MA141-004 (1)

Tuesday, January 29

- finish ch 1 and 2.1
- test review

THURS, JAN. 31st, TEST #1
(ch 0; ch 1; 2.1)

<u>ch 1:</u>

<u>ave velocity</u> <u>vs</u> inst. velocity

$$s(t) = -16t^2 + 64t + 100$$

$\frac{s(t): \text{feet}}{t: \text{seconds}}$

dist; ht; pos.

$(1, s(1))$ & $(2, s(2))$

(1) ave vel. from $\boxed{t = 1 \text{ to } t = 2}$

$$\text{AVE VEL} = \boxed{\frac{s(2) - s(1)}{2 - 1}} \quad \frac{ft}{sec}$$

$$= \frac{[-16(2)^2 + 64(2) + 100] - [-16(1)^2 + 64(1) + 100]}{1}$$

$-64 \quad +128 \qquad -16 \quad +64 \quad +100$

$$= \frac{[164 \text{ ft}] - [148 \text{ ft}]}{1 \text{ sec}}$$

$$= \frac{16 \text{ ft}}{1 \text{ sec}} = 16 \text{ ft/sec}$$

$$s(t) = -16t^2 + 64t + 100$$

$$v(t) = s'(t) = \lim_{h \to 0} \frac{s(t+h) - s(t)}{h}$$

↑ instantaneous velocity

$$v(t) = \lim_{h \to 0} \frac{\left[-16(t+h)^2 + 64(t+h) + 100\right] - \left[-16t^2 + 64t + 100\right]}{h}$$

$$v(t) = \lim_{h \to 0} \frac{-16(t^2 + 2th + h^2) + 64t + 64h + 100 + 16t^2 - 64t - 100}{h}$$

$$v(t) = \lim_{h \to 0} \frac{\cancel{-16t^2} - 32th - 16h^2 \cancel{+64t} + 64h \cancel{+100} \cancel{+16t^2} \cancel{-64t} \cancel{-100}}{h}$$

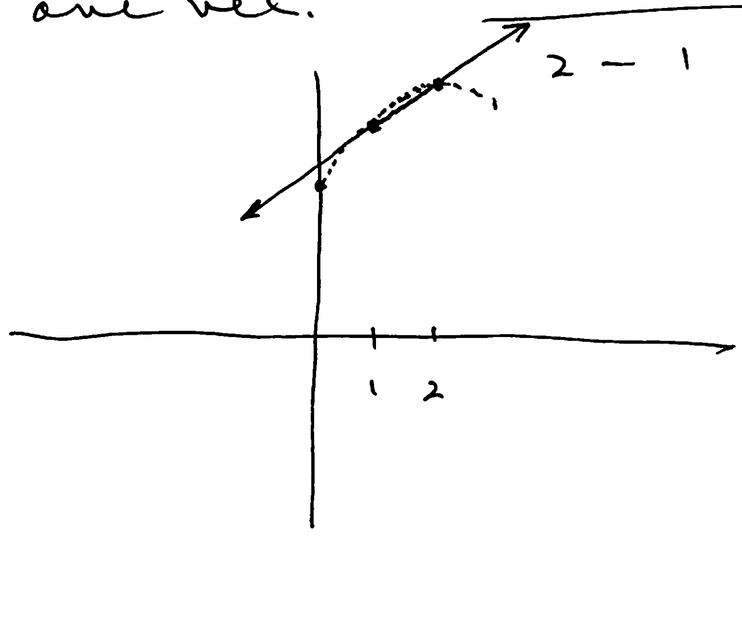
$$v(t) = \lim_{h \to 0} \frac{\cancel{h}(-32t - 16h + 64)}{\cancel{h}} \quad (h \neq 0)$$

$$v(t) = \lim_{h \to 0} (-32t - 16h + 64) = -32t + 64$$

$$s'(t) = \boxed{v(t) = -32t + 64}$$

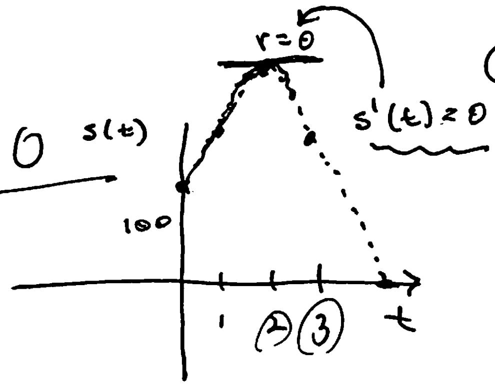
---

ave vel. $\dfrac{\overset{164 \; - \; 148}{s(2) - s(1)}}{2 - 1} = \underline{16 \text{ ft/sec}}$ ← m_{sec}



$s(t) = -16t^2 + 64t + 100$

dist; ht; pos.



1. $t=0$

$s(0) = -16(0)^2 + 64(0) + 100 = 100$ ft.

$(s_0 = 100$ ft.$)$

2. $t=1$ $\quad s(1) = -16(1)^2 + 64(1) + 100 = 148$ ft.

3. $t=2$ $\quad s(2) = -16(2)^2 + 64(1) + 100 = 164$ ft

4. $t=3$ $\quad s(3) = -16(3)^2 + 64(3) + 100 = 148$ ft

$= -144 + 192 + 100$

$v(t) = s'(t) = -32t + 64$

inst. vel

1. ~~$v=0$~~ $t=0$ $\quad v=?$ (initial vel.)

$v(0) = -32(0) + 64 = \boxed{64 \text{ ft/sec}}$

initial vel $= v_0 = 64$ ft/sec

2. $t=1$ $\quad v=?$

$v(1) = -32(1) + 64 = \boxed{32 \text{ ft/sec}}$

lost $\dfrac{32 \text{ ft/sec}}{\text{sec}}$

ACCEL DUE TO GRAVITY $= \dfrac{-32 \text{ ft/sec}}{\text{sec}}$

peak 3. $t=2$, $\quad v=?$

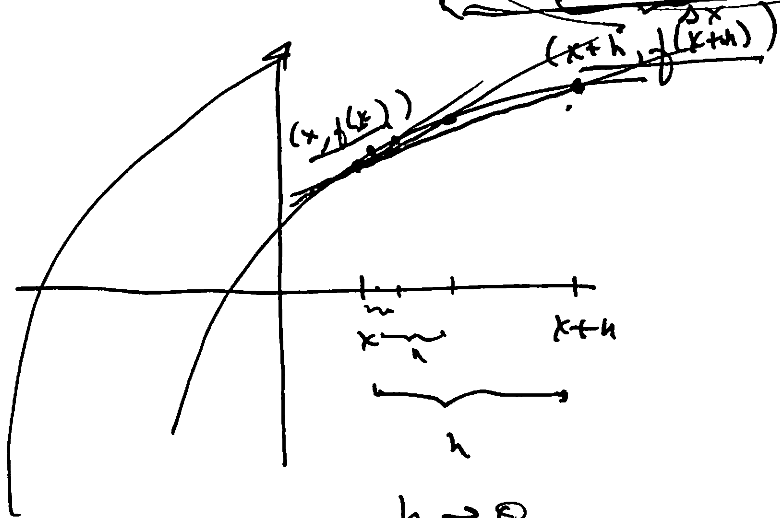
$v(2) = -32(2) + 64 = 0$ ft/sec

Ch 2:

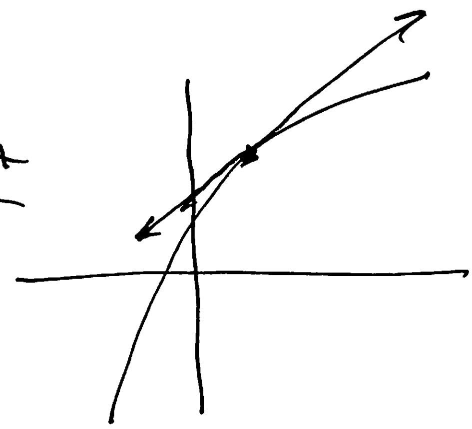
2.1:

DEF: (of DERIV.)

$$f'(x) = \lim_{h \to 0} \boxed{\frac{\overbrace{f(x+h) - f(h)}^{\Delta y}}{\underbrace{h}_{\Delta x}}} = m_{TAN}$$



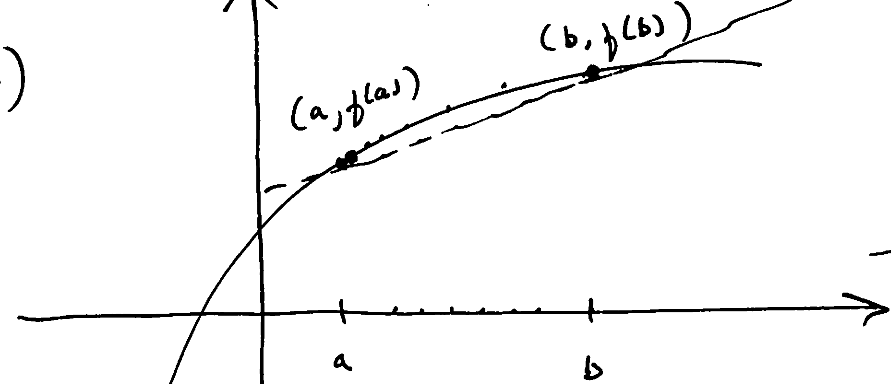
h = .000000000001

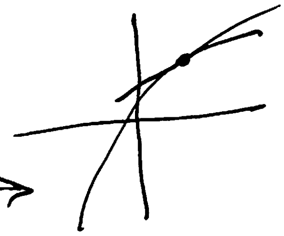


2 pts are closer tog. $\xrightarrow{h \to 0}$ 1 pt

ALT DEF (of DERIV.)

2 pts $(a, f(a))$ & $(b, f(b))$





$$f'(x) = \lim_{b \to a} \left[\frac{f(b) - f(a)}{b - a} \right]$$

v: gallons

t: minutes

$$\underline{V(t)} = \underline{50(t)^2}$$

(1) find ave rt. of change: $t=1$ to $t=2$

(2) find inst rt of change: $t=1$

(1) $$\frac{V(2)-V(1)}{2-1}\ \frac{\text{gal}}{\text{min}} = \frac{50(2)^2-50(1)^2}{1}\ \frac{\text{gal}}{\text{min}}$$

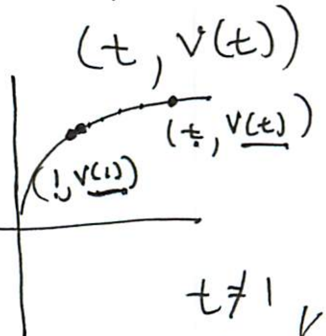
$$= \frac{200-50}{1} = 150\ \frac{\text{gal}}{\text{min}}$$

(2) $V'(t)$ = inst. rate of change of $\underline{\text{VOL}}$ at $t=1$

$(1, V(1))$

ACT. DEF

$$V'(t) = \lim_{t\to 1} \boxed{\frac{V(t)-V(1)}{t-1}}$$



$$V'(t) = \lim_{t\to 1} \frac{50t^2-50}{t-1}$$

$t \neq 1$

$$= \lim_{t\to 1} \frac{50(t^2-1)}{(t-1)} = \lim_{t\to 1} \frac{50(t+1)\cancel{(t-1)}}{\cancel{(t-1)}}$$

$$= \lim_{t\to 1} 50(t+1) = 100\ \frac{\text{gal}}{\text{min}}$$

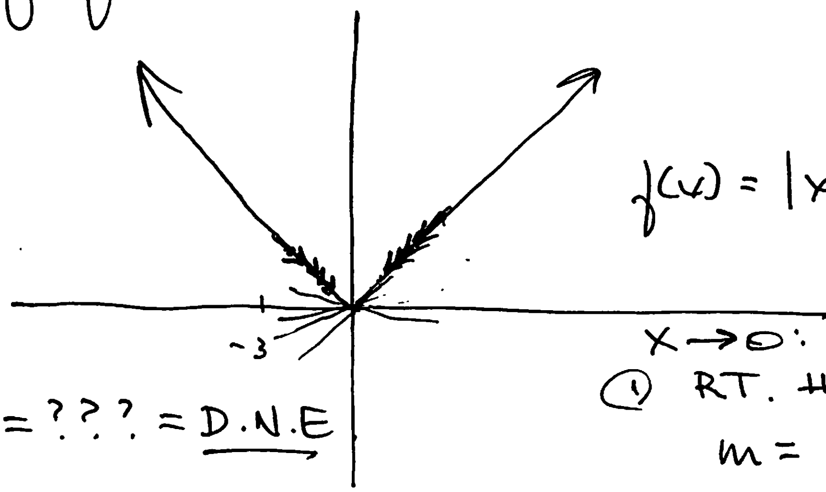
* if $f(x)$ is $\underbrace{\text{DIFF}}_{\text{able to find its deriv.}}$, then $f(x)$ is CONTIN.

(if P, then Q)

?? **CONVERSE:** (if Q, then P)

(NOT TRUE) switch hypothesis (if....) and the conclusion (then...)

if $f(x)$ is CONTIN, then $f(x)$ is ~~DIFF.~~



$f(x) = |x|$

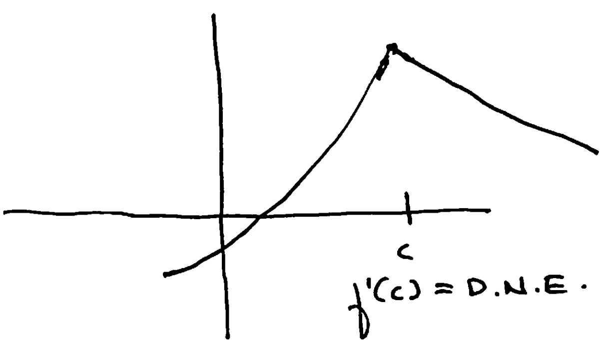
$f'(0) = ??? = \text{D.N.E}$

$x \to 0$:

(1) RT. HAND DERIV: $m = 1$

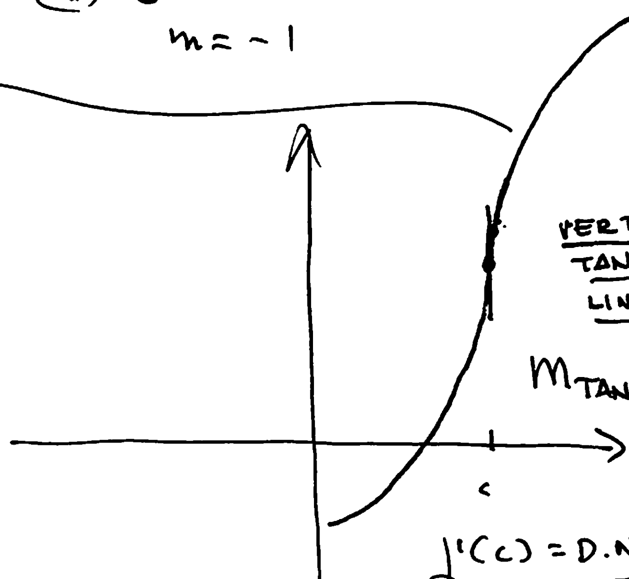
(2) LFT HAND DERIV: $m = -1$

NOT DIFF.



corner or cusp

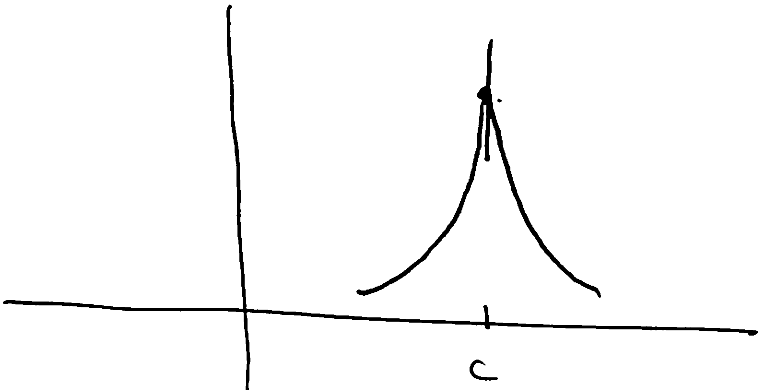
$f'(c) = \text{D.N.E.}$



VERT TANGENT LINE

$m_{TAN} = \text{UNDEF}$

$f'(c) = \text{D.N.E.}$



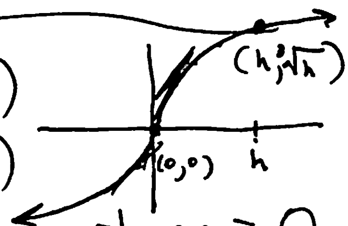
$f'(c) = $ D.N.E

(VERTICAL TANGENT LINE THERE)

$f(x) = \sqrt[3]{x} = \underline{x^{1/3}}$

$(0, \underline{0})$

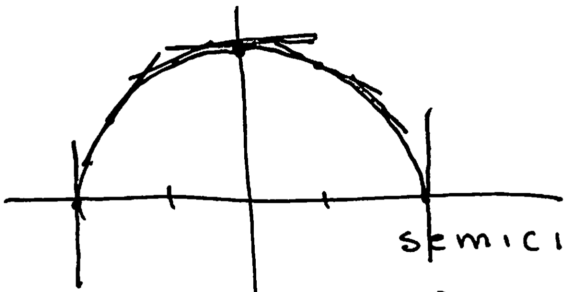
$(h, \underline{\sqrt[3]{h}})$



show that it is <u>not</u> diff. at $x = 0$

$$f'(x) = \lim_{h \to 0} \frac{h^{1/3} - 0}{h - 0} = \lim_{h \to 0} \frac{h^{1/3}}{h^1}$$

$$= \lim_{h \to 0} h^{-2/3} = \lim_{h \to 0} \frac{1}{h^{2/3}} = \underset{(\infty)}{\text{D.N.E}}$$



semicircle:

$f'(-2) = D.N.E.$

$f'(2) = D.N.E.$

$$s(t) = \frac{t}{t+1} \qquad t: \sec \qquad t \geq 0$$

$s(t): ft.$

$v(t) = ?$ $\quad v(t) = s'(t)$

$$v(t) = s'(t) = \lim_{h \to 0} \frac{s(t+h) - s(t)}{h}$$

$$= \lim_{h \to 0} \frac{\boxed{\frac{t+h}{t+h+1}} - \boxed{\frac{t}{t+1}}}{h}$$

$$= \lim_{h \to 0} \left[\frac{(t+h)(t+1)}{(t+h+1)(t+1)} - \frac{t \cdot (t+h+1)}{(t+1)\cdot(t+h+1)} \right] \cdot \frac{1}{h}$$

$$= \lim_{h \to 0} \left[\frac{\overset{\cancel{t^2}+\cancel{t}+th+h - \cancel{t^2} - th - \cancel{t}}{(t+h)(t+1) - (t)(t+h+1)}}{(t+h+1)(t+1)\cdot h} \right] = \lim_{h \to 0} \frac{\cancel{h}}{(t+\cancel{h}+1)(t+1)\cancel{h}}$$

$$= \frac{1}{(t+1)^2}$$

$$v(t) = \frac{1}{(t+1)^2}$$

$$v(8) = \frac{1}{(8+1)^2} = \frac{1}{81} \; ft/sec$$

domain:

$$f(x) = \frac{4-2x}{\boxed{10x-18}} = \frac{2(2-x)}{2(5x-9)}$$

$$10x - 18 = 0$$
$$10x = 18$$
$$x = \frac{18}{10} = \frac{9}{5} \quad (x \neq 9/5)$$

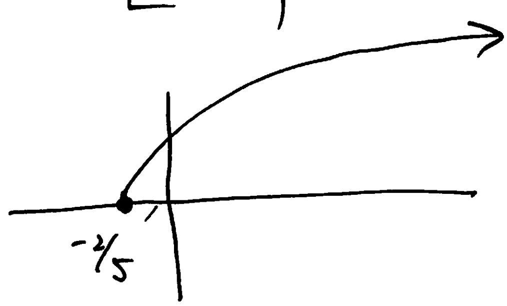
$$(-\infty, 9/5) \cup (9/5, \infty)$$

$x \in \mathbb{R},\ x \neq 9/5$

$$f(x) = \sqrt{5x+2}$$

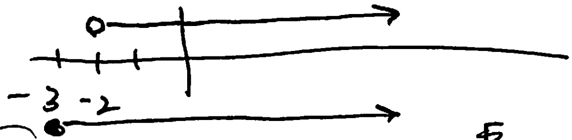
DOM:
$$5x + 2 \geq 0$$
$$\frac{5x}{5} \geq \frac{-2}{5}$$
$$x \geq -2/5$$

$$[-2/5, \infty)$$



RANGE: $y \geq 0$

$$\cancel{(x+2)}\,\frac{x-5}{\cancel{x+2}} \le 8(x+2)$$



① if $x+2>0$

$\boxed{x>-2}$

$x-5 \le 8(x+2)$

$x-5 \le 8x+16$

$\frac{-21}{7} \le \frac{7x}{7}$

$x \ge -3$

$\boxed{x \ge -3}$

$\boxed{(-2,\infty)}$

② if $x+2<0$

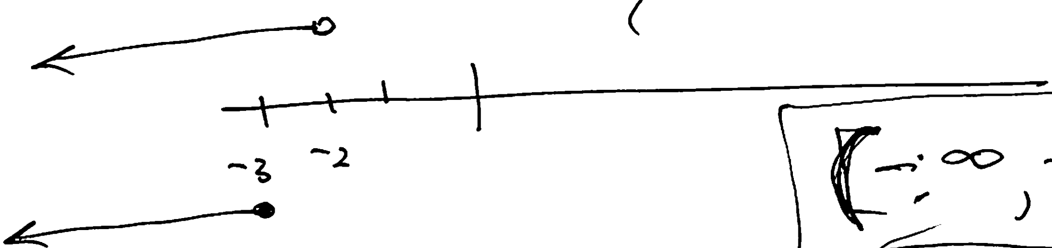
$\boxed{x<-2}$

$x-5 \ge 8(x+2)$

$x-5 \ge 8x+16$

$\frac{-21}{7} \ge \frac{7x}{7}$

$\boxed{x \le -3}$



$\boxed{(-\infty,-3]}$