MA141-004

Tuesday, February 26

- TEST #2 (first hour)
- 3.1 (second hour)
  - linear approx
  - NEWTON'S METHOD

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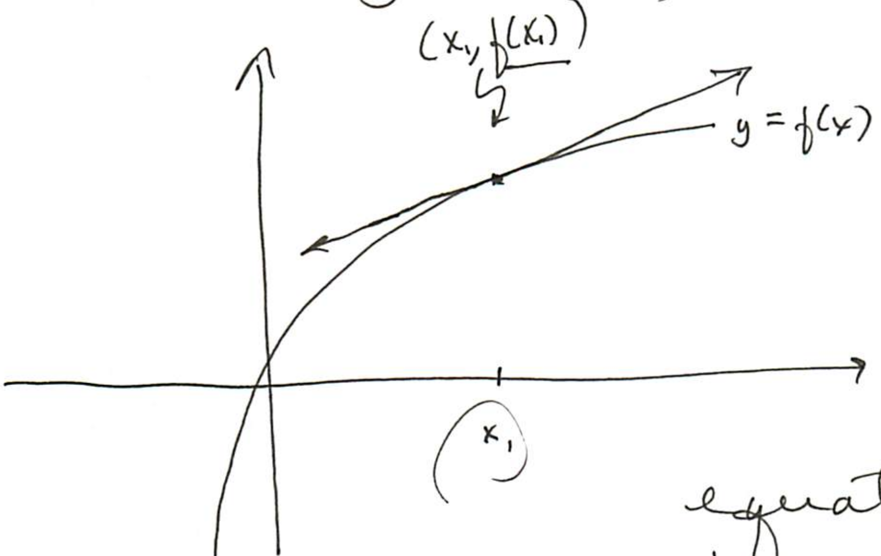
# LINEAR APPROXIMATION

(tangent line approx)

(linearization)




equation of that tangent line:

$$y - y_1 = m(x - x_1)$$

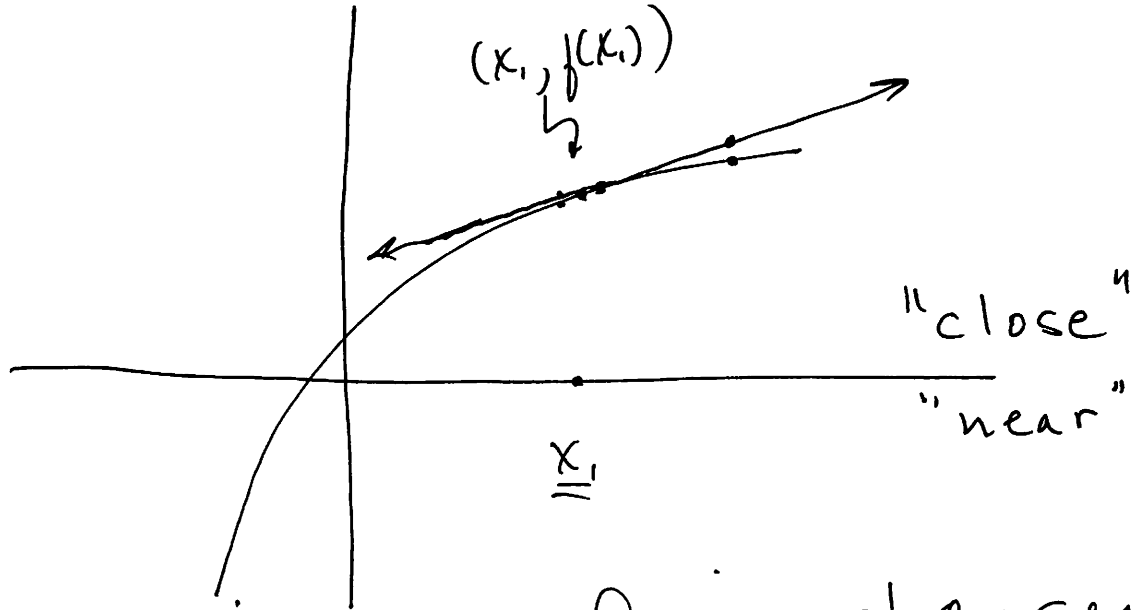
$$y - f(x_1) = m(x - x_1)$$

$$m = m_{TAN} = f'(x) = f'(x_1)$$

EQ OF TANGENT LINE:

$$y - f(x_1) = f'(x_1)(x - x_1)$$

$$y = f(\underline{x_1}) + f'(\underline{x_1})(x - \underline{x_1})$$

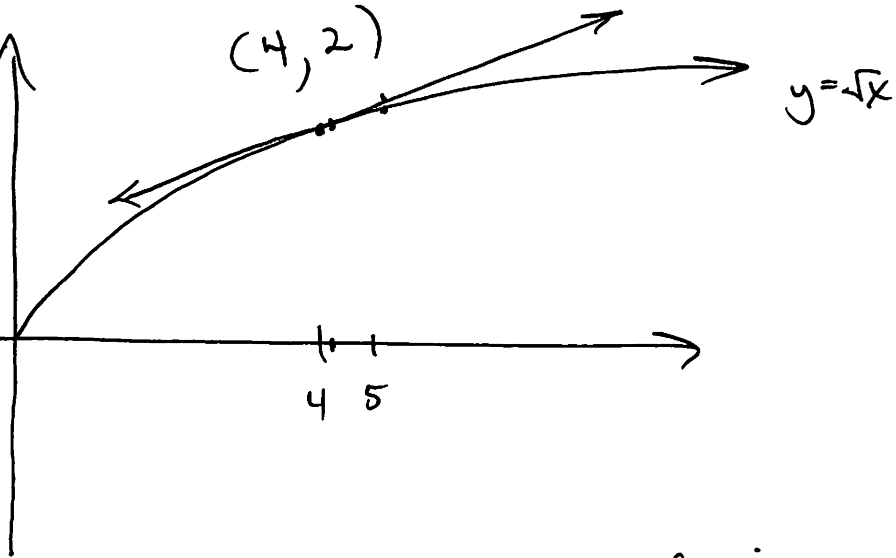



– using the line tangent to the curve to approx y-values ON THE CURVE.

$\sqrt{4.3}$

$f(x) = \sqrt{x} = x^{1/2}$

$f'(x) = \frac{1}{2} x^{-1/2} = \frac{1}{2\sqrt{x}}$




eq of tangent line:

$$y = f(x_1) + f'(x_1)(x - x_1)$$

$$(x_1 = 4)$$

$$y = f(4) + f'(4)(x - 4)$$

$$y = 2 + \frac{1}{4}(x - 4)$$

$$y = 2 + \frac{1}{4}x - 1$$

$$\boxed{y = \frac{1}{4}x + 1}$$

"close" to 4

$$\boxed{\sqrt{x}} \approx \boxed{\frac{1}{4}x + 1}$$

$$\sqrt{4.3} \approx \frac{1}{4}(4.3) + 1$$

$$\approx 1.075 + 1$$

$$\approx 2.075$$

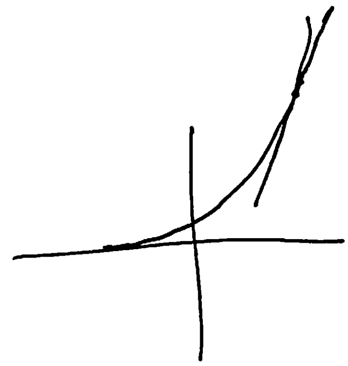
from calculator:

$\sqrt{4.3} \approx 2.074$

$\sqrt{5} = ??$

$\cancel{y = f(x_1) +}$

$y = \frac{1}{4}x + 1$



$\sqrt{x} \approx \frac{1}{4}x + 1$

$\sqrt{5} \approx \frac{1}{4}(5) + 1$

$\approx 1.25 + 1$

$\sqrt{5} \approx 2.25$

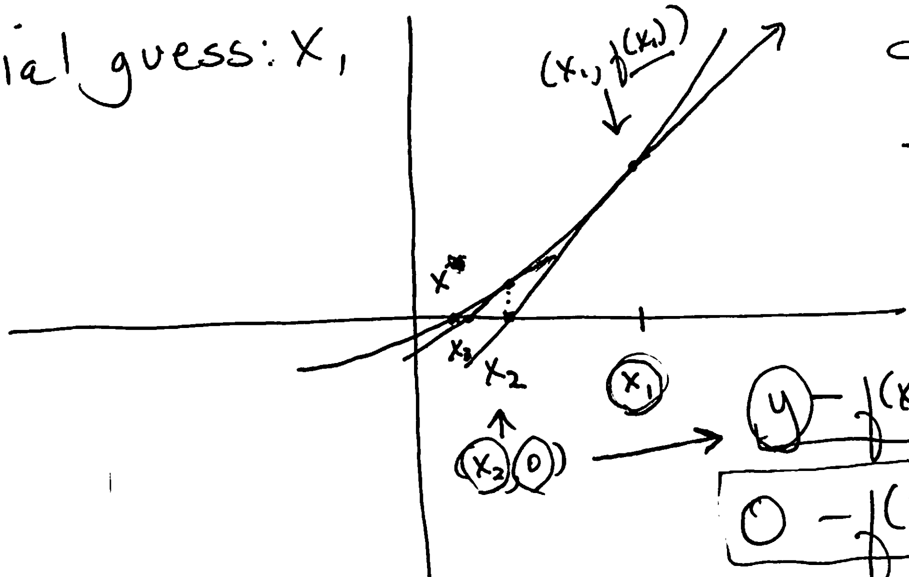
from calc.

$\sqrt{5} \approx 2.236$

NEWTON'S METHOD:

initial guess: $x_1$

find the root (zero) of a function




$y - f(x_1) = f'(x_1)(x - x_1)$

$\boxed{0 - f(x_1) = f'(x_1)(x_2 - x_1)}$

solve for $x_2$:

$-\frac{f(x_1)}{f'(x_1)} = x_2 - x_1$

$$x_1 - \frac{f(x_1)}{f'(x_1)} = \underline{x_2}$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

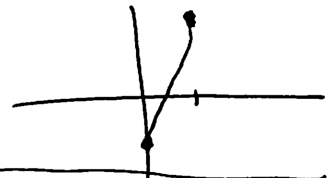
$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$$

in general ....

$$\boxed{x_{\underline{n+1}} = x_n - \frac{f(x_n)}{f'(x_n)}}$$

finding roots of functions

$$\boxed{f(x) = x^3 + 5x - 3 = 0}$$



$f(0) = -3 \qquad f(1) = 3$

$$\boxed{f'(x) = 3x^2 + 5}$$

root in $(0, 1)$

INTERMED. VALUE THM.

contin (polynom)

guess:

$x_1 = .5$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$x_2 = .5 - \frac{f(.5)}{f'(.5)} = .5 - \frac{[(.5)^3 + 5(.5) - 3]}{[3(.5)^2 + 5]}$$

$$x_2 = .5 - \frac{(-.375)}{5.75} = .5 + .0652 = \underline{.5652}$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = .5652 - \frac{[(.5652)^3 + 5(.5625) - 3]}{[3(.5652)^2 + 5]}$$

$$x_3 = .5652 - \frac{.0066}{5.958} = .5641$$

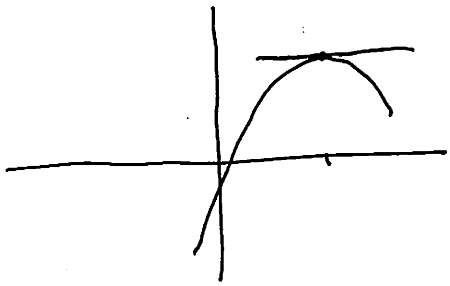
.00110

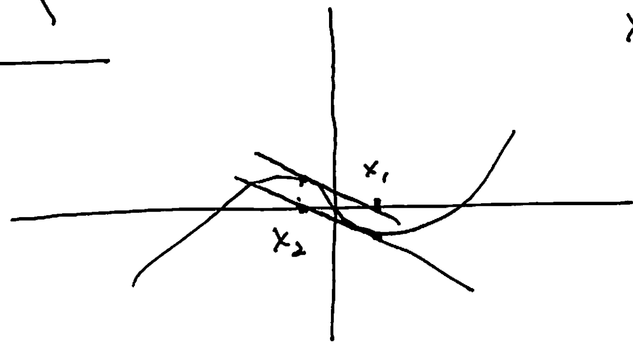
$x_4 =$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

FAILS:

1. $f'(x_n) = 0$
2. loop






$x_1 \ldots x_2 \ldots x_1$

Use Newton's method to approximate the positive root of $x^3 + 5x - 3 = 0$ to three consistent decimal places.

Use Newton's method to approximate $\sqrt{7}$ to four consistent decimal places.

$f(x) = x^2 - 7$
$f'(x) = 2x$

Use Newton's method to approximate the smallest positive root of $\cos x = e^{-x}$ to three consistent decimal places.