MA 141 – 004 

Tuesday, February 5

today:

1. product rule
2. quotient rule
3. higher deriv.
4. "normal" (from exercises)
5. deriv. of TRIG

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product rule:

ex: $y = (2x-1)(5x+4)$

$y = f(x) \cdot g(x)$, $y' = ??$

$$y' = \lim_{h \to 0} \frac{f(x+h)\cdot g(x+h) - f(x)\cdot g(x)}{h}$$

$$y' = \lim_{h \to 0} \left[\frac{f(x+h)\cdot g(x+h) - f(x)\cdot g(x+h) + f(x)\cdot g(x+h) - f(x)\cdot g(x)}{h}\right]$$

$$y' = \lim_{h \to 0} \left( \frac{\boxed{g(x+h)}\left[f(x+h) - f(x)\right]}{h} + \frac{f(x)\left[g(x+h) - g(x)\right]}{h} \right)$$

$$\boxed{y' = g(x)\cdot f'(x) + f(x)\cdot g'(x)}$$ prod. rule

or

$$y' = f(x)\cdot g'(x) + g(x)\cdot f'(x)$$

ex: $y = (2x-1)(5x+4)$

product rule:

$$\rightarrow y' = (2x-1) \cdot \frac{d(5x+4)}{dx} + (5x+4) \cdot \frac{d(2x-1)}{dx}$$

$$\boxed{y' = (2x-1) \cdot (5) + (5x+4) \cdot (2)}$$

$$y' = \cancel{10x - 5 + 10x - 8 = 20x - 13 = y'}$$

$$y' = 10x - 5 + 10x + 8 = \boxed{20x + 3 = y'}$$

$m_{TAN}$

$y = (2x-1)(5x+4)$

other way:

$$y = (2x-1)(5x+4)$$

$$y = 10x^2 - 5x + 8x - 4$$

$$y = 10x^2 + 3x - 4$$

$$y' = 20x + 3$$

$$y = \left(16x - \frac{5}{x} + 8\sqrt{x}\right)\left(42x^5 - x^{2/3} + 5\right)$$

quotient rule:

$$y = \frac{f(x)}{g(x)} \qquad y' = ??$$

$$y' = \lim_{h \to 0} \frac{\frac{f(x+h)}{g(x+h)} - \frac{f(x)}{g(x)}}{h}$$

common denom...

$$y' = \lim_{h \to 0} \left[ \frac{f(x+h)}{g(x+h)} \cdot \frac{g(x)}{g(x)} - \frac{f(x)}{g(x)} \cdot \frac{g(x+h)}{g(x+h)} \right] \cdot \frac{1}{h}$$

$$y' = \lim_{h \to 0} \frac{[f(x+h) \cdot g(x) - f(x) \cdot g(x)] + [f(x) \cdot g(x) - f(x) \cdot g(x+h)]}{g(x+h) \cdot g(x) \cdot h}$$

$$y' = \lim_{h \to 0} \left( \frac{g(x)[f(x+h) - f(x)]}{g(x+h) \cdot g(x) \cdot h} + \frac{f(x)[g(x) - g(x+h)]}{g(x+\underset{0}{h}) \cdot g(x) \cdot h} \right)$$

$$y' = \cancel{\lim_{h \to 0}} \frac{g(x) \cdot f'(x)}{[g(x)]^2} - \frac{f(x) \cdot g'(x)}{[g(x)]^2}$$

$$\boxed{y' = \frac{g(x) \cdot f'(x) - f(x) \cdot g'(x)}{[g(x)]^2}}$$

QUOTIENT RULE

$$y = \frac{f(x)}{\boxed{g(x)}}$$

$$y = \frac{(3x+5)}{(2x-1)}$$

quotient rule:

$$y' = \frac{(2x-1)\cdot(3) - (3x+5)\cdot 2}{(2x-1)^2}$$

$$y' = \frac{6x-3-(6x+10)}{(2x-1)^2}$$

$$y' = \frac{\cancel{6x} - 3 - \cancel{6x} - 10}{(2x-1)^2} = \frac{-13}{(2x-1)^2}$$

$$y' = m_{TAN} = \frac{-13}{(2x-1)^2}$$

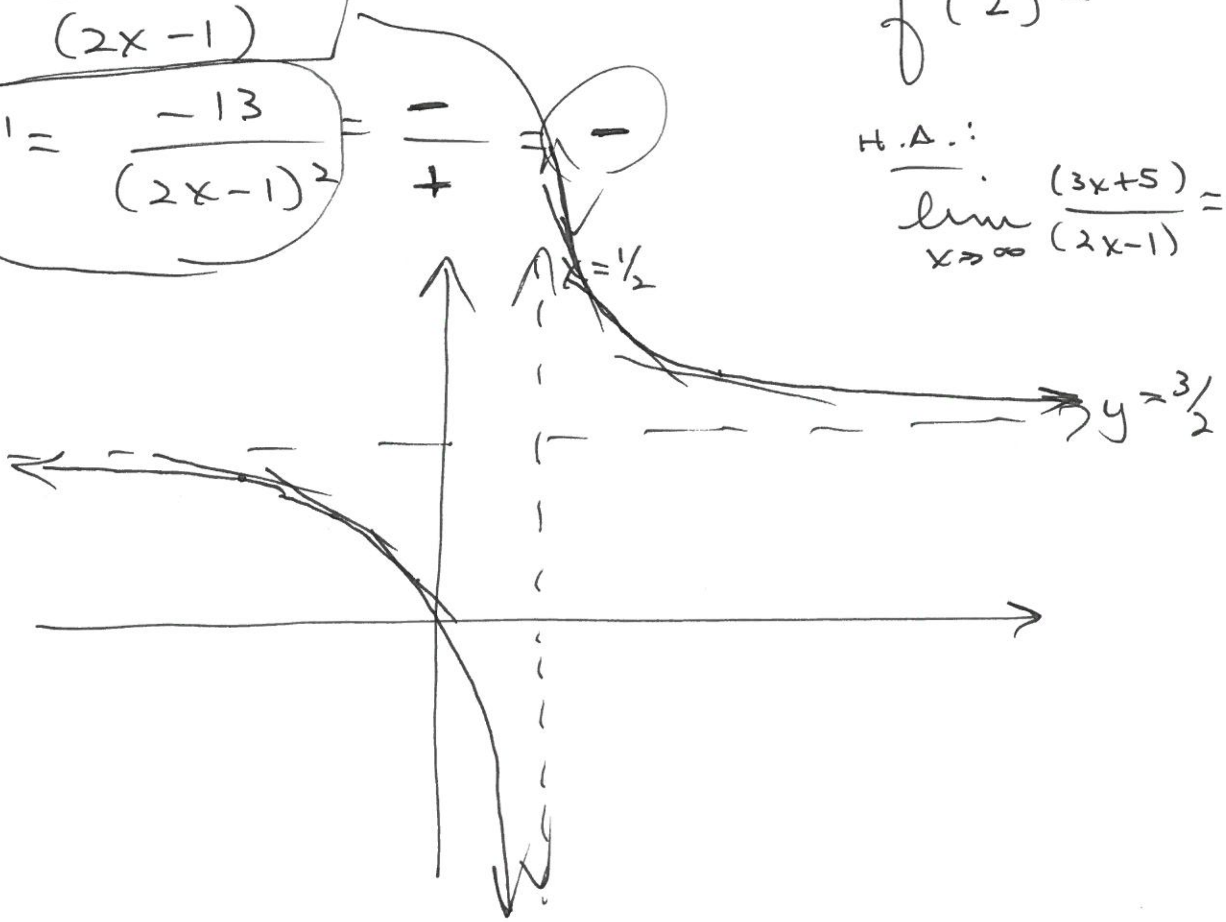
$$\boxed{y = \frac{(3x+5)}{(2x-1)}}$$

$f'(2) =$

$$y' = \frac{-13}{(2x-1)^2} = \frac{-}{+} = -$$

H.A.:

$$\lim_{x \to \infty} \frac{(3x+5)}{(2x-1)} = \frac{3}{2}$$



HIGHER DERIVATIVES:

$y = 3x^2 + 5x - 4 = f(x)$

$y' = 6x + 5 = f'(x)$

$y'' = 6 = f''(x)$

$y = \sim$

$\dfrac{dy}{dx} = y'$

$\dfrac{d^2y}{dx^2} = y''$

$s(t) = -16t^2 + 64t + 100$

$s(t)$: ~~MILES~~ FT.

$t$: ~~HR~~ SEC

DIST; HT; POS

GRAV

free falling (GRAVITY)

① $s(0) = 100$ $\quad t = 0$ sec $\quad s = 100$ ft.

$s(1) = -16 \cdot 1^2 + 64(1) + 100$

$t = 1$ sec. $\quad s = 148$ ft

② $s'(t) = v(t) = -32t + 64$

$t = 0 \quad v(0) = 64 \dfrac{ft}{sec.}$

$t = 1 \quad v(1) = -32(1) + 64 = 32 \dfrac{ft}{sec}$

⋮

③ $s''(t) = v'(t) = a(t) = -32$

ACCE

$t = 0 \quad a(0) = \boxed{-32 \dfrac{ft/sec}{sec}} = -32 \dfrac{ft}{sec^2}$

$t = 1 \quad a(1) = -32$ ✓

$t = 2 \quad a(2) = -32$ ✓

$$y = \frac{2x+1}{5x+3}$$

$$\frac{d[(5x+3)^2]}{dx} = \frac{d[25x^2+30x+9]}{dx}$$

quotient rule:

$$y' = \frac{(5x+3)(2) - (2x+1)(5)}{(5x+3)^2}$$

$$y' = \frac{10\cancel{x} + 6 \; - 10\cancel{x} - 5}{(5x+3)^2} = \frac{1}{(5x+3)^2}$$

$$y'' = \frac{\cancel{(5x+3)^2 \cdot (0)} - (1) \cdot \frac{d[(5x+3)^2]}{dx}}{[(5x+3)^2]^2}$$

$$y'' = \frac{-(1)(50x+30)}{[5x+3]^4}$$

$y = x^3 - 3x$

$y' = 3x^2 - 3 \quad = m_{TAN}$

slope of the normal: $\dfrac{-1}{3x^2-3}$

which are parallel to the line: $2x + 18y - 9 = 0$

$y = \dfrac{-2}{18}x + \dfrac{9}{18}$

$18y = -2x + 9$

same slope:

$$\frac{-1}{3x^2-3} = \frac{-1}{9}$$

$$3x^2 - 3 = 9$$
$$3x^2 = 12$$
$$x^2 = 4$$
$$x = \pm 2$$

$(2, f(2))$

$(-2, f(-2))$

PTS: $(2, 2)$, $(-2, -2)$

$(2, 2)$
$m = \frac{-1}{9}$

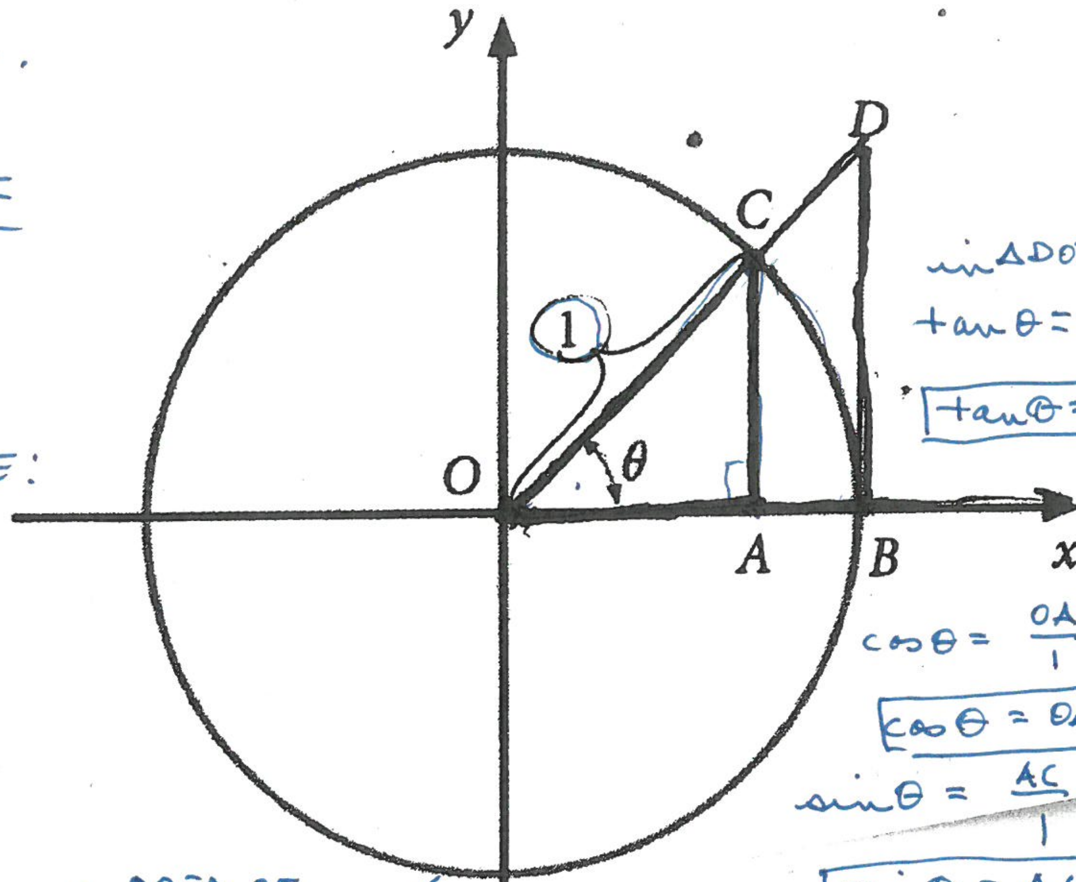
$(-2, -2)$
$m = \frac{-1}{9}$

$$y - y_1 = m(x - x_1)$$

⋮

prelim. to DERIV OF TRIG:

(UNIT CIRCLE: R = 1)




in ΔDOB:

$\tan\theta = \frac{DB}{1}$

$\boxed{\tan\theta = DB}$

$\cos\theta = \frac{OA}{1}$

$\boxed{\cos\theta = OA}$

$\sin\theta = \frac{AC}{1}$

$\boxed{\sin\theta = AC}$

AREAS:

AREA OF ΔCOA < AREA OF SECTOR COB < AREA OF ΔDOB

$$\tfrac{1}{2}(\text{base})(\text{ht}) < \frac{\theta}{2\pi}(\pi \cdot r^2) < \tfrac{1}{2}(\text{base})(\text{ht})$$

$$\tfrac{1}{2}(OA)(AC) \qquad \frac{\theta}{2\cancel{\pi}}(\cancel{\pi} \cdot 1^2) \qquad \tfrac{1}{2}(OB)(DB)$$

$$\tfrac{1}{2}\theta \qquad \tfrac{1}{2}(1)(DB)$$

$$\tfrac{1}{2}(DB)$$

$$\tfrac{1}{2}(OA)(AC) < \tfrac{1}{2}\theta < \tfrac{1}{2}(DB)$$

$$(OA)(AC) < \theta < (DB)$$

$$(\cos\theta)(\sin\theta) < \theta < (\tan\theta)$$

$$\cos\theta \cdot \sin\theta < \theta < \frac{\sin\theta}{\cos\theta}$$

$$\frac{\cos\theta \cdot \cancel{\sin\theta}}{\cancel{\sin\theta}} < \frac{\theta}{\sin\theta} < \frac{\cancel{\sin\theta}}{\cos\theta} \cdot \frac{1}{\cancel{\sin\theta}}$$

|

$$\boxed{* \qquad \cos\theta < \frac{\theta}{\sin\theta} < \frac{1}{\cos\theta}}$$

$$* \qquad \frac{1}{\cos\theta} > \frac{\sin\theta}{\theta} > \cos\theta$$

$$\lim_{\theta \to 0} \cos\theta < \boxed{\lim_{\theta \to 0} \frac{\theta}{\sin\theta}} < \lim_{\theta \to 0} \frac{1}{\cos\theta}$$

$$1 < \lim_{\theta \to 0} \frac{\theta}{\sin\theta} < 1$$

$$\lim_{\theta \to 0} \frac{\theta}{\sin\theta} = 1$$ (by the squeeze theorem)

$$1 > \lim_{\theta \to 0} \frac{\sin\theta}{\cancel{\sin\theta}} > 1$$

$$\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$$

(9)

$$\lim_{\theta \to 0} \frac{\theta}{\sin\theta} = \lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$$

$$\lim_{\theta \to 0} \frac{\sin\theta}{\theta}$$

$$\underline{\theta = \pi/6} \qquad \frac{\sin \pi/6}{\pi/6} = \frac{\frac{1}{2}}{.5235}$$

$$\theta = \pi/100 \qquad \frac{\sin(\pi/100)}{\frac{\pi}{100}} = \frac{.0031415}{.0031415} \approx 1$$

$\cancel{.000548}$

$5.48 \times 10^{-4}$