ma 141-004 ①

Tuesday, February 12

- this week (2/12 & 2/14)

  - chain rule (2.5)
  - implicit diff (2.6)
  - logs; log diff (2.7)

CHAIN RULE:

$f(x) = (2x+1)^2$

power rule....

$f'(x) = 2(2x+1)^1 = 4x+2$

$f(x) = (2x+1)^2 = 4x^2+4x+1$

$f'(x) = \underline{8x+4}$

$f(x) = (\underline{2x+1})^2 \qquad f'(x) = 2(2x+1)^1 \cdot (2) = 8x+4$

① $y = [f(x)]^n \qquad y' = n[f(x)]^{n-1} \cdot \underline{f'(x)}$

deriv of "INSIDE" function

extended power rule:

$$f(x) = (3x^2 - x + 11)^{8}$$

$$f'(x) = 8(3x^2 - x + 11)^{7} \cdot (6x - 1)$$

or

$$f'(x) = (48x - 8)(3x^2 - x + 11)^{7}$$

2 COMPOSITION OF FUNCTIONS

$$f(x) = x^{8}$$

$$g(x) = (3x^2 - x + 11)$$

$$(f \circ g)(x) = f(g(x)) = f(3x^2 - x + 11)$$

$$= (3x^2 - x + 11)^{8}$$

CHAIN RULE:

$$y = (f \circ g)(x) = f(g(x))$$

$$y' = f'(g(x)) \cdot \boxed{g'(x)}$$

$$= 8(3x^2 - x + 11)^{7} \cdot (6x - 1)$$

ex:

$$f(x) = 3x + 5$$

$$3x+5$$

$$g(x) = 8x^2 - x$$

$$y = g(f(x)) = g(f(x))$$

$$y' = g'(f(x)) \cdot f'(x)$$

$$y' = \left[8(2(3x+5)) - 1(3\cancel{x+5})\right] \cdot 3$$

or

$$y = (g \circ f)(x) = g(f(x))$$

$$= g(3x+5)$$

$$= 8(3x+5)^2 - (3x+5)$$

$$y' = 8\left[2(3x+5)^1(3)\right]$$

$$y(x) = [\sin(8x)]^{5}$$

$$y'(x) = \underbrace{5[\sin 8x]^{4}} \cdot \underbrace{(\cos 8x)} \cdot \underline{8}$$

$$y'(x) = 40(\sin 8x)^{4} \cdot (\cos 8x)$$

3 y in terms of u

u in terms of x

$$\boxed{y = 3u^{2} + 5u - 11} \quad \text{find } \frac{dy}{du}$$

$$u = (8x^{2}+1) \quad \text{find } \frac{du}{dx}$$

$$\frac{dy}{du} \cdot \frac{du}{dx} = \frac{dy}{dx}$$

$$(6u+5) \cdot (16x) = \frac{dy}{dx}$$

$(8x^{2}+1)$

$$[6(8x^{2}+1)+5] \cdot [16x] = \frac{dy}{dx}$$

$$[48x^{2}+11] \cdot (16x) = \frac{dy}{dx}$$

$$\underline{(48)(16)x^{3}} + \underline{(11)(16)x} = \frac{dy}{dx}$$

$$y = 3(8x^2+1)^2 + 5(8x^2+1) - 11$$

$$\frac{dy}{dx} = 3\left[2(8x^2+1)^1 \cdot 16x\right] + 5\left[16x\right]$$

$$= 48x(16x^2+2) + 80x$$

$$= (48)(16)x^3 + (48)(2)x + 80x$$

$$= \underline{(48)(16)x^3} + \left[(48)(2) + 80\right] \cdot x$$

"chain" rule:

$$\frac{dy}{dr} \cdot \frac{dr}{dw} \cdot \frac{dw}{dq} \cdot \frac{dq}{dx} = \frac{dy}{dx}$$

$x^{1/2}$ $\quad (x^2-x+7)^{1/2}$

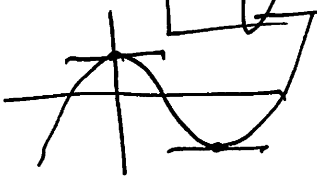
from ex 2.5:

$$f(x) = \sqrt{x^2-x+7} = (x^2-x+7)^{1/2}$$

$$f'(x) = \frac{1}{2}(x^2-x+7)^{-1/2} \cdot (2x-1)$$

$$\boxed{f'(x) = \frac{(2x-1)}{2\sqrt{x^2-x+7}}}$$

① $y'=0$ ✓
② y' undef.



$$y = \left(\frac{x^2+17}{2x^2+x+1}\right)^3$$

$$y' = 3\cdot\left(\frac{x^2+17}{2x^2+x+1}\right)^2 \cdot \frac{d\left[\frac{x^2+17}{2x^2+x+1}\right]}{dx}$$

$$y' = 3\cdot\left(\frac{x^2+17}{2x^2+x+1}\right)^2 \cdot \left[\frac{(2x^2+x+1)(2x)-(x^2+17)(4x+1)}{(2x^2+x+1)^2}\right]$$

$$y' = 3\cdot\frac{(x^2+17)^2}{(2x^2+x+1)^2}\cdot\frac{\quad}{(2x^2+x+1)^2}$$

$$y' = \frac{3(x^2+17)(\quad)}{(2x^2+x+1)^4}$$

① $y'=0$
② y' undef.

$$f(x) = \frac{2\sin 5x}{1 - 3\sec 5x}$$

$$f'(x) = \frac{(1-3\sec 5x)\left[2\cdot(\cos 5x)\cdot 5\right] - (2\sin 5x)\cdot\left[(-3)\cdot(\sec 5x\cdot\tan 5x)\cdot 5\right]}{(1-3\sec 5x)^2}$$

$$f'(x) = \frac{(10\cos 5x)(1-3\sec 5x) + (30\sin 5x)(\sec 5x\tan 5x)}{(1-3\sec 5x)^2}$$

find the points on the graph of $f(x) = (4x^2-8x+3)^4$ at which the tangent line is horizontal

$$m_{TAN} = \boxed{0 = f'(x)}$$

$$f'(x) = \boxed{4(4x^2-8x+3)^3\cdot(8x-8) = 0}$$

$4x^2-8x+3=0$ $\quad$ $8x-8=0$

$x=1$

$4x^2 - 8x + 3 = 0$

$\cancel{(4x \quad)(x \quad) = 0}$

$(2x - 1)(2x - 3) = 0$

$2x - 1 = 0$ or $2x - 3 = 0$

$x = 1/2$ $\qquad$ $x = 3/2$

from earlier $x = 1$

$(\frac{1}{2}, 0)$ & $(\frac{3}{2}, 0)$ & $(1, 1)$

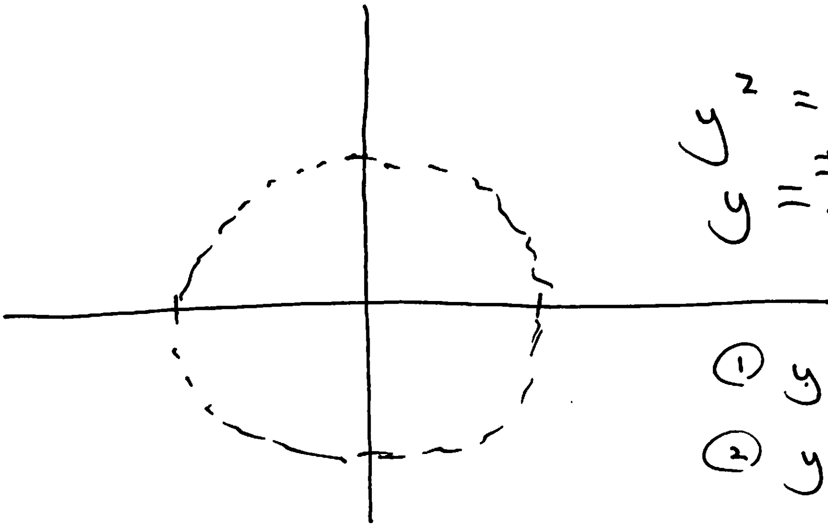
$f(1/2)$ $\qquad$ $f(3/2)$ $\qquad$ $f(1)$

$f(x) = (\underline{4x^2 - 8x + 3})^4$

$f(1) = [4(1)^2 - 8(1) + 3]^4 = 1$

IMPLICIT DIFFERENTIATION

$x^2+y^2=1$ circle; center $(0,0)$; $r=1$

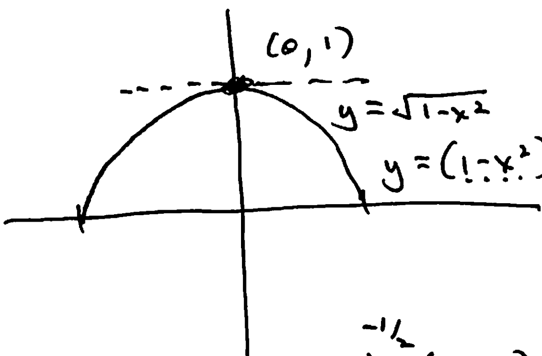


$$y^2=1-x^2$$

$$y=\pm\sqrt{1-x^2}$$

① $y=\sqrt{1-x^2}$

② $y=-\sqrt{1-x^2}$



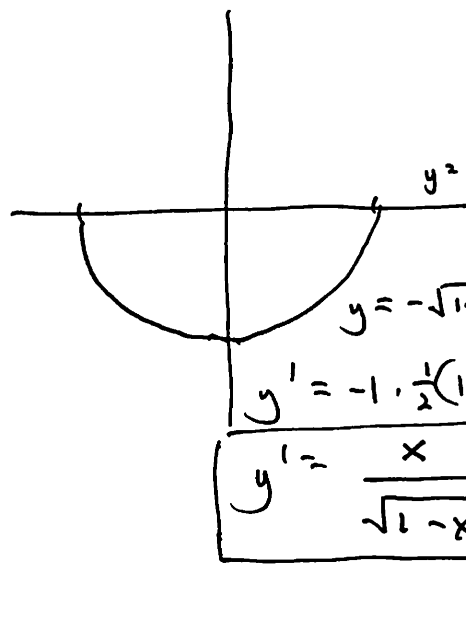
$(0,1)$

$y=\sqrt{1-x^2}$

$y=(1-x^2)^{1/2}$

$$y'=\frac{1}{2}(1-x^2)^{-1/2}(-2x)$$

$$\boxed{y'=\frac{-x}{\sqrt{1-x^2}}}=m_{TAN}$$



$y=-1\cdot(1-x^2)^{1/2}$

$y=-\sqrt{1-x^2}$

$$y'=-1\cdot\frac{1}{2}(1-x^2)^{-1/2}(-2x)$$

$$\boxed{y'=\frac{x}{\sqrt{1-x^2}}}=m_{T}$$

$(x)^2$

d: $2(x)^1\cdot\frac{dx}{dx}$

$(y)^2$

d: $2(y)^1\cdot\frac{dy}{dx}$

$$\underbrace{x^2}+\underbrace{y^2}=\underbrace{1}\quad \text{find } \frac{dy}{dx}$$

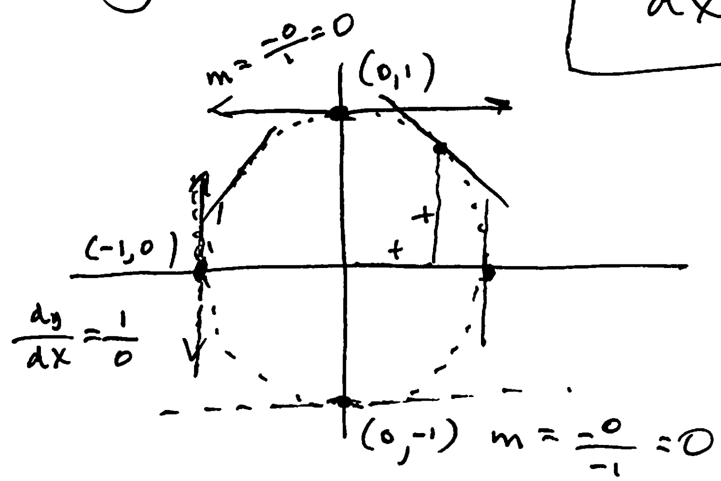
$$2x+2\cdot y^1\cdot\frac{dy}{dx}=0$$

$$2y\cdot\frac{dy}{dx}=-2x$$

$$\frac{\cancel{2y}\frac{dy}{dx}}{\cancel{2y}} = \frac{-2x}{2y}$$

$x^2+y^2=1$

$$\boxed{\frac{dy}{dx} = \frac{-x}{y}}$$



① $y' = \frac{-x}{\sqrt{1-x^2}}$ for $y=\sqrt{1-x^2}$

$$\boxed{y' = \frac{-x}{y}}$$

② $y' = \frac{-x}{-\sqrt{1-x^2}}$ for $y=-\sqrt{1-x^2}$

$$\boxed{y' = \frac{-x}{y}}$$

$$\boxed{x^2 + y^2 = 1}$$

derive with respect to "t"

$(x)^2$ DER $\underline{2(x)^1 \frac{dx}{dt}}$ $(y)^2$ DER $2(y)^1 \cdot \frac{dy}{dt}$

$$2x \cdot \frac{dx}{dt} + 2y \cdot \frac{dy}{dt} = 0$$

$$2xy^3 + 5\frac{x}{y} = 8$$

find $\frac{dy}{dx}$

$$\left[(2x)\frac{d(y)^3}{dx} + (y)^3 \cdot \frac{d(2x)}{dx}\right] + 5 \cdot \left[\frac{(y) \cdot \frac{d(x)}{dx} - (x) \cdot \frac{d(y)}{dx}}{(y)^2}\right] =$$

$$\left[(2x) \cdot 3y^2 \frac{dy}{dx} + y^3 \cdot (2)\right] + 5 \left[\frac{y \cdot (1) - x \cdot \frac{dy}{dx}}{y^2}\right] = 0$$

$$6xy^2 \cdot \frac{dy}{dx} + 2y^3 + \frac{5y - 5x\frac{dy}{dx}}{y^2} = 0$$

$$6xy^2 \cdot \frac{dy}{dx} + 2y^3 + \frac{5}{y} - \frac{5x \cdot \frac{dy}{dx}}{y^2} = 0$$

$$6xy^2 \cdot \frac{dy}{dx} - \frac{5x}{y^2} \cdot \frac{dy}{dx} = -2y^3 - \frac{5}{y}$$

$$\frac{\frac{dy}{dx}\cancel{\left(6xy^2 - \frac{5x}{y^2}\right)}}{\cancel{\left(6xy^2 - \frac{5x}{y^2}\right)}} = \frac{-2y^3 - \frac{5}{y}}{\left(6xy^2 - \frac{5x}{y^2}\right)}$$

$$\frac{dy}{dx} = \boxed{\frac{\left(-2y^3 - \frac{5}{y}\right)}{\left(6xy^2 - \frac{5x}{y^2}\right)}} \cdot \frac{y^2}{y^2}$$