

Put all work and answers in the stamped blue book provided. Please put your **name**, **row letter** and **seat number** on the front of the blue book. One problem per page. The back of a sheet can serve as a new page. Scientific calculators may be used ; **no** graphing calculators. Fold the test copy and put it inside your blue book before turning it in. (8 questions – 12 points each; 4 points for following directions)

1.) Find $f'(x)$ using the **DEFINITION OF DERIVATIVE** ($\lim \dots$) : $f(x) = \frac{3}{2x-1}$

2.) Find the **vertex**, **all intercepts** and **graph**: $f(x) = x^2 - 8x + 13$

3.) Evaluate the following **limits**: a.) $\lim_{x \rightarrow \infty} \frac{3x+5}{2x^2-4x+1}$ b.) $\lim_{x \rightarrow 5} \frac{2x^2-10x}{x^2-4x-5}$

4.) **Graph** the following: $f(x) = \begin{cases} 1+x^2 & x < 1 \\ 4-x & x \geq 1 \end{cases}$

For this function, find: $\lim_{x \rightarrow 1^+} f(x)$ $\lim_{x \rightarrow 1^-} f(x)$ $\lim_{x \rightarrow 1} f(x)$ Is this function **continuous** at $x=1$? (verify; 3 possible steps)

5.) a.) Find the **domain** of the function (in interval notation): $f(x) = \sqrt{3x-2}$

b.) **Simplify** the difference quotient completely for $h \neq 0$: $\frac{3(x+h)^3 - 3x^3}{h}$

6.) Find the **average rate of change** of $f(x)$ from $x = 1$ to $x = 4$; find the **instantaneous rate of change** of $f(x)$ at $x=3$: $f(x) = 5x^2 - 6x + 4$

7.) a.) Find the **equation of the line** containing the points $(0,5)$ and $(2,-7)$.

(answer in “ $y = mx + b$ ” form)

b.) Find the **equilibrium point** for the following supply and demand functions:

$S(x) = 15x + 7000$ and $D(x) = -30x + 8800$

8.) Find the equation of the line tangent to $f(x) = 3x + \frac{8}{x}$ at the point $(2,10)$.

Bonus (5 points): Write out the words to NC State's Alma Mater.

(8 QUESTIONS - 12 POINTS EACH)

1.) $f(x) = \frac{3}{2x-1}$ find $f'(x)$ using the DEF OF DERIV
12 PTS

$$\begin{aligned}
 f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{\left[\frac{3}{2(x+h)-1} \right] - \left[\frac{3}{2x-1} \right]}{h} \\
 &= \lim_{h \rightarrow 0} \frac{\left[\frac{3}{2(x+h)-1} \right] \cdot \frac{(2x-1)}{(2x-1)} - \left[\frac{3}{2x-1} \right] \cdot \frac{(2(x+h)-1)}{(2(x+h)-1)}}{h} \\
 &= \lim_{h \rightarrow 0} \frac{3(2x-1) - 3(2(x+h)-1)}{(2(x+h)-1) \cdot (2x-1)} \cdot \frac{1}{h} \\
 &= \lim_{h \rightarrow 0} \frac{\cancel{6x} - \cancel{3} - \cancel{6x} - 6h + \cancel{3}}{(2(x+h)-1) \cdot (2x-1) \cdot h} \\
 &= \lim_{h \rightarrow 0} \frac{-6h}{(2(x+h)-1) \cdot (2x-1) \cdot h} \quad (h \neq 0) \\
 &= \lim_{h \rightarrow 0} \frac{-6}{(2(x+h)-1)(2x-1)} = \frac{-6}{(2x-1)^2}
 \end{aligned}$$

(page 2)

2.) $f(x) = x^2 - 8x + 13$

1-PTS find the vertex & all intercepts & graph

vertex: $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$ $\frac{-b}{2a} = \frac{-(-8)}{2(1)} = \frac{8}{2} = 4$

$f(4) = 4^2 - 8(4) + 13 = 16 - 32 + 13 = -3$

$V(4, -3)$

intercepts:

① y-int: (set $x=0$)
 $y = 0^2 - 8(0) + 13 = 13$

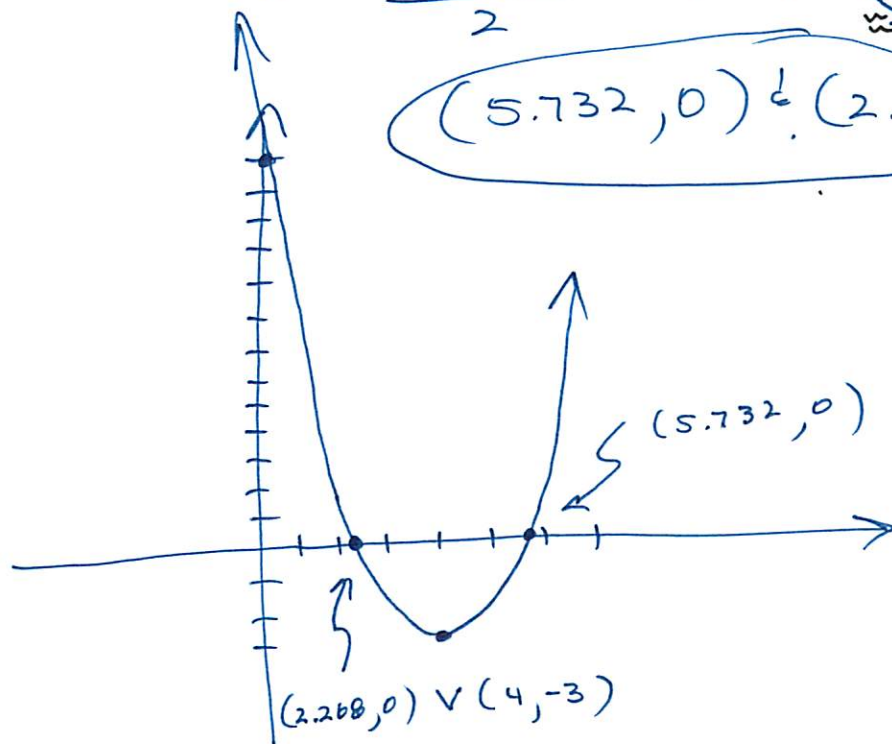
$(0, 13)$

② x-int: (set $y=0$)
 $0 = x^2 - 8x + 13$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{8 \pm \sqrt{64 - 4(1)(13)}}{2}$

$x = \frac{8 \pm \sqrt{12}}{2} = 4 \pm \sqrt{3} \Rightarrow \underline{5.732}$
 $\Rightarrow \underline{2.268}$

$(5.732, 0) \text{ \& } (2.268, 0)$



(page 3)

3.) a.) 6 PTS $\lim_{x \rightarrow \infty} \frac{3x+5}{2x^2-4x+1} = \lim_{x \rightarrow \infty} \frac{\frac{3x}{x^2} + \frac{5}{x^2}}{\frac{2x^2}{x^2} - \frac{4x}{x^2} + \frac{1}{x^2}}$

$= \lim_{x \rightarrow \infty} \frac{\frac{3}{x} + \frac{5}{x^2}}{2 - \frac{4}{x} + \frac{1}{x^2}} = \frac{0}{2} = 0$

(work not necessary; OK to use "shortcut")

b.) 6 PTS $\lim_{x \rightarrow 5} \frac{2x^2-10x}{x^2-4x-5} = \lim_{x \rightarrow 5} \frac{2x \cancel{(x-5)}}{\cancel{(x-5)}(x+1)}$

$= \lim_{x \rightarrow 5} \frac{2x}{x+1} = \frac{10}{6} = \frac{5}{3}$

$(x \neq 5)$

4.) 12 PTS $f(x) = \begin{cases} 1+x^2, & x < 1 \\ 4-x, & x \geq 1 \end{cases}$

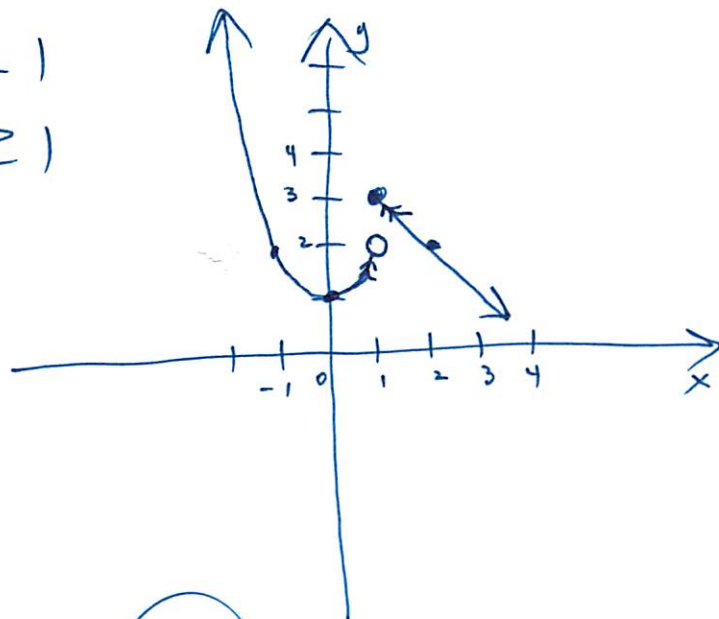
$(x < 1)$
 $y = 1+x^2$

x	y
1	2
0	1
-1	2
-2	5

delete

$(x \geq 1)$
 $y = 4-x$

x	y
1	3
2	2
3	1



$\lim_{x \rightarrow 1^+} f(x) = 3$ $\left\{ \begin{array}{l} \lim_{x \rightarrow 1^-} f(x) = 2 \\ \lim_{x \rightarrow 1} f(x) = \text{DOES NOT EXIST} \end{array} \right.$

NO
limit

(page 4)

continuous at $x=1$?

1.) $f(1)$ exists? yes, $f(1)=2$

2.) $\lim_{x \rightarrow 1} f(x)$ exists? no, $\lim$ D.N.E.

$\therefore f(x)$ is DISCONTINUOUS

5.) a.) find the domain: $y = \sqrt{3x-2}$

6 PTS

$$3x-2 \geq 0$$

$$3x \geq 2$$

$$x \geq \frac{2}{3}$$

$$\left[\frac{2}{3}, \infty \right)$$

b.) 6 PTS simplify $\frac{3(x+h)^3 - 3x^3}{h}$

$$\left[(x+h)^3 = x^3 + 3x^2h + 3xh^2 + h^3 \right]$$

$$= \frac{3(x^3 + 3x^2h + 3xh^2 + h^3) - 3x^3}{h}$$

$$= \frac{\cancel{3x^3} + 9x^2h + 9xh^2 + 3h^3 - \cancel{3x^3}}{h}$$

$$= \frac{\cancel{h}(9x^2 + 9xh + 3h^2)}{\cancel{h}} \quad (h \neq 0)$$

$$= \underline{9x^2 + 9xh + 3h^2}$$

(page 5)

6.) $f(x) = 5x^2 - 6x + 4$
12 PTS

average rate of change from $x=1$ to $x=4$:
 $(1, f(1))$ & $(4, f(4))$

$$\begin{aligned}\frac{f(4) - f(1)}{4 - 1} &= \frac{[5(4)^2 - 6(4) + 4] - [5(1)^2 - 6(1) + 4]}{3} \\&= \frac{[80 - 24 + 4] - [5 - 6 + 4]}{3} = \frac{[60] - [3]}{3} = \frac{57}{3} \\&= 19\end{aligned}$$

instantaneous rate of change at
 $x=3$

$$\begin{aligned}f'(x) = m_{\text{TAN}} &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\&= \lim_{h \rightarrow 0} \frac{[5(x+h)^2 - 6(x+h) + 4] - [5x^2 - 6x + 4]}{h} \\&= \lim_{h \rightarrow 0} \frac{5x^2 + 10xh + 5h^2 - 6x - 6h + 4 - 5x^2 + 6x - 4}{h} \\&= \lim_{h \rightarrow 0} \frac{10xh + 5h^2 - 6h}{h} \quad (h \neq 0) \\&= \lim_{h \rightarrow 0} (10x + 5h - 6) = 10x - 6\end{aligned}$$

$$\begin{aligned}m_{\text{TAN}} &= 10x - 6 = f'(x) \\f'(3) &= 10(3) - 6 \\&= 30 - 6 = 24\end{aligned}$$

(page 6)

7.) a.) $(0, 5) \dot{\sim} (2, -7)$ $m = \frac{5 - (-7)}{0 - 2} = \frac{12}{-2} = -6$
6PTS

$$y - y_1 = m(x - x_1) \quad y - 5 = -6(x - 0)$$

$$\boxed{y = -6x + 5}$$

b.) equilibrium point: $S(x) = D(x)$
6PTS

$$15x + 7000 = -30x + 8800$$

$$+30x \quad -7000 \quad +30x \quad -7000$$

$$45x = 1800$$

$$x = \frac{1800}{45} = 40$$

$$D(40) = -30(40) + 8800 = -1200 + 8800 = \underline{7600}$$

or
 $S(40) = 15(40) + 7000 = 600 + 7000 = \underline{7600}$

$$\boxed{(40, 7600) \dot{\sim} \text{EQ PT}}$$

(page 7)

$$8.) f(x) = 3x + \frac{8}{x} = 3x + 8x^{-1}$$

$$f'(x) = 3 + 8(-1 \cdot x^{-2}) = 3 - \frac{8}{x^2} = m_{\text{TAN}}$$

at the point $(2, 10)$

$$f'(2) = 3 - \frac{8}{2^2} = 3 - \frac{8}{4} = 1$$

EQ:

$$y - y_1 = m(x - x_1)$$

$$y - 10 = 1(x - 2)$$

$$y = x - 2 + 10$$

$$y = x + 8$$

BONUS (5 pt):

The Alma Mater of NC State

Where the winds of Dixie softly blow o'er the fields of Caroline,
There stands ever cherished, N.C. State, as thy honored shrine
So lift your voices! Loudly sing from hill to oceanside!
Our hearts ever hold you, N.C. State in the folds of our love
and pride

Words by Alvin Fountain : Class of '22

Music by Bonnie Norris: Class of '23