NORTH CAROLINA STATE UNIVERSITY Department of Mathematics
MA121-002 Test #1 Form B Thursday, January 31, 2019 Dr. J. Griggs

Put all work and answers in the stamped blue book provided. Please put your **name, row letter** and **seat number** on the front of the blue book. One problem per page. The back of a sheet can serve as a new page. Scientific calculators may be used; **no** graphing calculators. Fold the test copy and put it inside your blue book before turning it in. (8 questions – 12 points each; 4 points for following directions)

1.) Find $f'(x)$ using the **DEFINITION OF DERIVATIVE** (lim...): $f(x)=\dfrac{2}{3x-1}$

2.) Find the **vertex, all intercepts** and **graph**: $f(x)=x^2-6x+3$

3.) Evaluate the following **limits**: a.) $\lim\limits_{x\to\infty}\dfrac{5x+3}{4x^2-2x+1}$ b.) $\lim\limits_{x\to 5}\dfrac{3x^2-15x}{x^2-4x-5}$

4.) **Graph** the following: $f(x)=\begin{cases} 2+x^2 & x<1 \\ 5-x & x\geq 1 \end{cases}$

For this function, find: $\lim\limits_{x\to 1^+} f(x)$ $\lim\limits_{x\to 1^-} f(x)$ $\lim\limits_{x\to 1} f(x)$ Is the function **continuous** at x=1? (verify; 3 possible steps)

5.) a.) Find the **domain** of the function (in interval notation): $f(x)=\sqrt{2x-3}$

b.) **Simplify** the difference quotient completely for $h\neq 0$: $\dfrac{4(x+h)^3-4x^3}{h}$

6.) Find the **average rate of change** of $f(x)$ from x = 1 to x = 4; find the **instantaneous rate of change** of $f(x)$ at x=3: $f(x)=6x^2-5x+4$

7.) a.) Find the **equation of the line** containing the points $(0,7)$ *and* $(2,-5)$.

(answer in "$y=mx+b$" form)

b.) Find the **equilibrium point** for the following supply and demand functions:

$S(x)=15x+7000$ *and* $D(x)=-30x+8800$

8.) Find the equation of the line tangent to $f(x)=2x+\dfrac{6}{x}$ at the point (2,7).

Bonus (5 points): Write out the words to NC State's Alma Mater.

MA 121-002 TEST 1 B

(8 QUESTIONS - 12 POINTS EACH)

1.) 12 PTS

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$\left(f(x) = \frac{2}{3x-1} \right)$$

$$f'(x) = \lim_{h\to 0} \frac{\frac{2}{3(x+h)-1} - \frac{2}{3x-1}}{h}$$

$$= \lim_{h\to 0} \frac{\left[\frac{2}{3(x+h)-1}\right]\cdot\frac{(3x-1)}{(3x-1)} - \left[\frac{2}{3x-1}\right]\frac{(3(x+h)-1)}{(3(x+h)-1)}}{h}$$

$$= \lim_{h\to 0} \frac{2(3x-1) - 2(3(x+h)-1)}{(3(x+h)-1)\cdot(3x-1)} \cdot \frac{1}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{6x} \cancel{-2} - \cancel{6x} -6h \cancel{+2}}{(3(x+h)-1)\cdot(3x-1)\cdot h}$$

$$= \lim_{h\to 0} \frac{-6\cancel{h}^{1}}{(3(x+h)-1)(3x-1)\cdot\cancel{h}} \quad (h \neq 0)$$

$$= \lim_{h\to 0} \frac{-6}{(3(x+\underset{0}{h})-1)(3x-1)} = \frac{-6}{(3x-1)^2}$$

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2.) $f(x) = x^2 - 6x + 3$

12 PTS

find the vertex & all intercepts & graph:

vertex $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$ $\quad \frac{-b}{2a} = \frac{-(-6)}{2(1)} = \frac{6}{2} = 3$

$f(3) = 3^2 - 6(3) + 3 = 9 - 18 + 3 = -9 + 3 = -6$

$V(3,-6)$

intercepts

① x-int: (set $y=0$)

$0 = x^2 - 6x + 3$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{6 \pm \sqrt{36 - 4(1)(3)}}{2} = \frac{6 \pm \sqrt{24}}{2}$

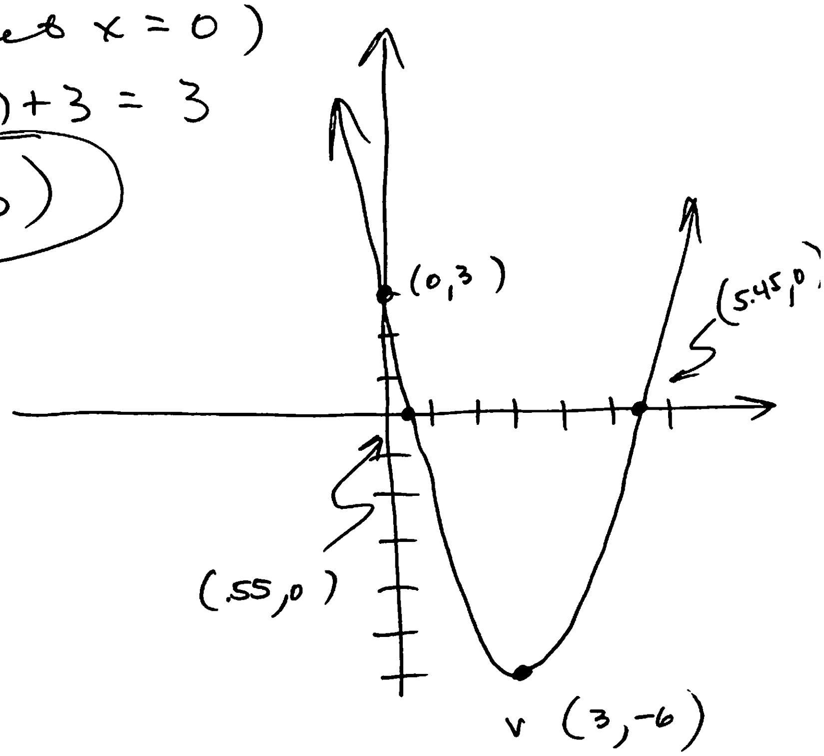
$x = \frac{6 \pm 2\sqrt{6}}{2} = 3 \pm \sqrt{6}$ ≈ 5.4495, $\approx .5505$

$(5.4495, 0)$ & $(.5505, 0)$

② y-int: (set $x = 0$)

$y = 0^2 - 6(0) + 3 = 3$

$(0, 3)$



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3.) a.) $\lim_{x\to\infty} \frac{5x+3}{4x^2-2x+1} = \lim_{x\to\infty} \frac{\frac{5x}{x^2}+\frac{3}{x^2}}{\frac{4x^2}{x^2}-\frac{2x}{x^2}+\frac{1}{x^2}}$

6 PTS

$= \lim_{x\to\infty} \frac{\frac{5}{x}+\frac{3}{x^2}}{4-\frac{2}{x}+\frac{1}{x^2}} = \frac{0}{4} = 0$

(~~work~~ not necessary, OK to use "shortcut")

b.) $\lim_{x\to 5} \frac{3x^2-15x}{x^2-4x-5} = \lim_{x\to 5} \frac{3x\cancel{(x-5)}}{\cancel{(x-5)}(x+1)}$ $(x \neq 5)$

6 PTS

$= \lim_{x\to 5} \frac{3x}{x+1} = \frac{15}{6} = \frac{5}{2}$

4.) $f(x) = \begin{cases} 2+x^2, & x<1 \\ 5-x, & x\geq 1 \end{cases}$

12 PTS

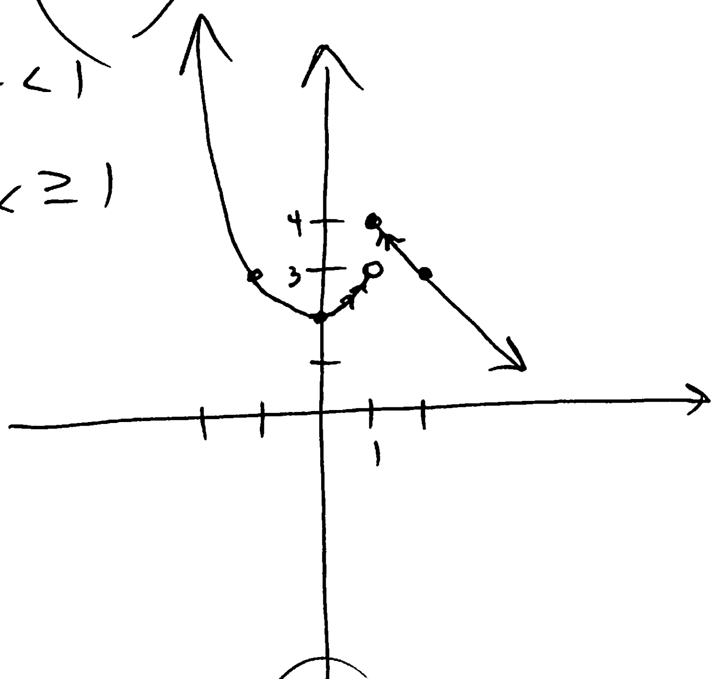
$(x<1)$
$y=2+x^2$

x	y
1	3
0	2
−1	3
−2	6

delete

$(x\geq 1)$
$y=5-x$

x	y
1	4
2	3
3	2



$\lim_{x\to 1^+} f(x) = 4$; $\lim_{x\to 1^-} f(x) = 3$; $\lim_{x\to 1} f(x)$ DOES NOT EXIST

NO limit

continuous at $x=1$?

1.) $f(1)$ exists? yes, $f(1)=2$

2.) $\lim_{x\to 1} f(x)$ exists? no, lim D.N.E.

$\therefore f(x)$ is DISCONTINUOUS

5.) a.) find the domain: $y=\sqrt{2x-3}$

6 PTS

$2x-3 \geq 0$

$2x \geq 3$

$x \geq 3/2$ $\quad [3/2, \infty)$

b.) simplify $\dfrac{4(x+h)^3 - 4x^3}{h}$

6 PTS

$[(x+h)^3 = x^3 + 3x^2h + 3xh^2 + h^3]$

$$= \frac{4(x^3+3x^2h+3xh^2+h^3)-4x^3}{h}$$

$$= \frac{\cancel{4x^3}+12x^2h+12xh^2+4h^3-\cancel{4x^3}}{h}$$

$$= \frac{\cancel{h}(12x^2+12xh+4h^2)}{\cancel{h}} \quad (h \neq 0)$$

$$= 12x^2+12xh+4h^2$$

3.) $f(x) = 6x^2 - 5x + 4$

12 PTS

average rate of change from $x=1$ to $x=4$:

$(1, f(1))$; $(4, f(4))$

$$\frac{f(4)-f(1)}{4-1} = \frac{[6(4)^2-5(4)+4]-[6(1)^2-5(1)+4]}{3}$$

$$= \frac{[96-20+4]-[6-5+4]}{3} = \frac{[80]-[5]}{3}$$

$$= \frac{75}{3} = 25$$

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instantaneous rate of change at $x=3$

$$f'(x) = m_{TAN} = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\{6(x+h)^2-5(x+h)+4\} - [6x^2-5x+4]}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{6x^2}+12xh+6h^2-\cancel{5x}-5h+\cancel{4}-\cancel{6x^2}+\cancel{5x}-\cancel{4}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(12x+6h-5)}{\cancel{h}} \quad (h \neq 0)$$

$$= \lim_{h\to 0} (12x+\underbrace{6h}_{0}-5) = 12x-5$$

$m_{TAN} = 12x - 5 = f'(x)$

$f'(3) = 12(3) - 5$

$= 36 - 5$

$= 31$

7.) a.) (0,7) & (2,-5) $m = \frac{7-(-5)}{0-2} = \frac{12}{-2} = -6$

6 PTS

$y - y_1 = m(x - x_1)$ $\quad y - 7 = -6(x - 0)$ $\quad \boxed{y = -6x + 7}$

b.) equilibrium point: $D(x) = S(x)$

6 PTS

$$\begin{aligned} -30x + 8800 &= 15x + 7000 \\ +30x - 7000 &\quad +30x - 7000 \\ \hline 1800 &= 45x \end{aligned}$$

$x = \frac{1800}{45} = 40$

$D(40) = -30(40) + 8800 = -1200 + 8800 = \underline{7600}$

or

$S(40) = 15(40) + 7000 = 600 + 7000 = \underline{7600}$

(40, 7600) is EQ. PT

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8.) $f(x) = 2x + \frac{6}{x} = 2x + 6x^{-1}$

$f'(x) = 2 + 6(-1 \cdot x^{-2}) = 2 - \frac{6}{x^2} = m_{TAN}$

at the point $(2, 7)$

$f'(2) = 2 - \frac{6}{2^2} = 2 - \frac{6}{4} = 2 - \frac{3}{2} = \frac{1}{2}$

EQ: $y - y_1 = m(x - x_1)$

$y - 7 = \frac{1}{2}(x - 2)$ $y = \frac{1}{2}x - 1 + 7$

$y = \frac{1}{2}x + 6$

BONUS (5 pt):

The Alma Mater of NC State

Where the winds of Dixie softly blow o'er the fields of Caroline,
There stands ever cherished, N.C. State, as thy honored shrine
So lift your voices! Loudly sing from hill to oceanside!
Our hearts ever hold you, N.C. State in the folds of our love and pride

Words by Alvin Fountain : Class of '22
Music by Bonnie Norris: Class of '23