MA141 - 004

Thursday, January 24

I.V.T.:

$y = 0$

$(f(x) = 0)$

Show that $f(x)$ has a root (zero) in $[1, 2]$

$g(x) = 3x^{4/3}$

$f(x) = 4x^3 - 6x^2 + 3x - 2$

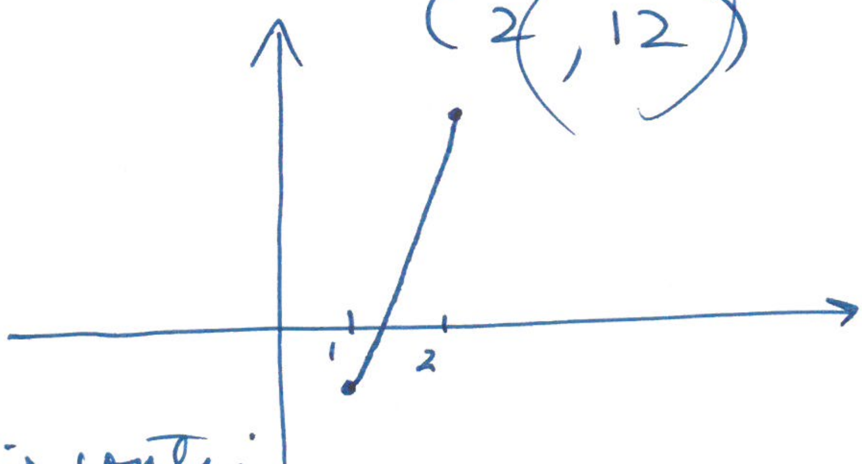
contin? polynomial $\therefore$ contin.

$f(1) = 4(1)^3 - 6(1)^2 + 3(1) - 2 = -1$

$(1, -1)$

$f(2) = 4(2)^3 - 6(2)^2 + 3(2) - 2 = 32 - 24 + 6 - 2 = 12$

$(2, 12)$



if contin
the function takes on all values ~~for~~ from -1 to 12 as x goes from 1 to 2.

$\rightarrow$ $\underline{0}$ is in that range from recitation worksheet:

$f(x) = \sin\left(\frac{\pi \cdot x}{4}\right)$ $\quad g(x) = x^2 - x$

if they intersect, they are equal at that point

$f(x) - g(x) = 0$

$h(x) = \sin\left(\frac{\pi \cdot x}{4}\right) - (x^2 - x)$

find the root (zero) of $h(x)$

(1) contin?

- ¿ $x^2 - x$ contin? polynomial
- ¿ $\sin\left(\frac{\pi \cdot x}{4}\right)$ contin? sine is contin

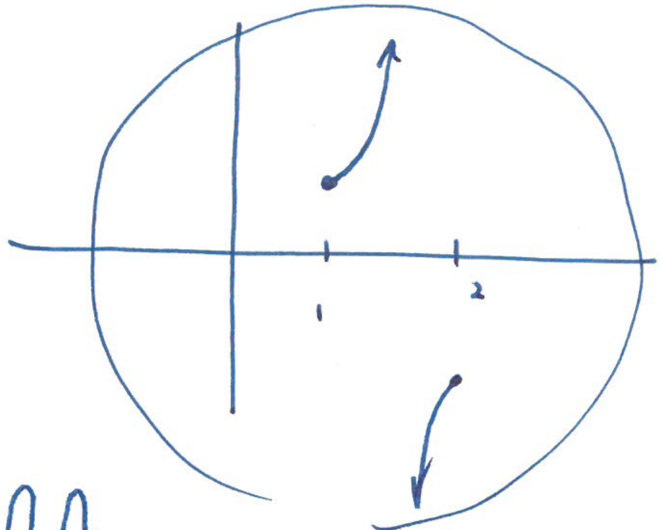
sum or diff of contin functions $\longrightarrow$ also contin.

$h(x) = \boxed{\sin\left(\frac{\pi \cdot x}{4}\right) - (x^2 - x) = 0}$

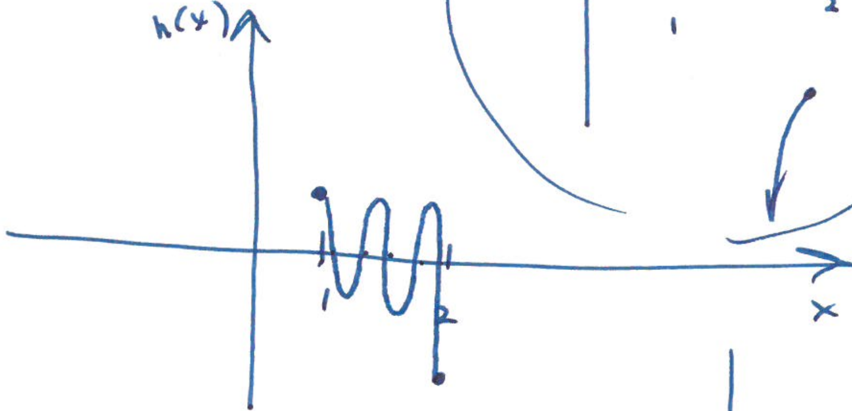
(2) $[1, 2]$

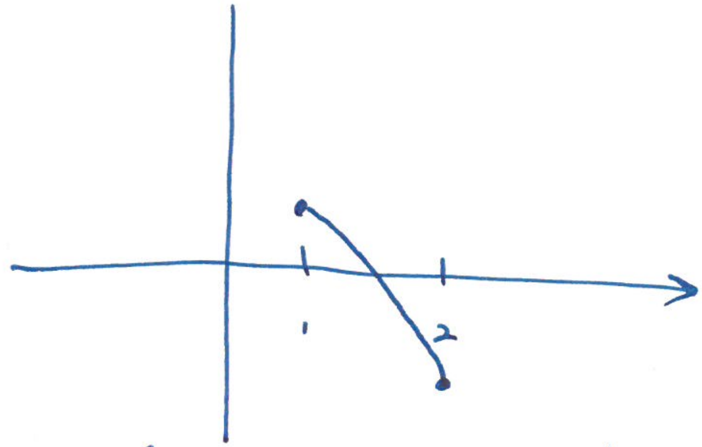
$h(1) = \sin\left(\frac{\pi \cdot 1}{4}\right) - (1^2 - 1) = \frac{\sqrt{2}}{2}$

$h(2) = \sin\left(\frac{\pi \cdot 2}{4}\right) - (2^2 - 2) = -1$



root in [1,2]?





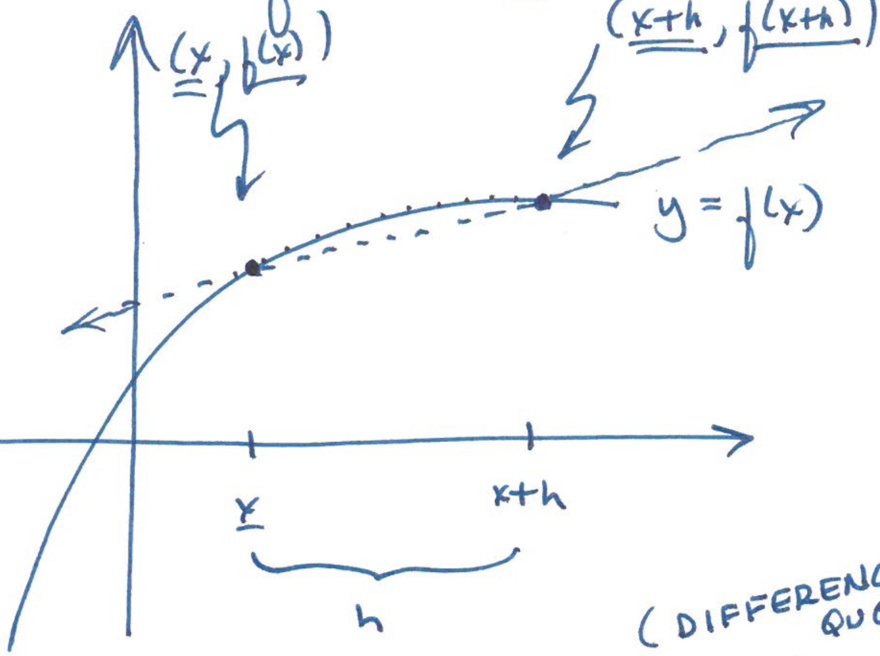
there is at least one pt in [1,2] where h(x) is zero

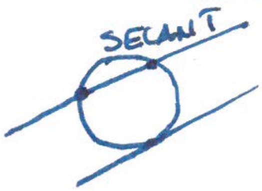
ch 1:

average rate of change

x: initial x-value

h: change in x





(DIFFERENCE QUOTIENT)

$$m_{SEC} = \text{average rate of change}$$

$$m_{SEC} = \frac{f(x+h) - f(x)}{(x+h) - x} = \frac{f(x+h) - f(x)}{h}$$

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ex: $f(x) = 3x^2 + 5x - 8$

find the average rate of change

$$m_{SEC} = \frac{f(x+h) - f(x)}{h}$$

$$= \frac{\left[3(x+h)^2 + 5(x+h) - 8\right] - \left[3x^2 + 5x - 8\right]}{h}$$

$$= \frac{3(x^2+2xh+h^2)+5x+5h-8-3x^2-5x+8}{h}$$

$$= \frac{\cancel{3x^2}+6xh+3h^2+\cancel{5x}+5h-\cancel{8}-\cancel{3x^2}-\cancel{5x}+\cancel{8}}{h}$$

$$= \frac{\cancel{h}(6x+3h+5)}{\cancel{h}} \quad (h \neq 0)$$

$$m_{sec} = \underline{6x+3h+5}$$

x: initial x-value

h: change in x

$f(x) = 3x^2+5x-8$ $\quad \boxed{m_{sec} = 6x+3h+5}$

$(1, 0)$ & $(3, 34)$ $\quad x=1$, $h=2$

$f(1) = 3(1)^2+5(1)-8$

$f(1) = 3+5-8$

$f(1) = 0$

$f(3) = 3(3)^2+5(3)-8$

$f(3) = 27+15-8$

$f(3) = 34$

$$m_{sec} = 6(1)+3(2)+5$$

$$= 6+6+5 = 17$$

$$m = \frac{34-0}{3-1} = \frac{34}{2} = 17$$

distance; height; position

t : sec

$s(t)$: feet

$$s(t) = -16t^2 + 48t + 50$$

$$\frac{\text{ft}}{\text{sec}}$$

$$m_{sec} = \frac{\overset{\text{ft}}{\boxed{s(t+h)}} - \overset{\text{ft}}{\boxed{s(t)}}}{\underset{\text{sec}}{\boxed{(t+h)}} - \underset{\text{sec}}{\boxed{t}}} = \frac{\left(-16(t+h)^2 + 48(t+h) + 50\right) - \left(-16t^2 + 48t + 50\right)}{h}$$

$$m_{sec} = \text{ave velocity} = \frac{\cancel{-16t^2} - 32th - 16h^2 + \cancel{48t} + 48h + \cancel{50} \cancel{+16t^2} \cancel{-48t} \cancel{-50}}{h}$$

$$= \frac{\cancel{h}(-32t - 16h + 48)}{\cancel{h}} \quad (h \neq 0)$$

$$= -32t - 16h + 48 \ \frac{\text{ft}}{\text{sec}}$$

$$f(x) = \frac{2x+1}{5x+3}$$

find the average rate of change

$$m_{sec} = \frac{f(x+h) - f(x)}{h}$$

$$\frac{5 \cdot 1}{5 \cdot 3} - \frac{1 \cdot 3}{5 \cdot 3}$$

$$\frac{5}{15} - \frac{3}{15}$$

$$\frac{5-3}{15}$$

$$= \frac{\left[\frac{2(x+h)+1}{5(x+h)+3}\right] - \left[\frac{2x+1}{5x+3}\right]}{h}$$

common denom

$$= \left[\frac{(2x+2h+1)}{[5(x+h)+3]}\frac{(5x+3)}{(5x+3)} - \frac{(2x+1)[5(x+h)+3]}{(5x+3)[5(x+h)+3]}\right] \cdot \frac{1}{h}$$

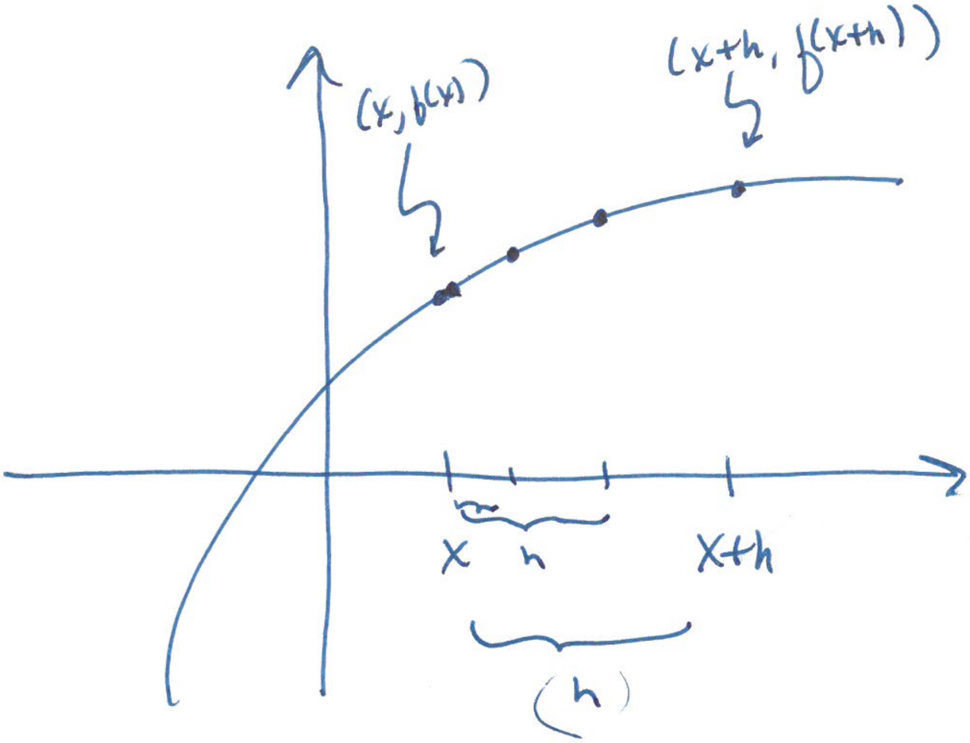
$$= \frac{(2x+2h+1)(5x+3) - (2x+1)(5x+5h+3)}{[5(x+h)+3] \cdot (5x+3) \cdot h}$$

$$= \frac{(10x^2+10xh+5x+6x+6h+3) - (10x^2+10xh+6x+5x+5h+3)}{[5(x+h)+3] \cdot (5x+3) \cdot h}$$

$$= \frac{\cancel{10x^2} + \cancel{10xh} + \cancel{11x} + 6h + \cancel{3} - \cancel{10x^2} - \cancel{10xh} - \cancel{11x} - 5h - \cancel{3}}{[5(x+h)+3] \cdot (5x+3) \cdot h}$$

$$= \frac{\overset{1}{\cancel{h}}}{[5(x+h)+3] \cdot (5x+3) \cdot \underset{1}{\cancel{h}}} \qquad (h \neq 0)$$

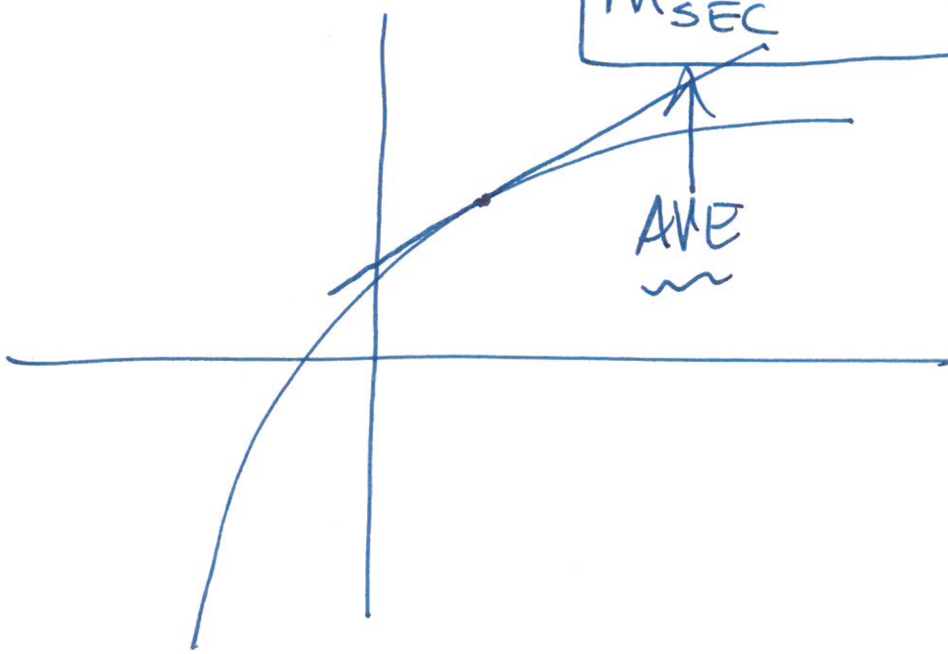
$$m_{sec} = \frac{1}{[5(x+h)+3] \cdot (5x+3)}$$



$h \to 0$

as $h \to 0$, the 2 points get closer together

$$\boxed{m_{SEC} \xrightarrow{h \to 0} m_{tan}}$$



AVE

INST. RATE OF CHANGE

$\boxed{\text{instant}}$aneous rate of change

or

m_{TAN}

or

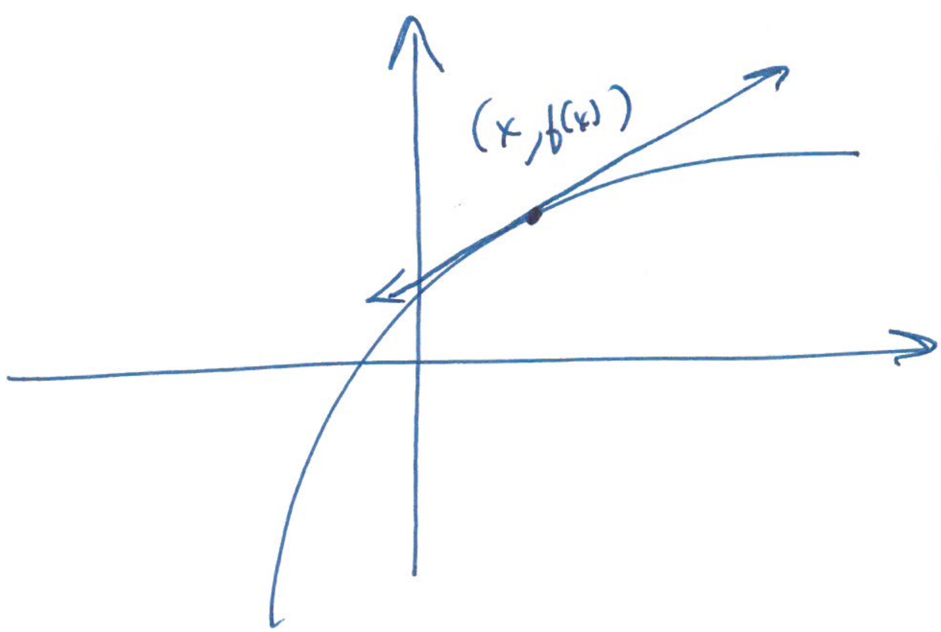
the DERIVATIVE

$$m_{TAN} = \lim_{h \to 0} \left[\frac{f(x+h) - f(x)}{h} \right] = f'(x)$$

f "prime" of x

m_{SEC}

=



$m_{TAN} = ??$

$f(x) = 3x^2 + 5x - 8$

(from earlier today)

$$m_{SEC} = \frac{f(x+h) - f(x)}{h}$$

$$= \frac{\left[3(x+h)^2 + 5(x+h) - 8\right] - \left[3x^2 + 5x - 8\right]}{h}$$

$$\vdots$$

$$m_{SEC} = 6x + 3h + 5$$

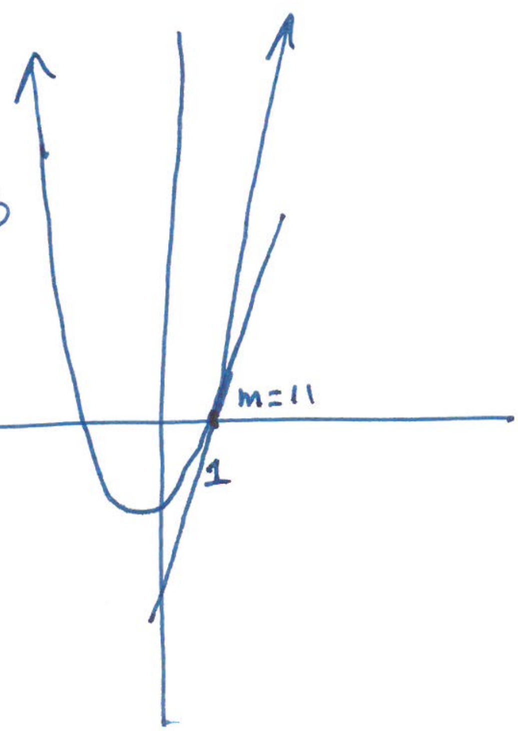
$$m_{TAN} = \lim_{h \to 0} (6x + \underbrace{3h}_{0} + 5)$$

$$\boxed{m_{TAN} = 6x + 5}$$

$f(x) = 3x^2 + 5x - 8$

$m_{TAN} = ? = 6(1) + 5 = 11$

(when $x = 1$)



$$f(x) = \frac{2x+1}{5x+3}$$

find m_{TAN} (inst. rate of change; derivative)

$$\boxed{m_{TAN}} = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \quad \text{DEF. OF DERIV.}$$

$$= \lim_{h\to 0} \frac{\left[\frac{2(x+h)+1}{5(x+h)+3}\right] - \left[\frac{2x+1}{5x+3}\right]}{h}$$

$$= \lim_{h\to 0} \quad \vdots \quad \text{(from earlier today)}$$

$$= \lim_{h\to 0} \left[\frac{1}{[5(x+\underbrace{h}_{0})+3]\cdot[5x+3]}\right]$$

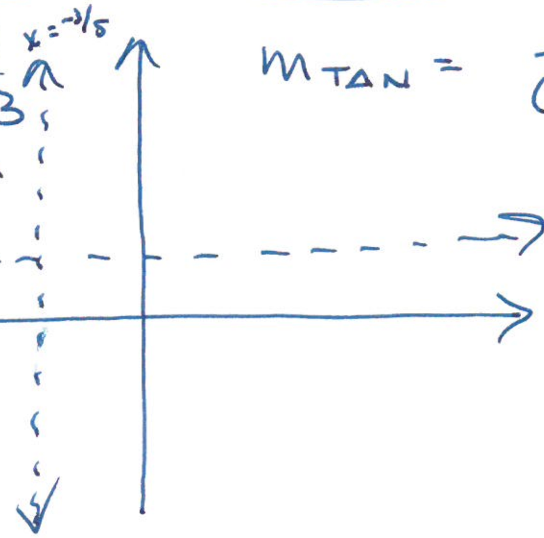
$$m_{TAN} = \frac{1}{[5x+3]\cdot[5x+3]} = \frac{1}{(5x+3)^2}$$

$$f(x) = \frac{2x+1}{5x+3} \qquad m_{TAN} = \frac{1}{(5x+3)^2} = f'(x)$$

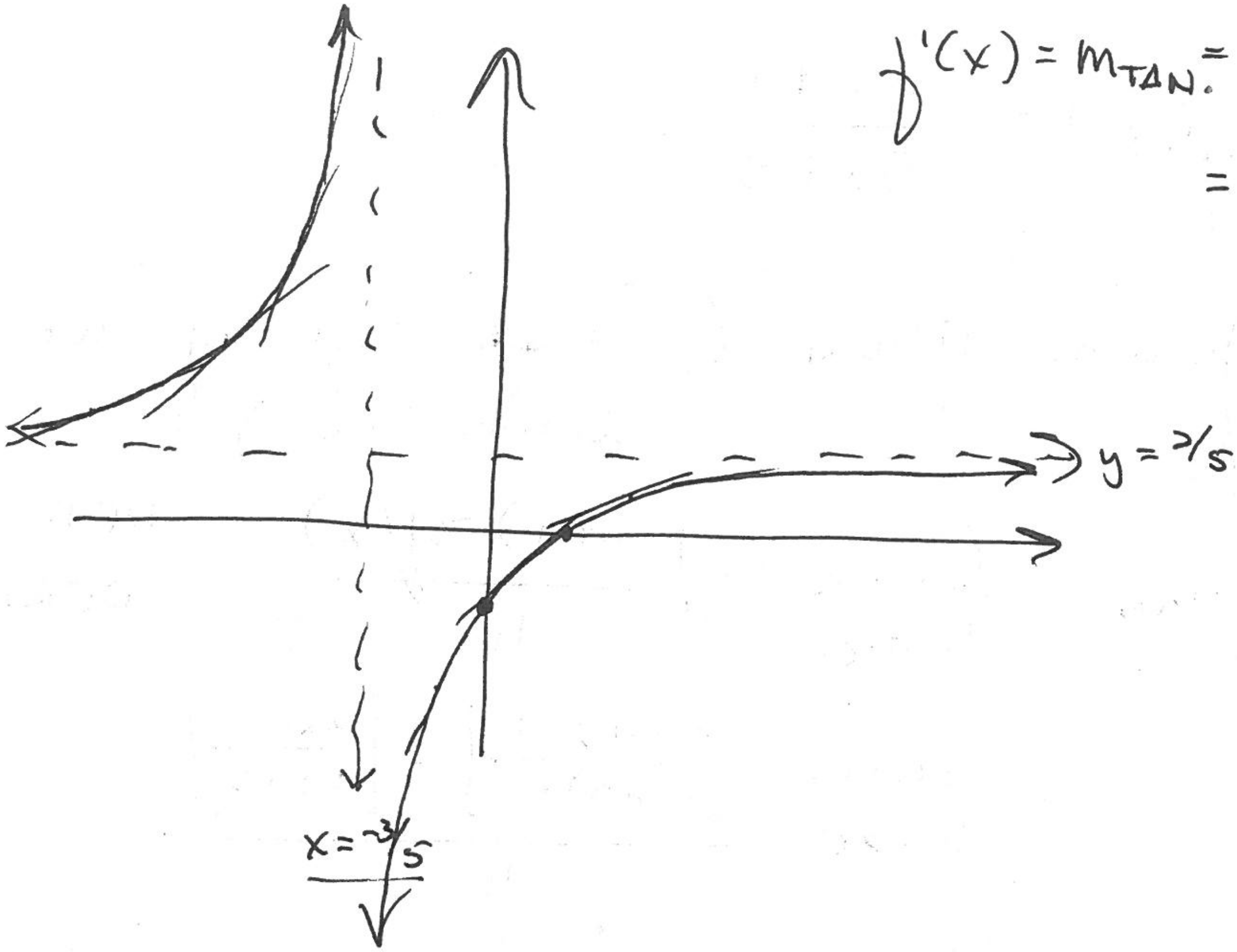
V.A.: $x = -3/5$

H.A.: $y = 2/5$

$$\lim_{x\to\infty} \frac{2x+1}{5x+3} = 2/5$$



$$f'(x) = m_{TAN} = \frac{1}{(5x+3)^2} = \frac{(+)}{(+)} = +$$



1. Determine the limits:

$$a)\ \lim_{x\to 5}\left[\frac{1}{2}(x-5)^2+4x\right] \qquad b)\ \lim_{\theta\to\frac{\pi}{2}}\frac{\pi\sin(\theta)}{\theta} \qquad c)\ \lim_{x\to\infty}\frac{x^2+3x-10}{2-3x}$$

2. Suppose $g(x)=\frac{x^2-16}{x^3-64}=\frac{(x+4)(x-4)}{(x-4)(x^2+4x+16)}$.

   (a) Explain why the line $y=0$ is a horizontal asymptote of the graph of g.

   (b) Is the line $x=4$ a vertical asymptote of the graph of g?

   (c) Let $h(x)=\begin{cases}\frac{x^2-16}{x^3-64} & \text{for } x\neq 4\\ C & \text{for } x=4\end{cases}$. What value C makes h continuous at $x=4$?

3. Use Squeeze Theorem to evaluate $\lim_{x\to 0}x^4\cos(\frac{\pi}{x})$.

4. Is the function $f(x)=\frac{3}{x^2-1}$ continuous on its domain? Notice that $f(0)=-3$ and $f(4)=\frac{1}{5}$. Does the Intermediate Value Theorem guarantee the existence of a root of $f(x)$ on the interval $[0,4]$?

5. Use the Intermediate Value Theorem to prove that there exists a value $x\in[1,2]$ where the functions $f(x)=\sin(\frac{\pi x}{4})$ and $g(x)=x^2-x$ intersect.

Relevant Information to Recall

Theorem 1. Theorems on Limits
Assume that functions f and g have limits L_1 and L_2 at the point x_0 so that

$$\lim_{x\to x_0}f(x)=L_1 \text{ and } \lim_{x\to x_0}g(x)=L_2$$

Then:

1. $\lim_{x\to x_0}(f\pm g)(x)=L_1\pm L_2$
2. $\lim_{x\to x_0}(fg)(x)=L_1L_2$
3. $\lim_{x\to x_0}\left(\frac{f}{g}\right)(x)=\frac{L_1}{L_2}$ *provided $L_2\neq 0$*
4. $\lim_{x\to x_0}(kf)(x)=kL_1$ *for every k in $\mathbb{R}$*
5. $\lim_{x\to x_0}b=b$ *for every b in $\mathbb{R}$*
6. $\lim_{x\to x_0}x=x_0$

Definition 5. **Horizontal Asymptote**
If $\lim_{x\to\infty} f(x) = L$ *or* $\lim_{x\to-\infty} f(x) = L$, *then the line* $y = L$ *is a* **horizontal asymptote** *of the graph of* f.

Definition 9. **Vertical Asymptote**
If f *increases or decreases without bound as* x *approaches* x_0 *from either the left or the right, then the vertical line* $x = x_0$ *is a* **vertical asymptote** *of the graph of* f.

Definition 10. **Continuity**
A function f *is* **continuous at the point** x_0 *if*

1. f *is defined at* x_0, *so* $f(x_0)$ *exists*
2. $\lim_{x\to x_0} f(x)$ *exists*
3. $\lim_{x\to x_0} f(x) = f(x_0)$

Theorem 7. **Squeeze Theorem**

Let $\mathcal{I}$ *be an open interval containing the point* x_0, *and let functions* f, g *and* h *satisfy the inequalities* $f(x) \leq g(x) \leq h(x)$ *for all* $x \in \mathcal{I}$ *except possibly at* x_0. *If*

$$\lim_{x\to x_0} f(x) = L = \lim_{x\to x_0} h(x)$$

then

$$\lim_{x\to x_0} g(x) = L$$

Theorem 14. **Intermediate Value Theorem**

Let f *be continuous on the interval* $[a, b]$ *and let* L *be a number between* $f(a)$ *and* $f(b)$. *Then there exists at least one number* c *with* $a < c < b$ *such that*

$$f(c) = L$$