MA141 – ♥♥4

Thursday, February 14

- inverse trig derivatives
- exponentials and logs – DERIV
- log. differentiation

INVERSE TRIG:

$\boxed{y = \sin x}$ $\quad y' = \cos x$

$y = \sin(4x)$ $\quad y' = (\cos 4x) \cdot 4$

$y' = 4\cos 4x$

$1^2 = x^2 + leg^2$

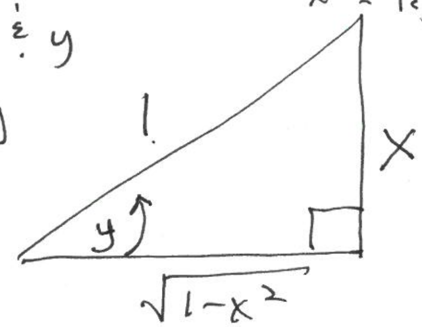
$1^2 - x^2 = leg^2$

INVERSE:

① switch x & y

② solve for y

* $\boxed{x = \sin y}$



$y = \arcsin x$

* $\boxed{y = \sin^{-1} x}$ $\quad y' = ??$ $\quad \frac{dy}{dx} = ??$

$\boxed{\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}}$

$x = \sin(y)$ (diff. implicitly)

$1 = \cos y \cdot \frac{dy}{dx}$ $\qquad \frac{dy}{dx} = \frac{1}{\boxed{\cos y}} = \frac{1}{\frac{\sqrt{1-x^2}}{1}}$

$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$

$y = \sin x$

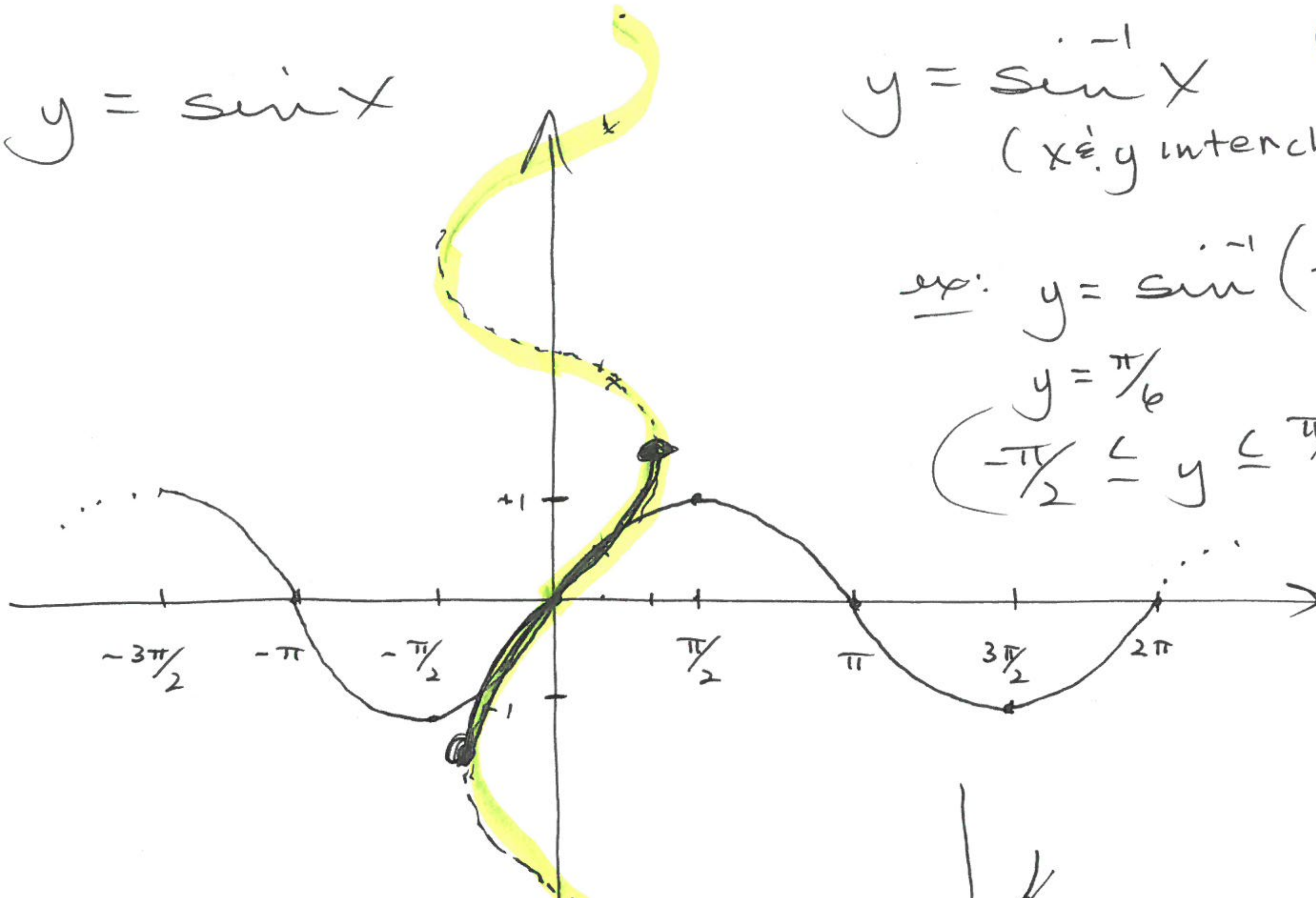
$y = \sin^{-1} x$

(x & y interch.)

ex: $y = \sin^{-1}\left(\frac{1}{2}\right)$

$y = \pi/6$

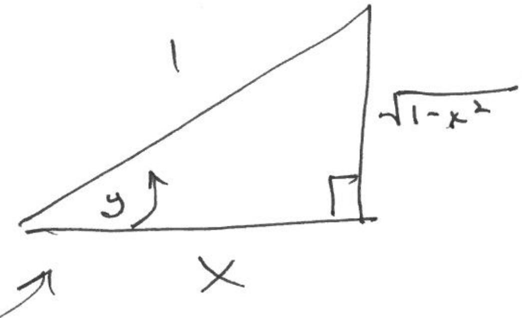
$\left(-\frac{\pi}{2} \le y \le \frac{\pi}{2}\right)$



$m_{TAN} =$

$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$

$y = \cos x$



INV:

$y = \cos^{-1} x \qquad \frac{dy}{dx} = ??$

$x = \cos(y)$ (take deriv.)

$1 = -\sin y \cdot \frac{dy}{dx}$ (solve for $\frac{dy}{dx}$)

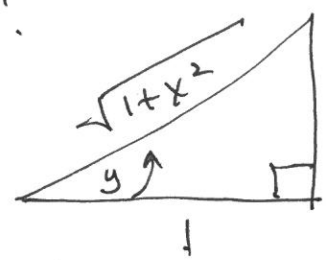
$$\frac{dy}{dx} = \frac{-1}{\sin y} = \frac{-1}{\frac{\sqrt{1-x^2}}{1}} = \frac{-1}{\sqrt{1-x^2}}$$

$y = \tan x$

INV:

$y = \tan^{-1} x \qquad \frac{dy}{dx} = ??$

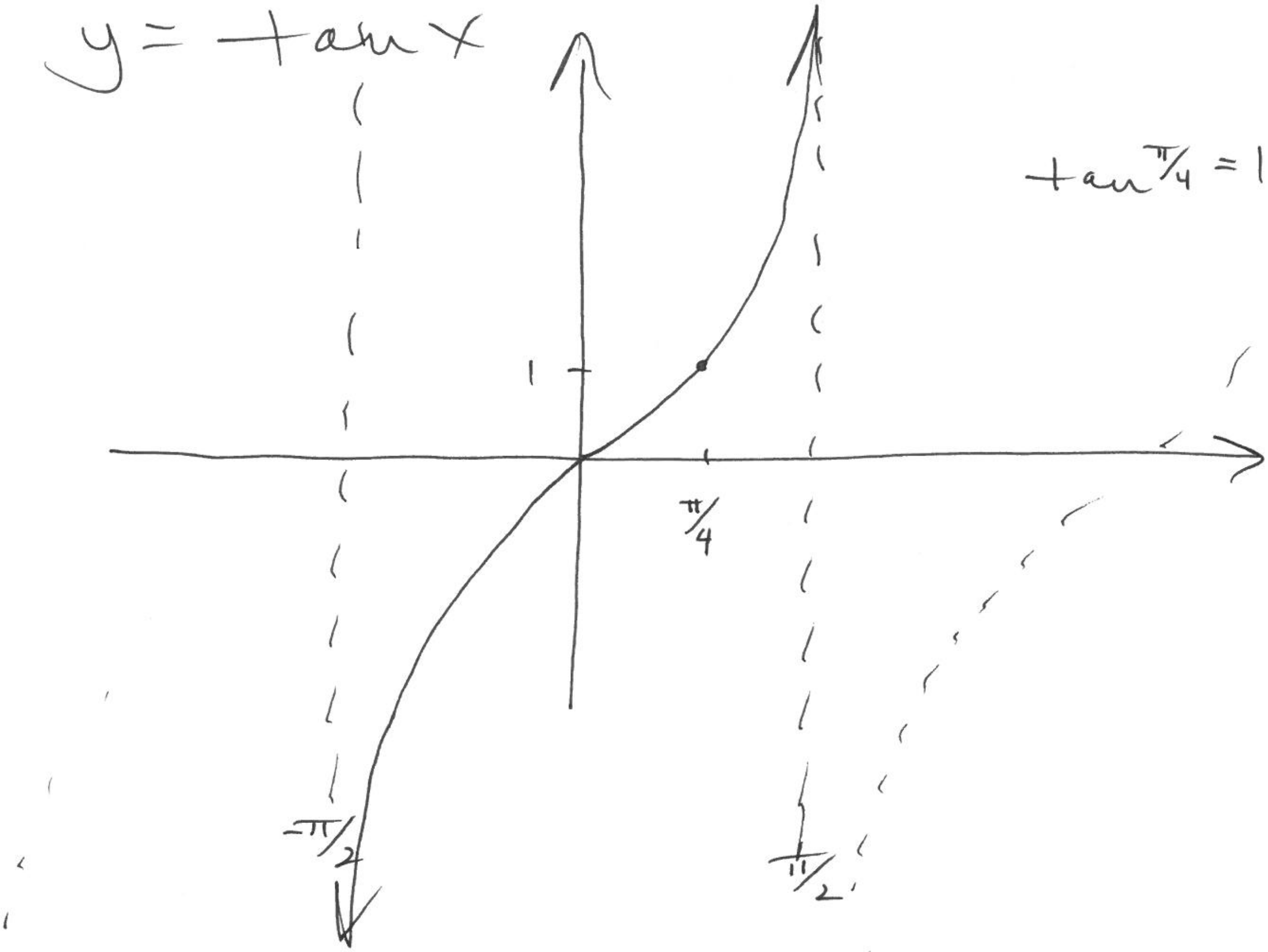
$= (x = \tan(y)) \longrightarrow$



$1 = \sec^2 y \cdot \frac{dy}{dx} \qquad \frac{dy}{dx} = \frac{1}{\sec^2 y} = \frac{1}{\left[\frac{\sqrt{1+x^2}}{1}\right]^2}$

$$\frac{dy}{dx} = \frac{1}{1+x^2}$$

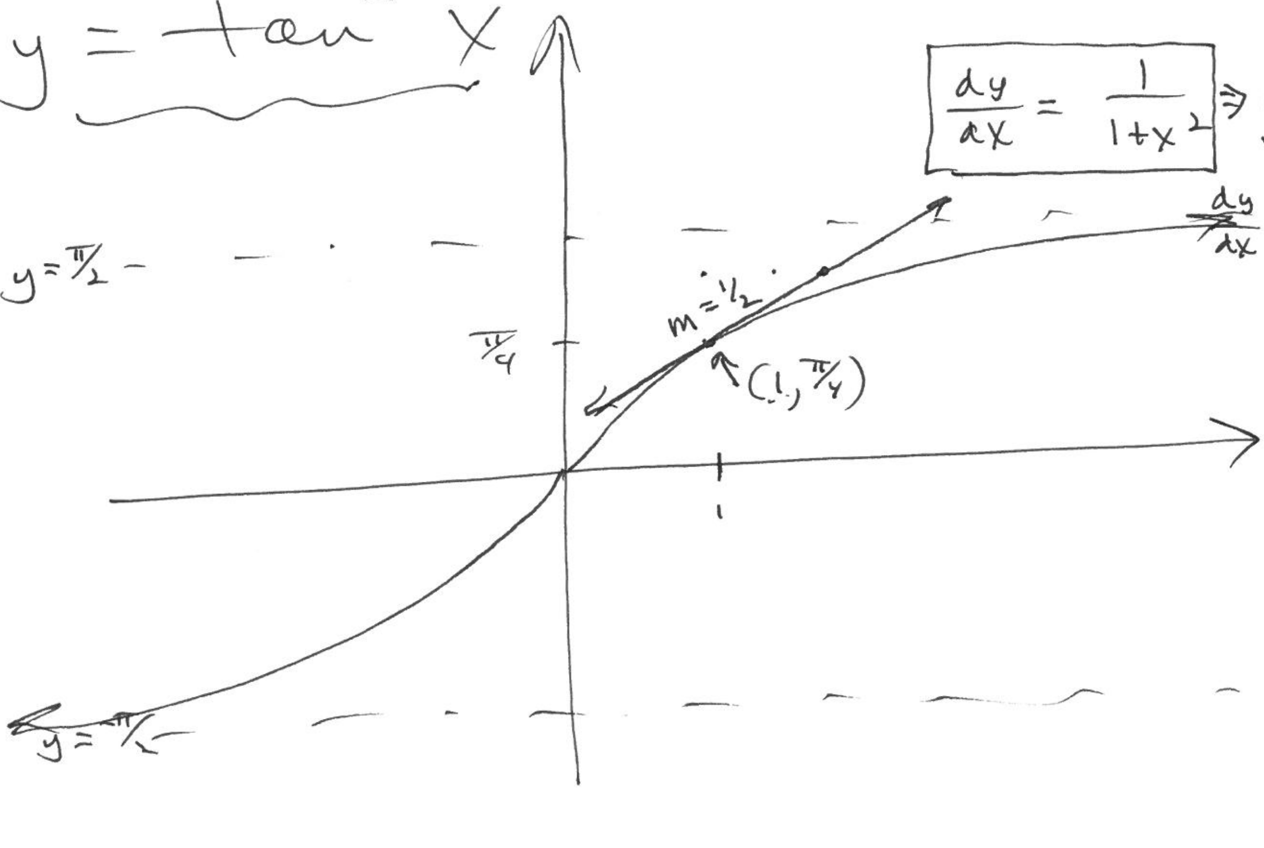
$y = \tan x$



$\tan \frac{\pi}{4} = 1$

$y = \tan^{-1} x$

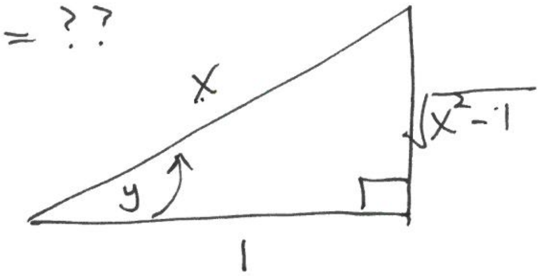
$\boxed{\frac{dy}{dx} = \frac{1}{1+x^2}} \Rightarrow$ when $x = 1$ $\frac{dy}{dx} = \frac{1}{2}$



$y = \sec x \qquad y' = \sec x \cdot \tan x$

INV! $\quad y = \sec^{-1} x \qquad \dfrac{dy}{dx} = ??$

$\left(\dfrac{x}{1} = \sec(y) \right) \longrightarrow$



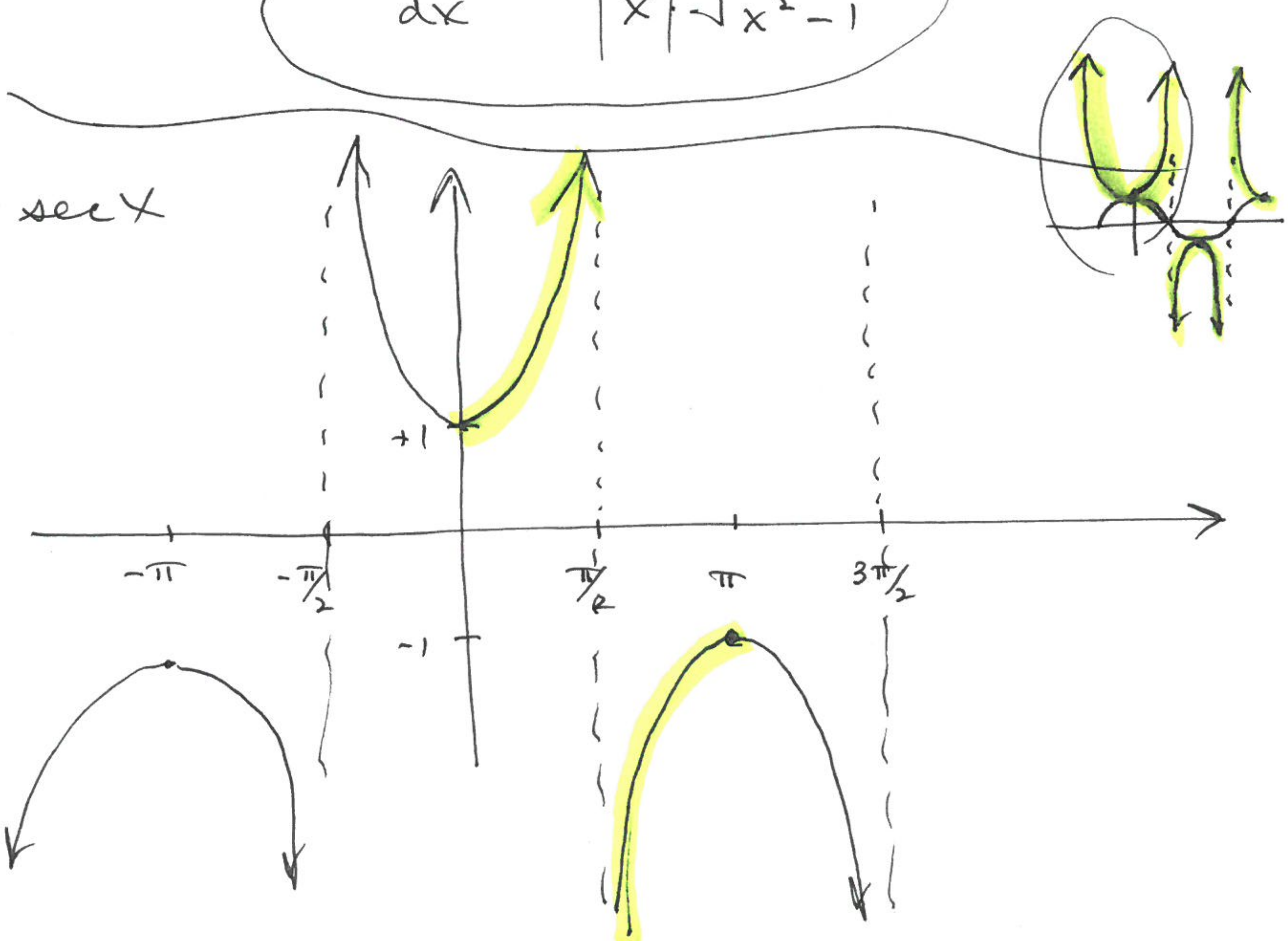
$1 = \sec y \cdot \tan y \cdot \dfrac{dy}{dx}$

$$\frac{dy}{dx} = \frac{1}{\sec y \cdot \tan y}$$

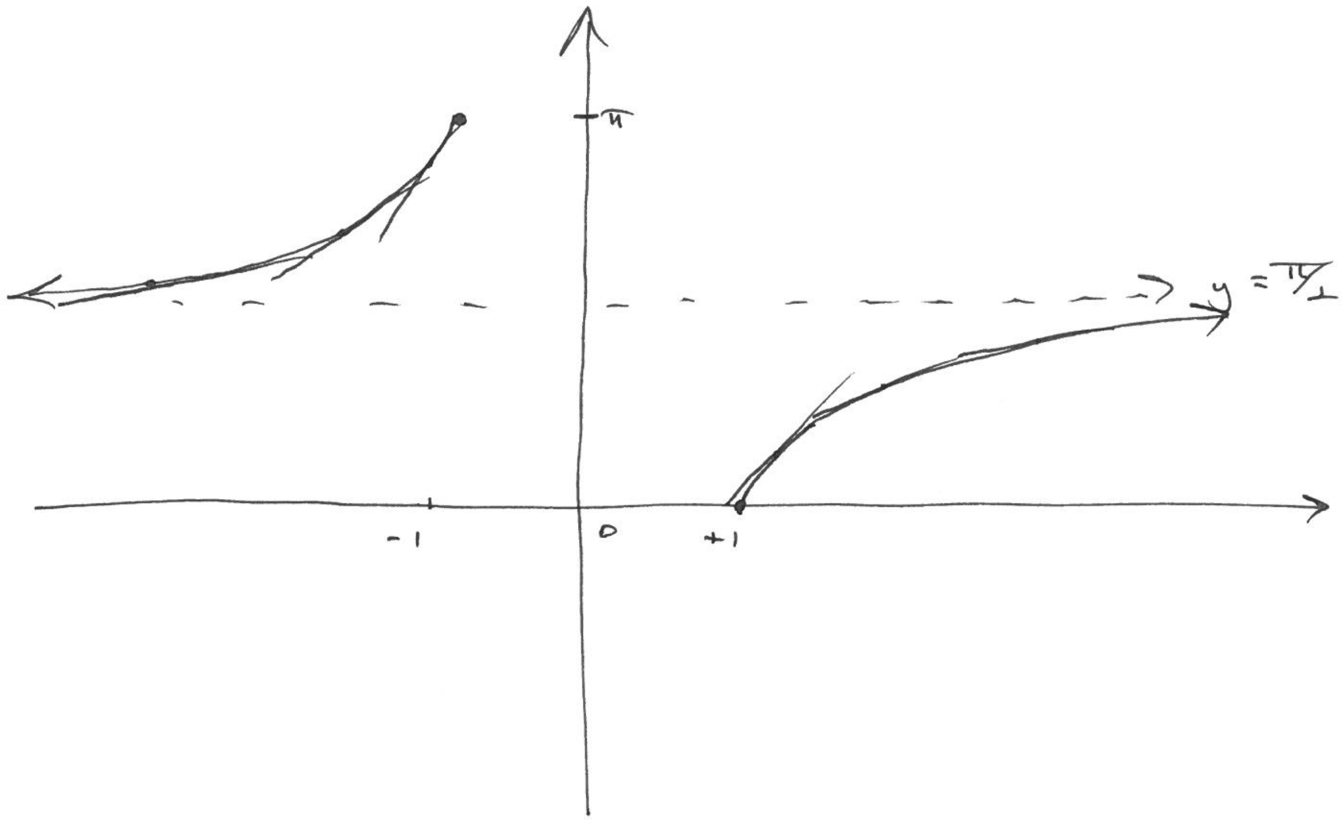
$$\frac{dy}{dx} = \frac{1}{x \cdot \frac{\sqrt{x^2-1}}{1}} = \frac{1}{x \cdot \sqrt{x^2-1}}$$

$$\frac{dy}{dx} = \frac{1}{|x| \cdot \sqrt{x^2-1}}$$

$y = \sec x$



$y = \sec^{-1} x$



$y = e^x \qquad (e \approx 2.718)$

$$\lim_{K\to\infty} \underbrace{\left(1+\frac{1}{K}\right)^{K}} = \underline{e} \qquad (1)^{\boxed{\text{huge}}}$$

$K = 100$

$\left(1+\frac{1}{100}\right)^{100} \approx 2.704$

$K = 10{,}000$

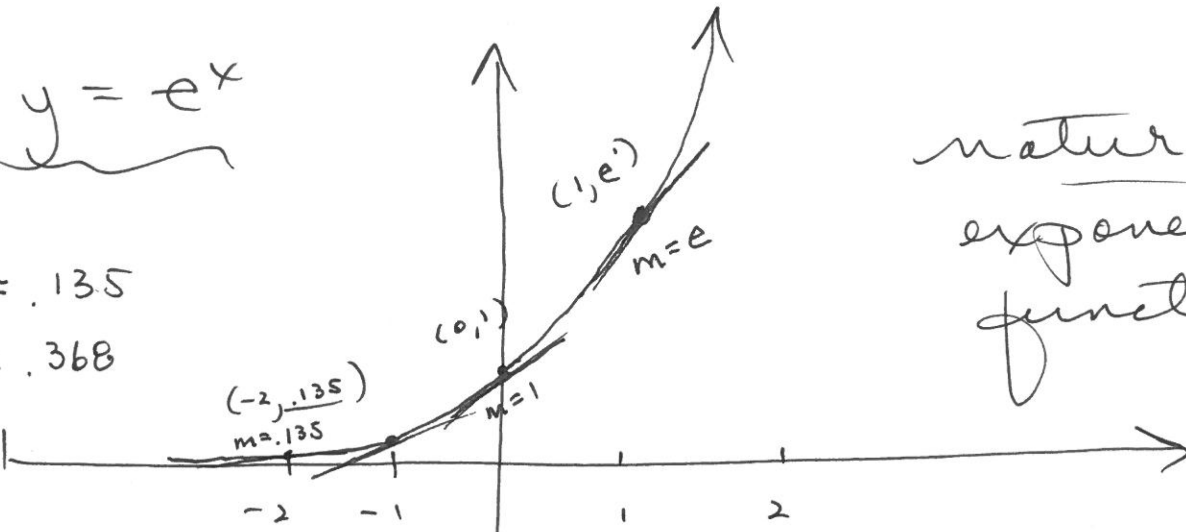
$\left(1+\frac{1}{10{,}000}\right)^{10{,}000} \approx 2.718$

$K = 1{,}000{,}000$

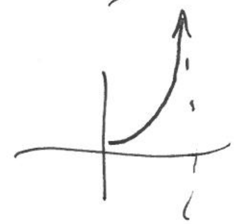
$\left(1+\frac{1}{1{,}000{,}000}\right)^{1{,}000{,}000} \approx 2.718$

$y = e^x$

x	y
-2	$e^{-2} \approx .135$
-1	$e^{-1} \approx .368$
0	$e^{0} = 1$
1	$e^{1} \approx 2.718$
2	$e^{2} \approx 7.389$



natural exponential function



DERIV OF $y = e^x$

$$y' = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$y' = \lim_{h \to 0} \frac{e^{x+h} - e^x}{h} \qquad e^{x+h} = e^x \cdot e^h$$

$$y' = \lim_{h \to 0} \frac{e^x \cdot e^h - e^x}{h} = \lim_{h \to 0} \frac{e^x(e^h - 1)}{h}$$

$$y' = e^x \cdot \boxed{\lim_{h \to 0} \frac{e^h - 1}{h}} \to 1$$

$h = .1$

$$\frac{e^{.1} - 1}{.1} \approx 1.051$$

$h = .001$

$$\frac{e^{.001} - 1}{.001} \approx 1.0005$$

$h = .000001$

$$\frac{e^{.000001} - 1}{.000001} \approx 1$$

$$\boxed{y' = e^x}$$

$y = e^x$

$y' = e^x$

$y'' = e^x$

$y''' = e^x$

$y = e^x \quad y' = e^x$

$y = e^{5x} \quad y' = e^{5x} \cdot \dfrac{d(5x)}{dx} \quad y = \exp(5x)$

$y' = e^{5x} \cdot 5 = 5 \cdot e^{5x}$

$y = 8^x$ or $y = 5^x$ or $y = 10^x$

in general:

$y = a^x \quad (a > 1)$

$e^{\boxed{\ln 5}} = 5 \quad e^{\ln 8} = 8 \quad \boxed{e^{\ln u} = u}$

$\ln 5 \approx 1.609 \quad e^{1.609} = 5$

$e^{(\text{the power that you raise } e \text{ to get } 5)} = 5$

$y = a^x = e^{\ln a^x} = e^{x \cdot \ln a}$

ex: $y = 2^x$ $\quad y = e^{\ln 2^x} = e^{x \cdot \ln 2}$

$y' = ?$

$y' = e^{x \cdot \ln 2} \cdot \dfrac{d(x \cdot \ln 2)}{dx}$

$y' = e^{x \cdot \ln 2} \cdot (\ln 2)$

$y' = 2^x \cdot \ln 2$

$y = \underbrace{2^x}$ $\quad y' = \underbrace{2^x} \cdot \ln 2$

in gen:

$$\boxed{y = \underline{a}^x \qquad y' = \underline{a}^x \cdot \underline{\ln a}}$$

$y = 10^x \quad y' = 10^x \cdot \ln 10$

$y = e^{5x} \qquad y' = e^{5x} \cdot 5$

$y = 4^{x^2+x} \qquad y' = 4^{x^2+x} \cdot \ln 4 \cdot (2x+1)$

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* generalized power rule:

$y = x^p \qquad y' = ? \quad = p \cdot x^{p-1}$

$y = e^{\ln x^p}$

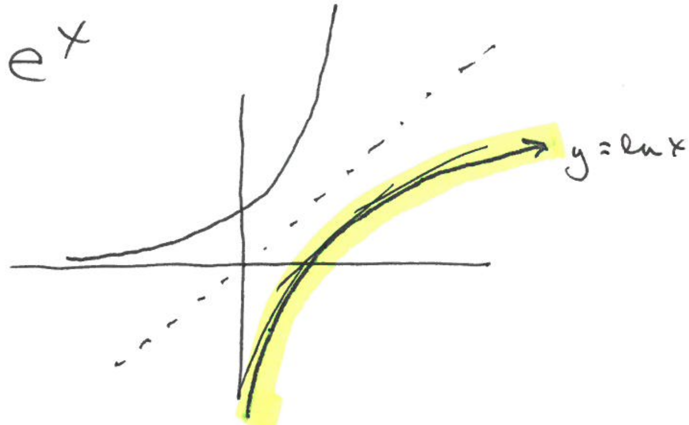
$y = \underbrace{e^{p \cdot \underline{\ln x}}}$

$y' = e^{p \cdot \ln x} \cdot \dfrac{d(p \cdot \underline{\ln x})}{dx}$

$y' = \underbrace{x^p \cdot p \cdot \dfrac{1}{x}} = p \boxed{x^p \cdot x^{-1}}$

$= p \cdot x^{p-1}$

$y = e^x \qquad y' = e^x$



INV:

$x = e^y$

y = the power you raise e to get x

$y = \log_e x$

$\boxed{y = \ln x}$ (natural log function)

$y' = ??$

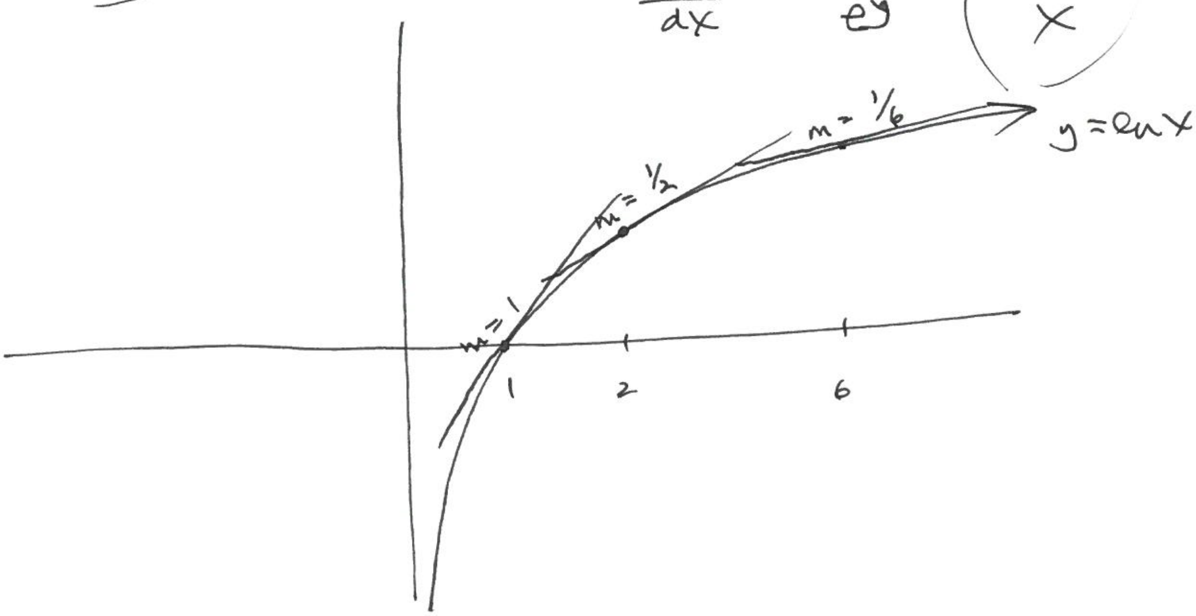
$y = \ln x$

$y' = \frac{1}{x}$

$\underbrace{x}_{} = \underbrace{e^y}_{}$ (take DERIV.)

$1 = e^y \cdot \frac{dy}{dx}$ (solve for $\frac{dy}{dx}$)

$\frac{dy}{dx} = \frac{1}{e^y} = \frac{1}{x}$



$$y = \ln(x) \qquad y' = \frac{1}{(x)}$$

$$y = \ln(x^2 - x + 5)$$

$$y' = \frac{1}{x^2 - x + 5} \frac{d(x^2 - x + 5)}{dx}$$

$$y' = \frac{1}{x^2 - x + 5} \cdot (2x - 1)$$

$$y' = \frac{2x - 1}{x^2 - x + 5}$$

MA 141-004 **S19** **Recitation 6, Feb 13**

1. Compute the limit:

$$\lim_{\theta \to 0} \frac{\cos\theta + 4\sin 3\theta - 1}{4\theta}$$

2. Fill in the table of derivatives for the 6 basic trigonometric functions.

$f(x)$	$f'(x)$	$f(x)$	$f'(x)$	$f(x)$	$f'(x)$
$\sin x$	$\cos x$		$\sec^2 x$	$\sec x$	
$\cos x$		$\cot x$			$-\csc x \cot x$

3. A right triangle with acute angle θ has a fixed opposite leg of 1 inch and adjacent leg with variable length x inches (x changes as θ changes). What is the rate of change of x with respect to θ?

4. Find the derivative $\frac{dy}{dx}$ of the given functions:

$$a)\ y = \sqrt[3]{(10-5x)\csc(x^3)} \qquad b)\ y = \frac{8\tan(x^2+x+1)}{7\sec(x^2+x+1)}$$

5. The curve (Maclaurin Trisectrix) defined by the equation $y^2(1+x) = x^2(3-x)$ is graphed below. Use implicit differentiation to determine $\frac{dy}{dx}$. Find the equation and sketch the tangent line to the curve at the point $(1,1)$.