Thursday, February 21

- finish 2.7 (related rates)

TEST #2: TUESDAY (FEB. 26th) CH. 2

$$y = (\sin x)^{e^x}$$

(1) $y = x^5$ $\quad y' = 5x^4$

(2) $y = 5^x$ $\quad y' = 5^x \cdot \ln 5$

(use log. diff.)

$$\ln y = \ln (\sin x)^{e^x}$$

$$\ln y = e^x \cdot \ln(\sin x)$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = e^x \left[\frac{1}{\sin x} \cdot \cos x \right] + \ln(\sin x) \cdot \left[e^x \right]$$

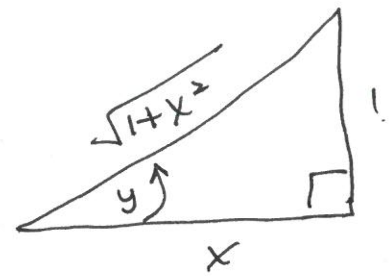
$$\frac{1}{y} \frac{dy}{dx} = e^x \cdot \cot x + e^x \cdot \ln(\sin x)$$

$$\longrightarrow \frac{dy}{dx} = y \left[e^x \cdot \cot x + e^x \cdot \ln(\sin x) \right]$$

$$\frac{dy}{dx} = (\sin x)^{e^x} \left[e^x \cdot \cot x + e^x \cdot \ln(\sin x) \right]$$

$y = \cot^{-1} x$ find y':

INV OF $y = \cot x$



$x = \cot(y)$

(now .... take DERIV.)

$$1 = -\csc^2 y \cdot \frac{dy}{dx}$$

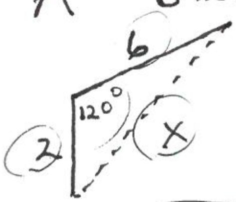
$$\frac{dy}{dx} = \frac{-1}{\csc^2 y} = \frac{-1}{\left[\frac{\sqrt{1+x^2}}{1}\right]^2} = \frac{-1}{1+x^2}$$

AT THIS INSTANT:

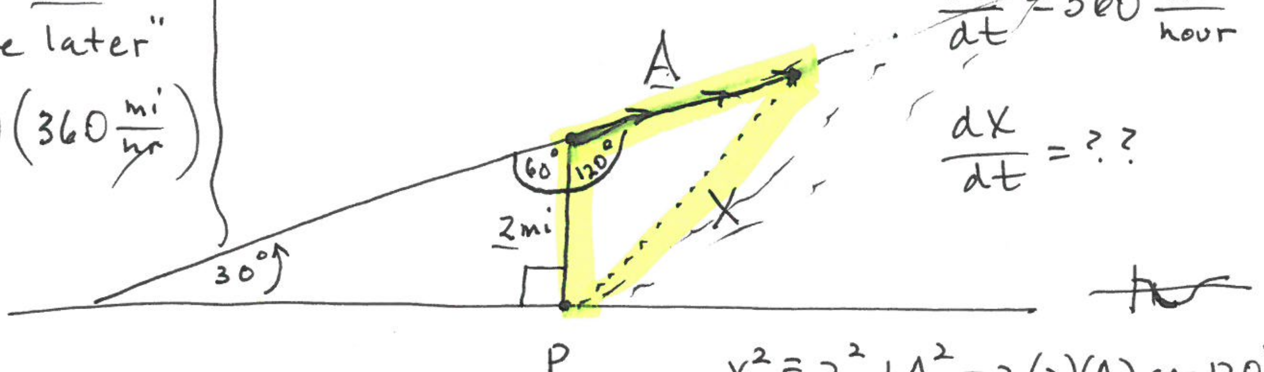
"one minute later"

$A = \left(\frac{1}{60}\,hr\right)\left(360\,\frac{mi}{hr}\right)$

$A = 6\,mi$



$X = \sqrt{52}$



$\frac{dA}{dt} = 360\,\frac{mi}{hour}$

$\frac{dX}{dt} = ??$

$X^2 = 2^2 + A^2 - 2(2)(A)\cos 120^\circ$

law of cosines:

$$X^2 = 2^2 + 6^2 - 2(2)(6)\cos 120^\circ$$

$$X^2 = 4 + 36 - 24\left(-\tfrac{1}{2}\right)$$

$$X^2 = 40 + 12 = 52$$

$$X = \sqrt{52}$$

$$x^2 = 2^2 + A^2 - 2(2)(A)\underline{\cos 120^\circ}$$

$(-1/2)$

$$x^2 = 4 + A^2 + 2A$$

(DERIV. WITH RESP. TO "t")

36
6
216

$$2x \cdot \frac{dx}{dt} = 0 + 2A\frac{dA}{dt} + 2 \cdot \frac{dA}{dt}$$

$$\cancel{2}x\,\frac{dx}{dt} = \cancel{2}(A)\cdot\frac{dA}{dt} + \frac{dA}{dt}$$

$$\sqrt{52}\cdot\frac{dx}{dt} = (6)\cdot(360) + (360)$$

$$\frac{dx}{dt} = \frac{2160 + 360}{\sqrt{52}}\ \frac{mi}{hr}$$

$$\frac{dx}{dt} = \underline{349.46}\ \frac{mi}{hr}$$

3.) $\underline{A} = \dfrac{\sqrt{3}}{4} \cdot \underline{(S)^2}$

$\dfrac{dA}{dt} = -4 \ \dfrac{cm^2}{min}$

$\dfrac{ds}{dt} = ??$

AT THIS INSTANT:

$A = 200 \ cm^2$

$\dfrac{4 \cdot 200}{\sqrt{3}} = \dfrac{\cancel{\sqrt{3}}}{\cancel{4}} (s^2) \cdot \cancel{4} \dfrac{1}{\cancel{\sqrt{3}}}$

$\dfrac{800}{\sqrt{3}} = s^2$

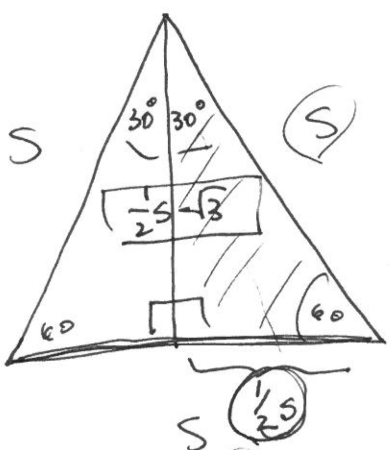
$s = \sqrt{\dfrac{800}{\sqrt{3}}}$

$\boxed{\dfrac{dA}{dt}} = \dfrac{\sqrt{3}}{\cancel{4}_{\,2}} \left(\cancel{2} s \dfrac{ds}{dt} \right)$

$\dfrac{dA}{dt} = \dfrac{\sqrt{3}}{2} \cdot s \cdot \dfrac{ds}{dt}$

$-4 = \dfrac{\sqrt{3}}{2} \left(\sqrt{\dfrac{800}{\sqrt{3}}} \right) \cdot \dfrac{ds}{dt}$

$\dfrac{ds}{dt} \approx -.214 \ \dfrac{cm}{min}$

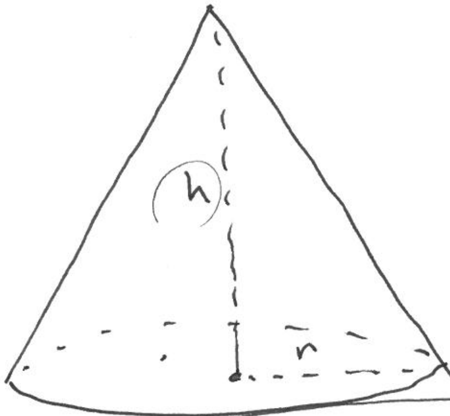


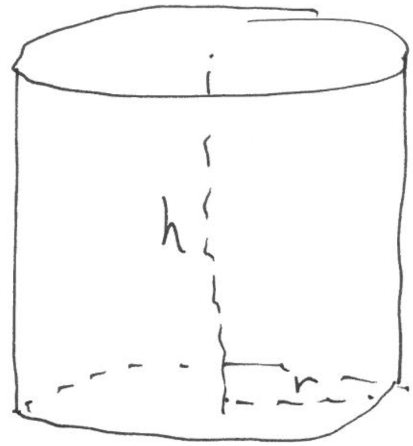
$A = \dfrac{1}{\cancel{2}} (\underline{s}) \left(\dfrac{1}{\cancel{2}} \underline{s} \sqrt{3}\right)$

$A = \dfrac{1}{4} s^2 \sqrt{3}$

14.)

$r = h$





cone:

$$V = \frac{1}{3}\boxed{\pi r^2 h}$$

$$V = \pi r^2 h$$

$$\boxed{V = \frac{1}{3}\pi h^2 \cdot h}$$

$$\frac{dh}{dt} = 6 \frac{in}{min}$$

$$V = \frac{1}{3}\pi h^3$$

$$\frac{dV}{dt} = ??$$

AT THIS INSTANT:

$h = 10$ in

$V =$ ___

$$\frac{dV}{dt} = \frac{1}{3}\pi \cdot 3h^2 \cdot \frac{dh}{dt}$$

$$\frac{dV}{dt} = \frac{1}{\cancel{3}}\pi \cancel{3}(10)^2 \cdot 6 \quad \frac{in^3}{min}$$

$$\frac{dV}{dt} = 600\pi \ \frac{in^3}{min}$$

$$\frac{dV}{dt} ; \frac{dh}{dt} ; \frac{dr}{dt}$$

$$V = \underbrace{\frac{1}{3}\pi r^2}\cdot \underbrace{h}$$

MA 141-004 S19 Recitation 7, Feb 20

1. Use logarithmic differentiation to determine $\frac{dy}{dx}$ for the given functions.

$$a)\ y = x^{x^2} \qquad\qquad b)\ y = \frac{e^{2x+3}\sin(x+4)}{\sqrt[3]{x^2+1}}$$

2. Claire starts at point P and runs east at a rate of 11 ft/sec. One minute later, Anna starts at P and runs north at a rate of 9 ft/sec. At what rate (in feet per second) is the distance between them changing after another minute?

3. Water is flowing at a rate of 6 ft^3/min into a tank that is the shape of a right circular cylinder whose base radius is 2 ft. How fast (in feet per minute) is the water level rising? Recall that the volume of a circular cylinder is $V = \pi r^2 h$, where r is radius and h is height.

4. Use implicit differentiation to show that

$$\frac{d}{dx}(\cot^{-1} x) = \frac{-1}{1+x^2}.$$