Thursday, February 28

- quick review (3.1)
- today: 3.2 (LOGS; $\boxed{\text{LN}}$; DERIVATIVES)
  3.3 (EXPONENTIAL GROWTH MODELS)
- tests returned on TUES
- drop/add/credit only (MONDAY)

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review 3.1:

$y = e^x$ (natural exponential function)
($e \approx 2.718$)

$m_{TAN} = y' = e^x$

$y = e^x \quad y' = e^x$

$y = e^{8x} \quad y' = e^{8x} \cdot 8 = 8 \cdot e^{8x}$

$y = e^{x^2} \quad y' = e^{x^2} \cdot (2x) = 2x e^{x^2}$

3.2:

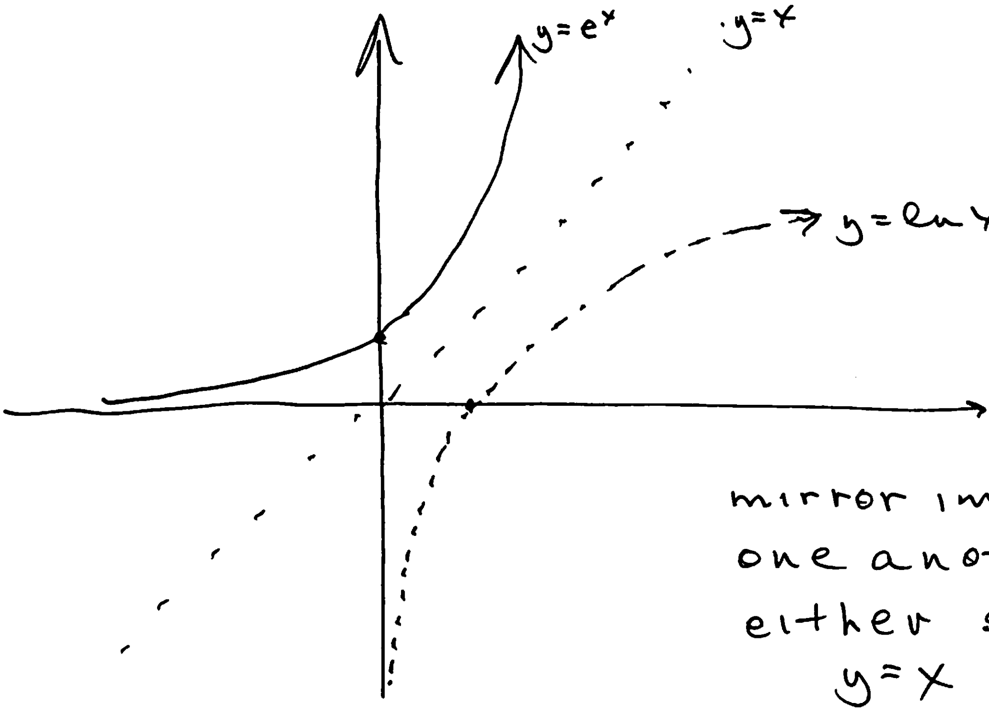
$y = e^x$ (take inverse)

$\boxed{x = e^y}$ (switch x & y)
(solve for y)

y is the power to which e is raised to get x

$y = \log_e x$ $\quad \boxed{y = \ln x}$ inverse of $y = e^x$

($e^y = x$)



mirror image of one another on either side of $y = x$

$y = \ln x \longrightarrow f(x) = \ln_e x$

$f'(x) = ??$

rewrite:

$\boxed{e^{f(x)} = x}$

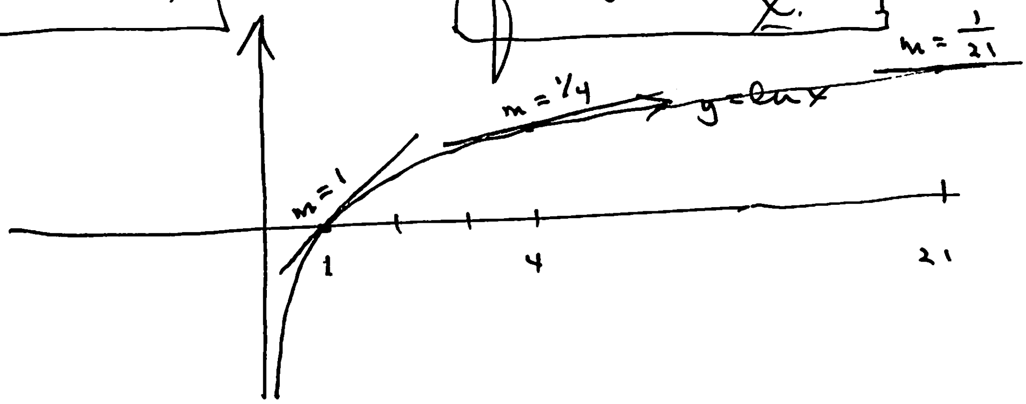
(take DERIV.)

$e^{f(x)} \cdot f'(x) = 1$

(solve for $f'(x)$)

$f'(x) = \dfrac{1}{e^{f(x)}} = \dfrac{1}{x}$

$\boxed{f(x) = \ln x}$ . . . $\boxed{f'(x) = \dfrac{1}{x}}$



$y = \ln(8x) \quad y' = \frac{1}{\cancel{8}x} \cdot \cancel{8} = \frac{1}{x}$

$\ln(a \cdot b) = \ln a + \ln b$

$y = \ln 8 + \ln x$

$y' = 0 + \frac{1}{x} = \frac{1}{x}$

$y = \ln(14x^2 - 5x + 3)$

$y' = \frac{1}{14x^2 - 5x + 3} \cdot (28x - 5)$

$y' = \frac{28x - 5}{14x^2 - 5x + 3}$

$y = \ln(\ln x)$

$y' = \frac{1}{\ln x} \cdot \frac{1}{x} = \frac{1}{x \cdot \ln x}$

3.3: EXPONENTIAL GROWTH MODELS

(basic or standard exponential growth)

$$\frac{dy}{dx} = k \cdot y$$

? ?

solution:

$$\boxed{y = c \cdot e^{kx}}$$

verify: (take deriv.)

$$\frac{dy}{dx} = \boxed{c \cdot e^{kx}} \cdot k$$

$$\frac{dy}{dx} = y \cdot k$$

$$\frac{dy}{dx} = k \cdot y$$

$$y = c \cdot e^{kx}$$

$$y = c \cdot e^{kt}$$

t = time

y_0 = y-value at t=0

t = 0: ① $y = y_0$

② ...

(t=0)

$$y_0 = c \cdot e^{k(0)}$$

$$y_0 = c \cdot 1$$

$$\boxed{y_0 = c} \implies \boxed{y = y_0 \cdot e^{kt}}$$

$$\boxed{A = P \cdot e^{rt}}$$

$$y = y_0 \cdot e^{kt}$$

basic exp. growth

y_0 : initial amt

y : future (final) amt

k : growth rate ($k > 0$)

INDY:

($t=0$) 2012 : 844,000

($t=6$) 2018 : 865,000

predict pop. in 2025 ($t = 13$)

$$\boxed{y = y_0 \cdot e^{kt}}$$

$$P = P_0 \cdot e^{kt}$$

$$P = (844{,}000)\, e^{kt}$$

2nd data point: (solve for k)

$$\frac{865{,}000}{844{,}000} = \frac{\cancel{844{,}000}\, e^{k \cdot 6}}{\cancel{844{,}000}}$$

$$\frac{865{,}000}{844{,}000} = e^{6k} \qquad e^{?} = e^{6k}$$

$$\ln\left(\frac{865{,}000}{844{,}000}\right) = \ln e^{6k}$$

$$\frac{\ln\left(\frac{865{,}000}{844{,}000}\right)}{6} = \frac{\cancel{6}k}{\cancel{6}}$$

$$k \approx .004096$$

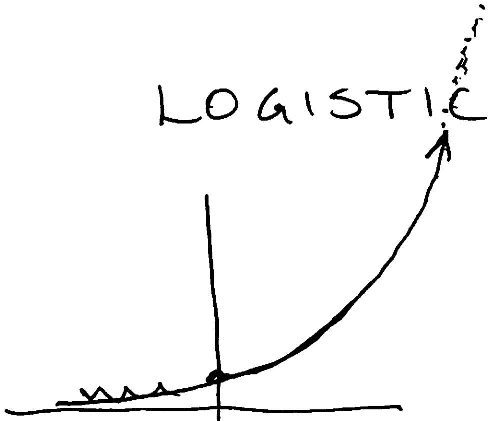
3rd data point:

$$P = (844{,}000)\, e^{(.004096)(13)}$$

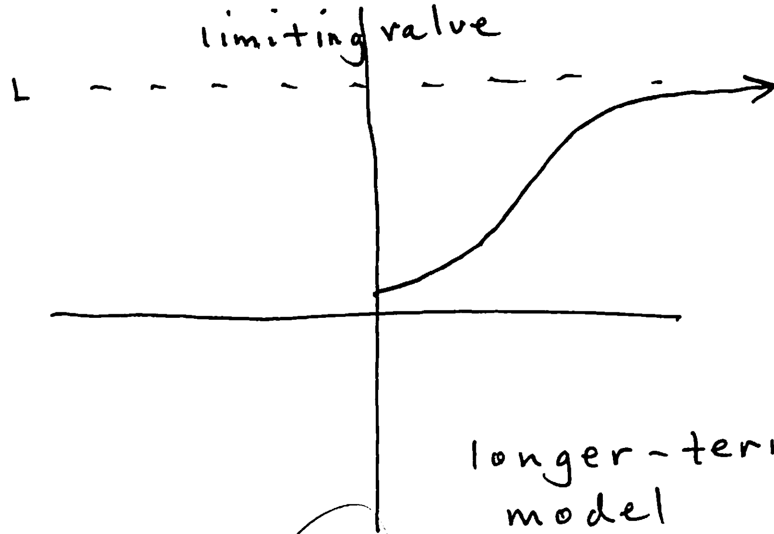
$$P \approx \cancel{1101703348}$$

$$\boxed{890{,}161}$$

LOGISTIC GROWTH:



$y = y_0 \cdot e^{kt}$



longer-term model

$(t=0)$ 2012: 844,000 ✓
$(t=6)$ 2018: 865,000

$\boxed{L: 1{,}000{,}000}$

predict pop. in 2025 $(t=13)$

$$y = \frac{L}{1 + b \cdot e^{kt}}$$

$$y = \frac{1{,}000{,}000}{1 + b \cdot e^{kt}}$$

① $t=0$ $\quad y = 844{,}000$

$$844{,}000 = \frac{1{,}000{,}000}{1 + b \cdot e^{k(0)}}$$

$$\frac{844{,}000}{1} = \frac{1{,}000{,}000}{1+b}$$

$$\frac{1{,}000{,}000}{844{,}000} = \frac{\cancel{844{,}000}(1+b)}{\cancel{844{,}000}}$$

$$\frac{1{,}000{,}000}{844{,}000} - 1 = \cancel{1} + b$$

$$\boxed{\frac{1{,}000{,}000}{844{,}000} - 1} = b \approx .18483$$

$$y = \frac{1{,}000{,}000}{1 + .18483\, e^{kt}}$$

(2) $t = 6$ $y = 865{,}000$

$$\frac{865{,}000}{1} = \frac{1{,}000{,}000}{1 + .18483\, e^{k(6)}}$$

$$\frac{1{,}000{,}000}{865{,}000} = \frac{\cancel{865{,}000}\left(1 + .18483\, e^{6k}\right)}{\cancel{865{,}000}}$$

$$\frac{1{,}000{,}000}{865{,}000} - 1 = \cancel{1} + .18483\, e^{6k}$$
$$\cancel{-1}$$

$$\frac{\frac{1{,}000{,}000}{865{,}000} - 1}{.18483} = \frac{\cancel{.18483}\, e^{6k}}{\cancel{.18483}}$$

$$\frac{\ln\left[\frac{\frac{1{,}000{,}000}{865{,}000} - 1}{.18483}\right]}{6} = \frac{\cancel{6}k}{\cancel{6}} \approx -.02819$$

$$y = \frac{1{,}000{,}000}{1 + .18483\, e^{-.02819t}}$$

pop in 2025: $(t=13)$ $y = \dfrac{1{,}000{,}000}{1 + .18483\, e^{-.02819(13)}}$