a}$$

$$f'(x) = \frac{1}{a^{f(x)} \cdot \ln a}$$

$$f'(x) = \frac{1}{x \cdot \ln a}$$

$y = \ln x$, $y' = \frac{1}{x}$

$y = \log_a x$

$y' = \frac{1}{x \cdot \ln a}$

ex: $y = \log_4 x$

$y' = \frac{1}{x \cdot \ln 4}$

$$y = \log_8 (3x-1)$$

$$y' = \frac{1}{(3x-1)\cdot \ln 8} \cdot 3$$

$$y' = \frac{3}{(3x-1)\cdot \ln 8}$$

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TEST #2 RESULTS

A's	50 (25.5%)	50.5%
B's	49 (25%)	
C's	42 (21.4%)	
D's	27 (13.8%)	28.1%
F's	28 (14.3%)	

AVE: 78.4

Quiz #1: Interview; documented with a **1 -2 page paper**

1.) paper should be typed (word processed), not handwritten (not emailed)
2.) Q & A format is OK; paragraph format is OK
3.) Interview a person (possibly a parent; not a student) who is currently doing for a living what you would eventually like to do for a living (based on your current major – or desired major)
4.) Face-to-face OK; Phone or skype OK; email OK
5.) One question you **MUST** ask: How is math used in this career?
6.) Other suggested questions you will probably want to ask: Average salary? Stress level? Is it seasonal (lots of work at certain times of the year) or fairly steady? Work/Life balance? Lifestyle/perks? Do you find your job rewarding? Is there continuing education associated with this job? Travel associated with this job? Do you have any summer jobs or internships for college students? Did your degree adequately prepare you for this job? And many, many other questions specific to your interviewee....
7.) DUE DATE: Tuesday, March 19, at the beginning of class