MA141-004

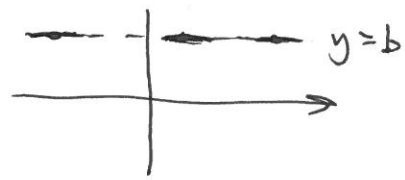
Thursday, January 31

- TEST #1 (first hour of class)
- CH 2 (second hour of class)
  - power rule
  - product rule
  - quotient rule

2.2: DERIVATIVE RULES

1. DERIV OF CONSTANT:

$$\boxed{f(x) = b \qquad f'(x) = 0}$$

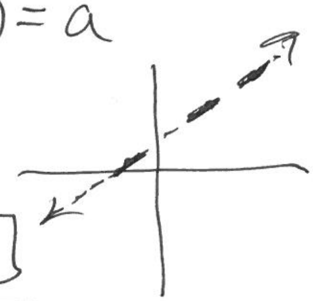


$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{b - b}{h}$$

$$= \lim_{h \to 0} \frac{0}{h} = 0$$

2. $f(x) = ax + b$ (LINEAR) $\qquad f'(x) = a$



$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{[a(x+h)+b] - [ax+b]}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{ax} + ah + \cancel{b} - \cancel{ax} - \cancel{b}}{h}$$

$$= \lim_{h \to 0} \frac{\boxed{a}\cancel{h}^{1}}{\cancel{h}_{1}} \quad (h \neq 0) = a$$

(3) CONSTANT MULTIPLE OF A FUNCTION:

$$\frac{d\left[k \cdot f(x)\right]}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{k \cdot f(x+h) - k \cdot f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\boxed{k}\left[f(x+h) - f(x)\right]}{h}$$

$$= k \cdot \boxed{\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$

$$\frac{d\left[k \cdot f(x)\right]}{dx} = k \cdot f'(x)$$

(4) ~~PRODUCT RULE~~: SUM/DIFFERENCE RULE:

$$\frac{d\left[f(x) + g(x)\right]}{dx} = \lim_{h \to 0} \frac{\left[f(x+h) + g(x+h)\right] - \left[f(x) + g(x)\right]}{h}$$

$$= \lim_{h \to 0} \frac{f(x+h) + g(x+h) - f(x) - g(x)}{h}$$

$$= \lim_{h \to 0} \frac{f(x+h) - f(x) + g(x+h) - g(x)}{h}$$

$$= \lim_{h \to 0} \left[\frac{f(x+h) - f(x)}{h} + \frac{g(x+h) - g(x)}{h}\right]$$

$$= \underline{f'(x) + g'(x)}$$

(5) ~~PR~~ POWER RULE:

$$\boxed{f(x) = x^n}$$

(n is a <u>non-neg</u> integer;
n is a <u>positive</u> integer)

<u>ex</u>: $y = x^2$

$y = \underline{x^3}$

$y = \underline{x^{11}}$

$y = \underline{14}x^3$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\boxed{(x+h)^n} - x^n}{h}$$

$$= \lim_{h\to 0} \frac{\left[\cancel{x^n} + \boxed{n x^{n-1}\cdot h} + \boxed{\frac{n(n-1)}{1\cdot 2}x^{n-2} h^2} + \dots\right] - \cancel{x^n}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}\left[n\cdot x^{n-1} + \frac{n(n-1)}{1\cdot 2}x^{n-2}\cdot h + (h\text{-terms})\right]}{\cancel{h}}$$

$(h \neq 0)$

$$= \lim_{h\to 0} \left[\underline{n\cdot x^{n-1}} + \cancel{\boxed{\frac{n(n-1)}{1\cdot 2}x^{n-2}\cdot h}}^{\,0} + \cancel{(h\text{-terms})}^{\,0}\right]$$

$$f'(x) = \underline{n}\cdot x^{\underline{n-1}} \qquad \underline{\underline{f(x) = x^n}}$$

POWER RULE:

(1) $f(x) = x^n \qquad f'(x) = n\cdot x^{n-1}$

(2) $\boxed{f(x) = a\cdot x^n \qquad f'(x) = a\cdot(n\cdot x^{n-1})}$

ex:

$f(x) = \underbrace{3x^5} \underbrace{-11x^3} + \underbrace{8x^2} - \underbrace{4x} + \underbrace{3}$

$f'(x) = 3\cdot(5\cdot x^{5-1}) - 11(3x^{3-1}) + 8(2x^{2-1}) - 4(1\cdot x^{1-1}) + 0$

$f'(x) = 15x^4 - 33x^2 + 16x - 4$

$(x+h)^5 \quad (x+h)^3$

generalizable:

ex: $f(x) = 8\sqrt{x} - \dfrac{5}{x^4} + 11x^{-2/3}$

$f(x) = 8x^{1/2} - 5x^{-4} + 11x^{-2/3}$

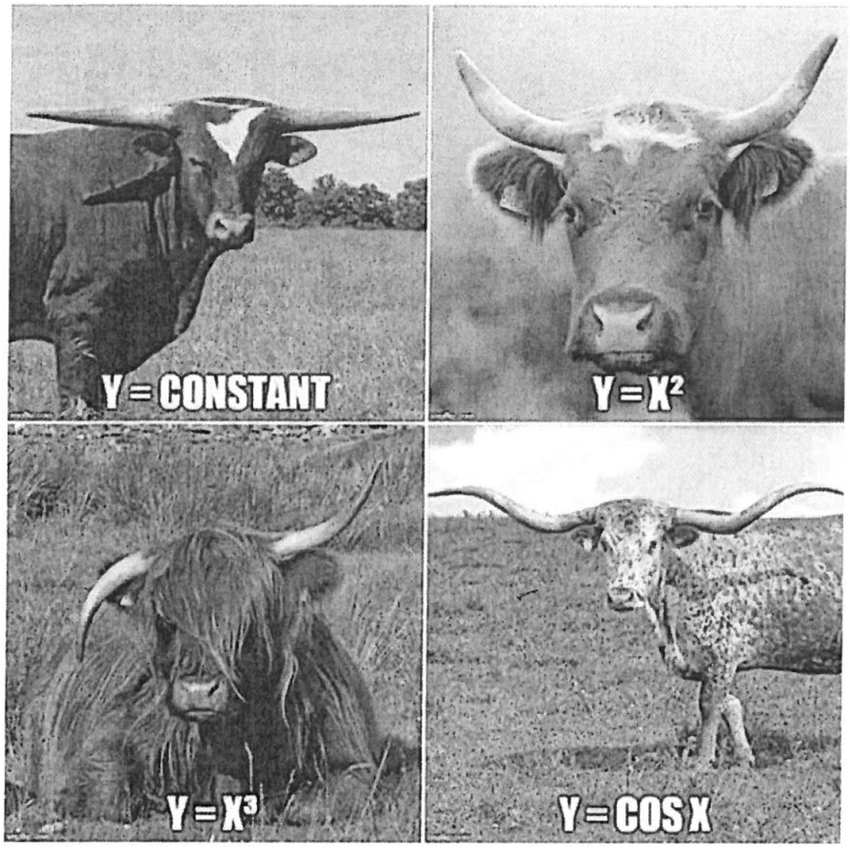
$f'(x) = 8\left[\frac{1}{2}x^{\frac{1}{2}-1}\right] - 5\left[-4\cdot x^{-4-1}\right] + 11\left[-\frac{2}{3}\cdot x^{-\frac{2}{3}-1}\right]$

$\boxed{f'(x) = \underline{4}x^{-1/2} + 20x^{-5} - \frac{22}{3}x^{-5/3}}$

$f'(1) =$

$f'(x) = \dfrac{4}{\sqrt{x}} + \dfrac{20}{x^5} - \dfrac{22}{3x^{5/3}}$

$f'(x) = \dfrac{4}{\sqrt{x}} + \dfrac{20}{x^5} - \dfrac{22}{3(\sqrt[3]{x})^5}$


Y = CONSTANT
Y = X²
Y = X³
Y = COS X

MA 141-004 S19 Recitation 4, Jan 30

1. Let $f(x) = 4x^2 + 2$.

   (a) Use the limit definition of derivative to determine $f'(x)$.

   (b) What is the relationship between the line tangent to f at the point $(a, f(a))$ and the derivative $f'(a)$?

   (c) What is the equation of the tangent line through the point $(-1, 6)$?

2. The volume of a spherical hot air balloon $V(r) = \frac{4}{3}\pi r^3$ changes as its radius changes. The radius is a function of time given by $r(t) = 3t$. Find the average rate of change of the volume with respect to t as t changes from $t = 1$ to $t = 2$. Find the instantaneous rate of change of the volume with respect to t at $t = 2$.

3. Suppose the function f is defined on the interval (a, b) where $f'(c)$ exists for all $c \in (a, b)$. Is f continuous on (a, b)?

4. Let g be a piecewise function defined by the following:

$$g(x) = \begin{cases} 4x^2 + 2 & \text{if } x \leq \frac{1}{2} \\ -2x + 4 & \text{if } x \geq \frac{1}{2} \end{cases}$$

   (a) Is g continuous at $x = \frac{1}{2}$?

   (b) Use right- and left- hand derivatives to show that g is not differentiable at $x = \frac{1}{2}$.

Relevant Information to Recall

- The function f is **differentiable at** x provided the following limit exists

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$

- Let the function f be defined on the interval (a, b) and $x \in (a, b)$. The **right-hand (left-hand) derivative of** f **at** x is the number $f'(x^+)$ $(f'(x^-))$ given by the following limit, provided the limit exists,

$$f'(x^+) = \lim_{h \to 0+} \frac{f(x+h) - f(x)}{h} \quad \left(f'(x^-) = \lim_{h \to 0-} \frac{f(x+h) - f(x)}{h}. \right)$$