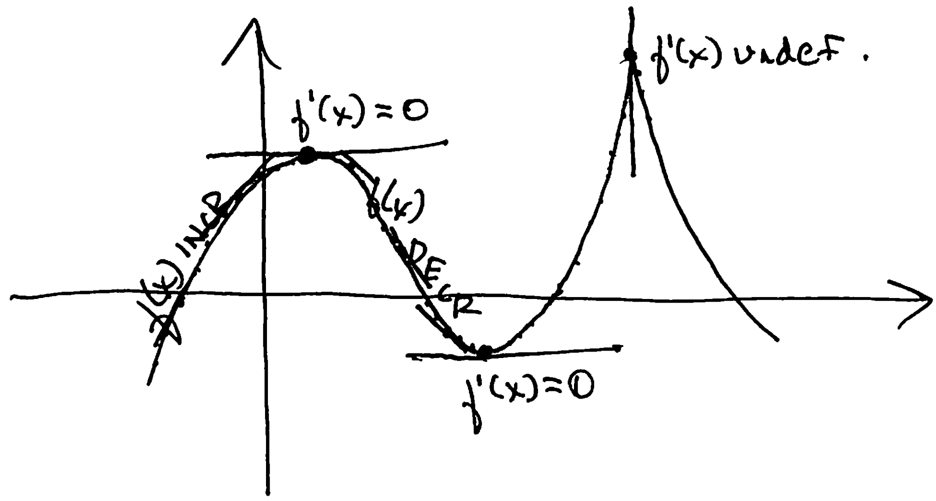
MA121-002

Tuesday, February 12

- this week (2.1; 2.2; 2.3; 2.4)
- Q1 (handed out today; due 2/14)

2.1: USE OF THE FIRST DERIVATIVE



$f'(x) = m_{TAN}$

(1) $f'(x) = 0$ ✓

HORIZONTAL TANGENT LINE

"FLAT"

(2) $f'(x)$ D.N.E. ✓

(undefined)

VERTICAL TANGENT LINE

"STEEP"

corner or cusp

(1) if $m_{TAN} = f'(x)$ is POS, then $f(x)$ is INCR.

(2) if $m_{TAN} = f'(x)$ is NEG., then $f(x)$ is DECR.

find the CRITICAL POINTS: $(x, f(x))$

critical values $\begin{cases} (1)\ f'(x) = 0 & \text{"FLAT"} \\ (2)\ f'(x) \text{ undef.} & \text{"STEEP"} \end{cases}$

ex: $f(x) = \underline{12} + \underline{9x} - \underline{3x^2} - \underline{x^3}$

polynomial
{ ① contin. ✓
② "smooth"

$f'(x) = 9 - 6x - 3x^2$

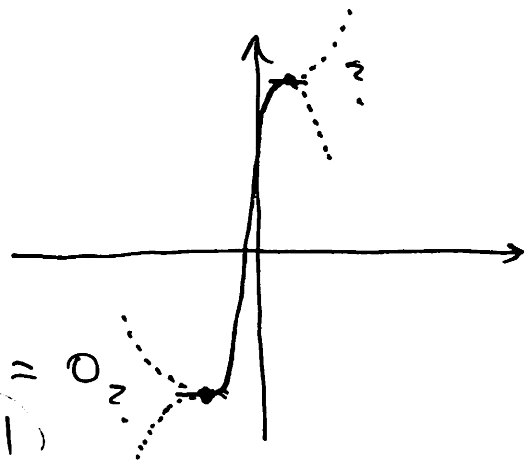
① $f'(x) = 0$ "FLAT" PLACES

$0 = 9 - 6x - 3x^2$ ✓

$0 = -1(3x^2 + 6x - 9)$

$0 = -3(x^2 + 2x - 3)$

$0 = \boxed{-3\,(x+3)(x-1)}$



$x+3 = 0$ or $x-1 = 0$

$x = -3$ $\quad$ $x = 1$

$(-3, \underline{-15})$ $\quad$ $(1, \underline{17})$

crit. pts:

$f(-3) = 12 + 9(-3) - 3(-3)^2 - (-3)^3$
$= 12 - 27 - \cancel{27} + \cancel{27} = -15$

$f(1) = 12 + 9(1) - 3(1)^2 - (1)^3$
$= 12 + 9 - 3 - 1 = 17$

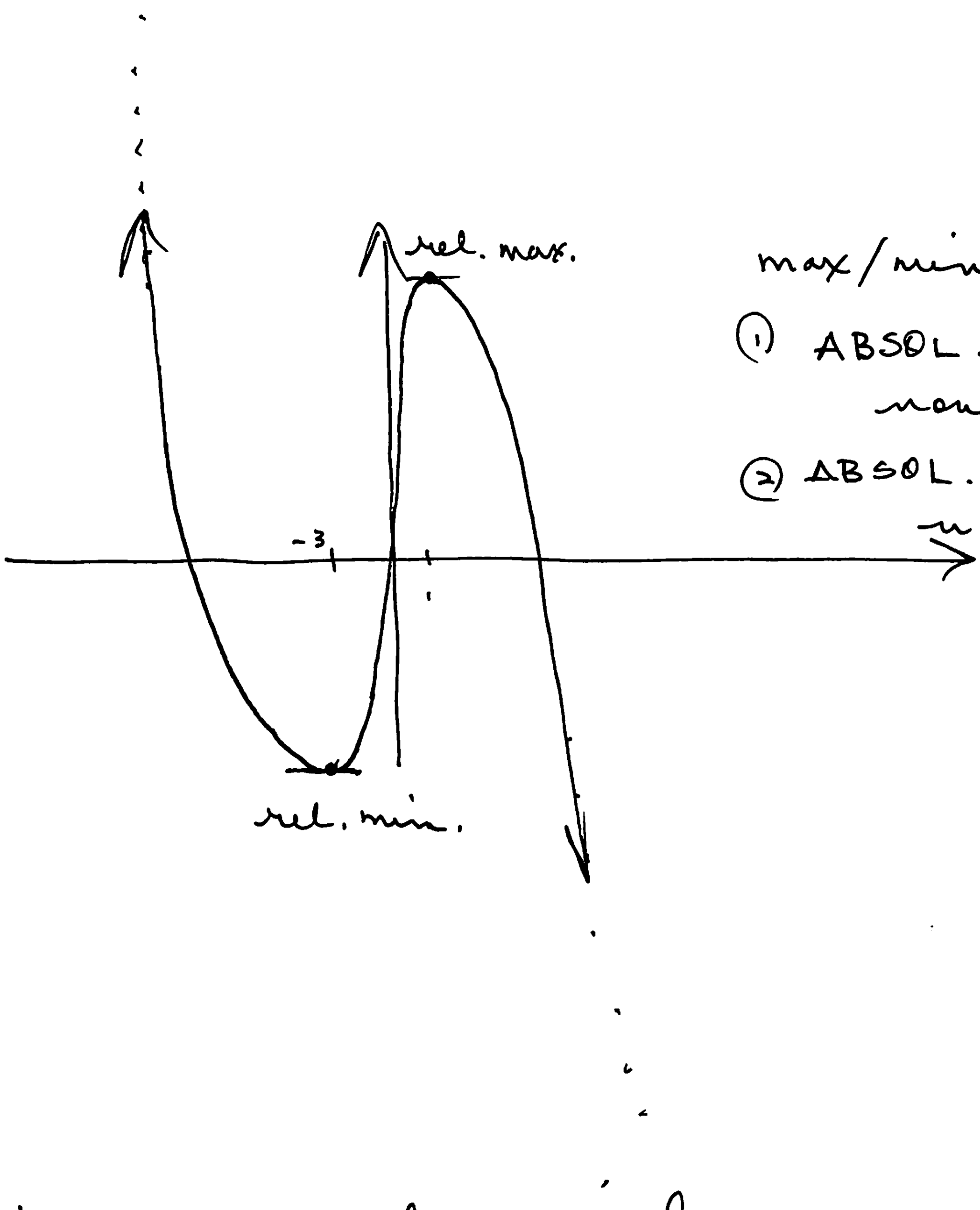
② $f'(x)$ ~~undef~~ ?? $\underline{\text{no}}$

$f'(x) = 9 - 6x - 3x^2$

$f'(x)$: $f'(x) = -3(x+3)(x-1)$

$f'(-4) = (-)(-)(-) = \text{NEG}$ | $f'(0) = (-)(+)(-) = \text{POS}$ | $f'(2) = (-)(+)(+) = \text{NEG}$

$f(x)$ DECR $\quad -3 \quad$ $f(x)$ INCR $\quad 1 \quad$ $f(x)$ DECR



max / min ? ?

① ABSOL. MIN ?
none

② ABSOL. MAX ?
none

ex: non-polynomial

$$y = 1 - x^{2/3}$$

① $y' = 0$ ② y' undef.

$$y' = f'(x) = m_{TAN} = 0 - 1 \cdot \left(\frac{2}{3} x^{-1/3}\right)$$

$$y' = \frac{-2}{3\sqrt[3]{x}}$$

$$y' \overset{?}{=} \boxed{0 \neq \frac{-2}{3\sqrt[3]{x}}}$$

$$-2 \overset{?}{\neq} 0$$

no "flat" places

$$y' = \text{undef} = \frac{-2}{3 \cdot \sqrt[3]{x}}$$

$m_{TAN} =$

$$3 \cdot \sqrt[3]{x} \overset{?}{=} 0$$

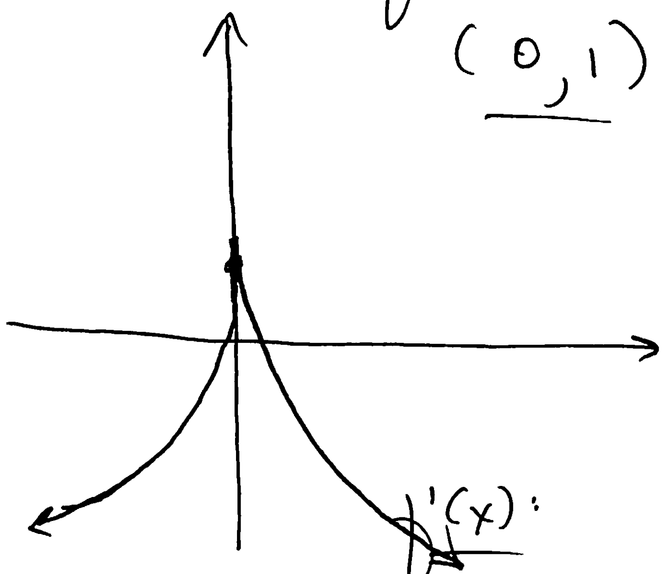
$$x = 0$$

VERTICAL TANGENT LINE

when $\underline{x=0}$ ... $f(0)$

$$f(x) = 1 - x^{2/3} = 1 - 0^{2/3} = \underline{1}$$

$\underline{(0,1)}$ vertical tangent line there



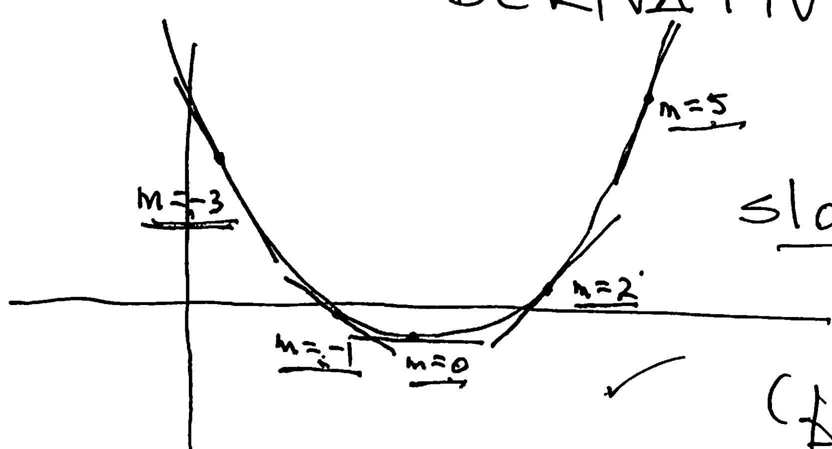
$$f'(x) = \frac{-2}{3\sqrt[3]{x}}$$

$f'(x)$:

$$f'(-1) = \frac{(-)}{(+)(-)} = + \qquad f'(1) = \frac{(-)}{(+)(+)} = -$$

$f(x)$ INCR. $\quad 0 \quad$ $f(x)$ DECR.

2.2: USE OF THE SECOND DERIVATIVE (shape...)

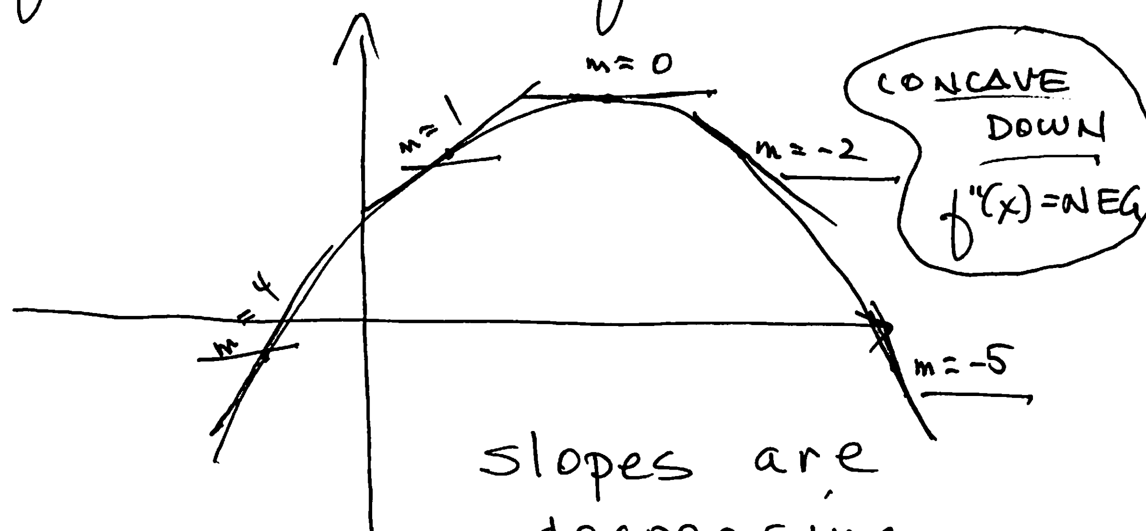


CONCAVE UP: $f''(x) = \text{POS}$

slopes are increasing

($f'(x)$ INCR.)

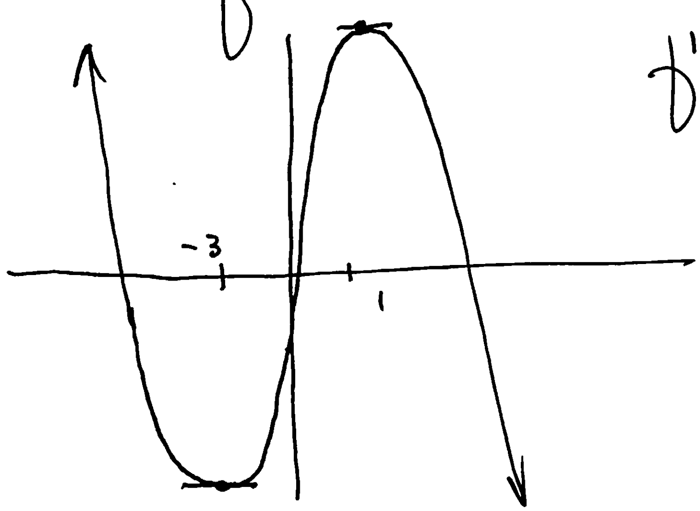
1. if $\boxed{f''(x) = +}$, then $\underline{f'(x) \text{ INCR}}$
2. if $\boxed{f''(x) = -}$, then $\underline{f'(x) \text{ DECR}}$.



slopes are decreasing
($f'(x)$ DECR.)

$f(-1) = 12 + 9(-1) - 3(-1)^2 - (-1)^3 = 12 - 9 - 3 + 1 = 1$

<u>ex</u>: $f(x) = 12 + 9x - 3x^2 - x^3$



$f'(x) = 9 \underline{- 6x} \underline{- 3x^2}$

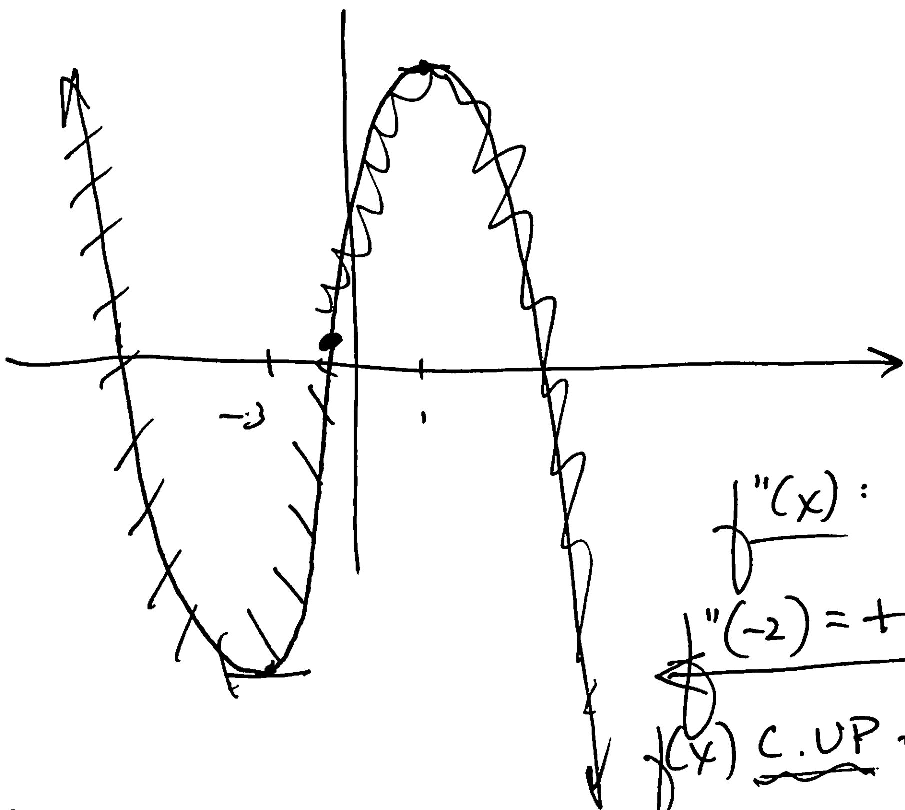
$f''(x) = \boxed{-6 - 6x}$

1. $\underline{f''(x) = 0}$ & $\underline{f''(x) \text{ undef.}}$

$-6 - 6x = 0$

$-6 = 6x$

$-1 = x \quad (-1, 1)$



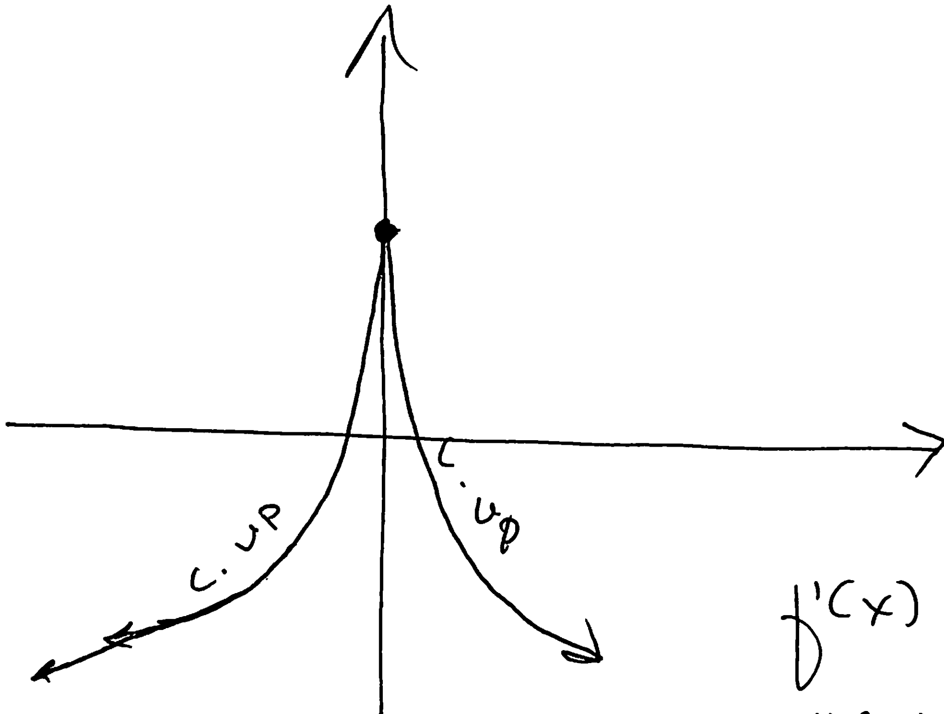
$f''(x)$: $f''(x) = -6-6x$

$f''(-2) = +$ | -1 | $f''(0) = -$

$f(x)$ C. UP | -1 | $f(x)$ C. DOWN

$(-1, 1)$ is a point of inflection

(changes concavity there)



$$f'(x) = -\frac{2}{3}x^{-1/3}$$

$$f''(x) = -\frac{2}{3}\left(-\frac{1}{3}x^{-4/3}\right)$$

$$f''(x) = \frac{2}{9}x^{-4/3}$$

$$f''(x) = \frac{2}{9(\sqrt[3]{x})^4}$$

① $\cancel{f''(x) = 0}$

$$\frac{2}{9(\sqrt[3]{x})^4} \stackrel{?}{=} 0$$

$2 \neq 0$

② $f''(x)$ undef.

$$\frac{2}{9(\sqrt[3]{x})^4} \text{ undef}$$

when $x = 0$

$(0, 1)$

$f''(x)$:

$f''(-1) = +$ | 0 | $f''(1) = +$

$f(x)$ C.UP | | $f(x)$ C.UP

NORTH CAROLINA STATE UNIVERSITY Department of Mathematics

MA121-002 Quiz #1 Tuesday, February 12, 2019 Dr. J. Griggs

In each problem, find $f'(x)$ using the recommended technique; simplify completely. Due at the beginning of class on Thursday, February 14. You can use the book, your notes. You are to work **individually** on this quiz. Turn in this sheet with work and answers:

1.) $f(x) = 8x^3 + 4\sqrt[3]{x} - \frac{5}{x^2} + 17$ (power rule)

2.) $f(x) = \frac{5x-3}{2x+1}$ (quotient rule)

3.) $f(x) = (2x+5)(3x^2 - 4x + 1)$ (product rule)

4.) $f(x) = \sqrt{3x^2 + 4x}$ (chain rule or extended power rule)