MA141-004 

Thursday, January 17

composition of functions & inverses

$$\lim_{x\to\infty} \frac{2x-1}{5x+4} =$$

$$\lim_{x\to\infty} \frac{\frac{2x}{x}-\frac{1}{x}}{\frac{5x}{x}+\frac{4}{x}}$$

$$\lim_{x\to\infty} \frac{2 - \frac{1}{x}\to 0}{5+\frac{4}{x}\to 0} = \frac{2}{5}$$

ex: $f(x) = \dfrac{2x-1}{5x+4}$ V.A.: $x = -4/5$ H.A.: $y = 2/5$ $y = \dfrac{2x-1}{5x+4}$

find $f^{-1}(x)$: "f inverse"

1. switch x & y
2. solve for y

INV:

$$\frac{x}{1} = \frac{2y-1}{5y+4}$$

$$x(5y+4) = 2y-1$$

$$5xy + 4x = 2y - 1$$

$$5xy - 2y = -1-4x$$

$$\frac{y(5x-2)}{5x-2} = \frac{-1-4x}{5x-2}$$

$$y = \frac{-1-4x}{5x-2} = f^{-1}(x)$$

H.A.: $y = -4/5$

V.A.: $x = 2/5$

$$y = \boxed{\frac{4x+1}{-5x+2} = f^{-1}(x)}$$

$$f(x) = \frac{2x - 1}{5x + 4}$$

$$f^{-1}(x) = \frac{4x+1}{-5x+2}$$

$$(f \circ f^{-1})(x) = (f^{-1} \circ f)(x) = x$$

$$f(f^{-1}(x))$$

$$f\left(\frac{4x+1}{-5x+2}\right)$$

$$= \frac{\boxed{2\left(\frac{4x+1}{-5x+2}\right)} - 1}{\boxed{5\left(\frac{4x+1}{-5x+2}\right)} + 4}$$

$$= \frac{\left(\frac{8x+2}{-5x+2}\right) - \left(\frac{-5x+2}{-5x+2}\right)}{\left(\frac{20x+5}{-5x+2}\right) + \left(\frac{4(-5x+2)}{(-5x+2)}\right)} = \frac{\frac{+13x}{-5x+2}}{\frac{13}{-5x+2}}$$

$$= \frac{13x}{\cancel{-5x+2}} \cdot \frac{\cancel{-5x+2}}{13} = x$$

$$f : \{ (8, 3) \ldots \}$$

$$f^{-1} : \{ (3, 8) \ldots \}$$

$$\begin{cases} x = 2\sec\theta \longrightarrow \dfrac{x}{2} = \sec\theta \\ y = 6\tan^2\theta \longrightarrow \dfrac{y}{6} = \tan^2\theta \end{cases}$$

$$\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

$$* \quad \boxed{\tan^2\theta + 1 = \sec^2\theta}$$

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$$\frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$$

$$* \quad 1 + \cot^2\theta = \csc^2\theta$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$\longrightarrow \left(\frac{y}{6}\right) + 1 = \left(\frac{x}{2}\right)^2$$

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$$(x+h)^5$$

$$* \quad \frac{f(x+h) - f(x)}{h}$$

$$= x^5 + 5x^4h + 10x^3h^2 + 10x^2h^3 + 5xh^4 + h^5$$

$$\frac{4\cdot 5}{2} \qquad \frac{\cancel{3}\cdot 10}{\cancel{3}} \qquad \frac{2\cdot 10}{4}$$

PASCAL'S TRIANGLE

$(x+h)^0 \qquad 1$

$(x+h)^1 \qquad 1 \quad 1$

$(x+h)^2 \qquad 1 \quad 2 \quad 1$

$(x+h)^3 \qquad 1 \quad 3 \quad 3 \quad 1$

$1 \quad 4 \quad 6 \quad 4 \quad 1$

$\underline{1} \quad \underline{5} \quad 10 \quad 10 \quad 5 \quad 1$

$\vdots$

$$(x+h)^{21}$$

$$= x^{21} + 21x^{20}h + 210x^{19}h^2 + 1330x^{18}h^3$$

$\dfrac{(21)(20)}{2} \longrightarrow \dfrac{(210)(19)}{3} \qquad \begin{array}{r} 19 \\ 70 \\ \hline 1330 \end{array} \qquad +$

# CH 1 : LIMITS

"closer & closer"

$$\lim_{x \to a} \boxed{f(x)} = L$$

(2-sided limit)

(1) $\lim_{x \to a^+} f(x) = \_\_\_\_\_\_$ (from the right)

(1-sided limit)

(2) $\lim_{x \to a^-} f(x) = \_\_\_\_\_\_$ (from the left)

(1-sided limit)

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$$\lim_{x \to 3^+} f(x)$$

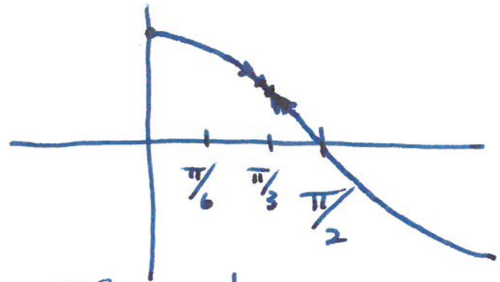
3.1, 3.01, 3.0001, 3.00000001, ....

$$\lim_{x \to 3^-} f(x)$$

2.9, 2.99, 2.999, 2.99999, ...

continuous

$y = \cos x$

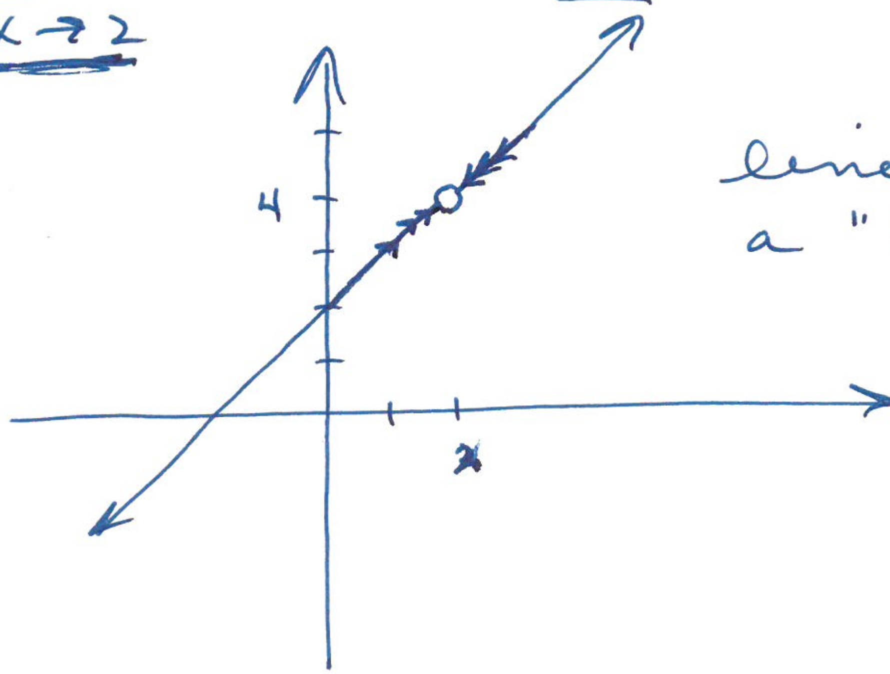



$$\lim_{x \to \pi/3} \cos x = \cos \pi/3 = \frac{1}{2}$$

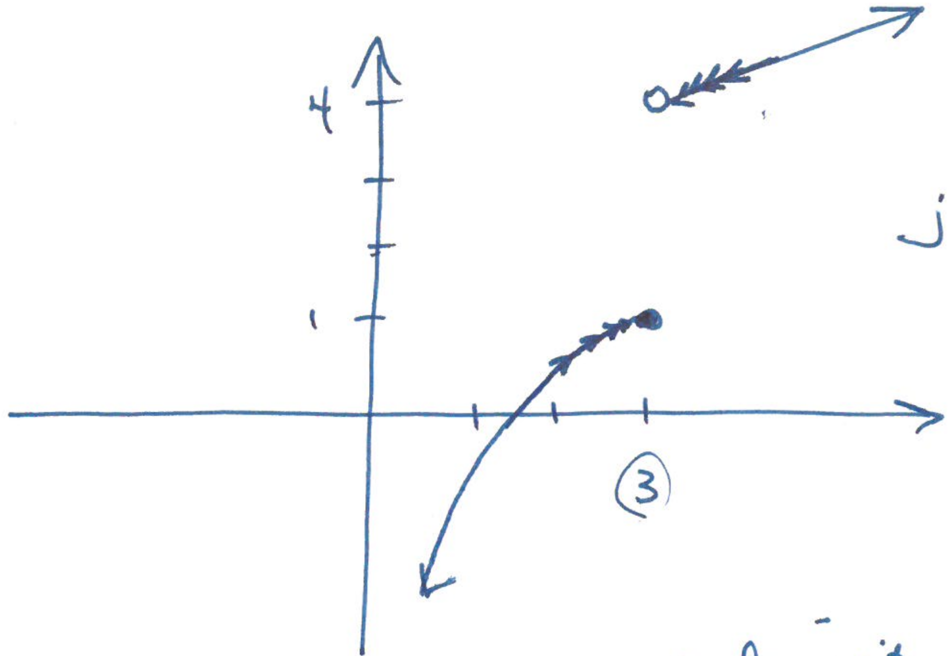
$$y = \frac{x^2-4}{x-2} = x+2 \quad (x \neq 2)$$

$$\lim_{x \to 2} \frac{x^2-4}{x-2} = \lim_{x \to 2} \frac{\cancel{(x-2)}(x+2)}{\cancel{(x-2)}} \quad (x \neq 2)$$

$$= \lim_{x \to 2} (x+2) = 4$$




line with a "hole" in it




jump discon.

$$\lim_{x \to 3} f(x) = \underline{\text{D.N.E.}}$$

no limit

① $\lim_{x \to 3^+} \boxed{f(x)} = \underline{4}$

② $\lim_{x \to 3^-} \boxed{f(x)} = \underline{1}$

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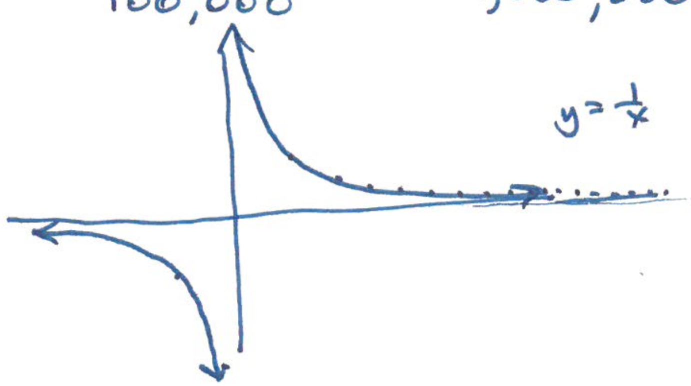
$$\lim_{\boxed{x \to \infty}} \frac{1}{x} = 0$$

$y = \frac{1}{x}$

$\boxed{xy = 1}$

$\frac{1}{10} \dots \frac{1}{100,000} \dots \frac{1}{1,000,000,000,000} \dots$



$y = \frac{1}{x}$    H.A.: $y = 0$

$$\lim_{x\to\infty} \frac{3x-7}{5x^2+2x-4}$$

$$\lim_{x\to\infty} \frac{\frac{3x}{x^2}-\frac{7}{x^2}}{\frac{5x^2}{x^2}+\frac{2x}{x^2}-\frac{4}{x^2}}$$

$$y=\frac{3x-7}{5x^2+2x-4}$$

H.A.: $y=0$

$$= \lim_{x\to\infty} \frac{\frac{3}{x}\to 0 - \frac{7}{x^2}\to 0}{5+\frac{2}{x}\to 0-\frac{4}{x^2}\to 0} = \frac{0}{5} = 0$$

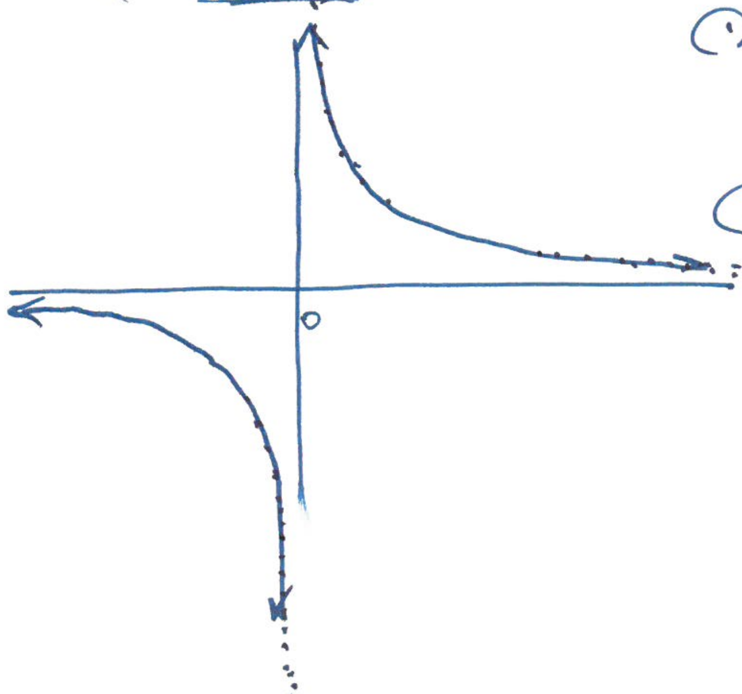
$$\lim_{x\to 0}\left(\frac{1}{x}\right) - \text{D.N.E}$$

$$\lim_{x\to 0^+}\left(\frac{1}{x}\right) = +\infty$$

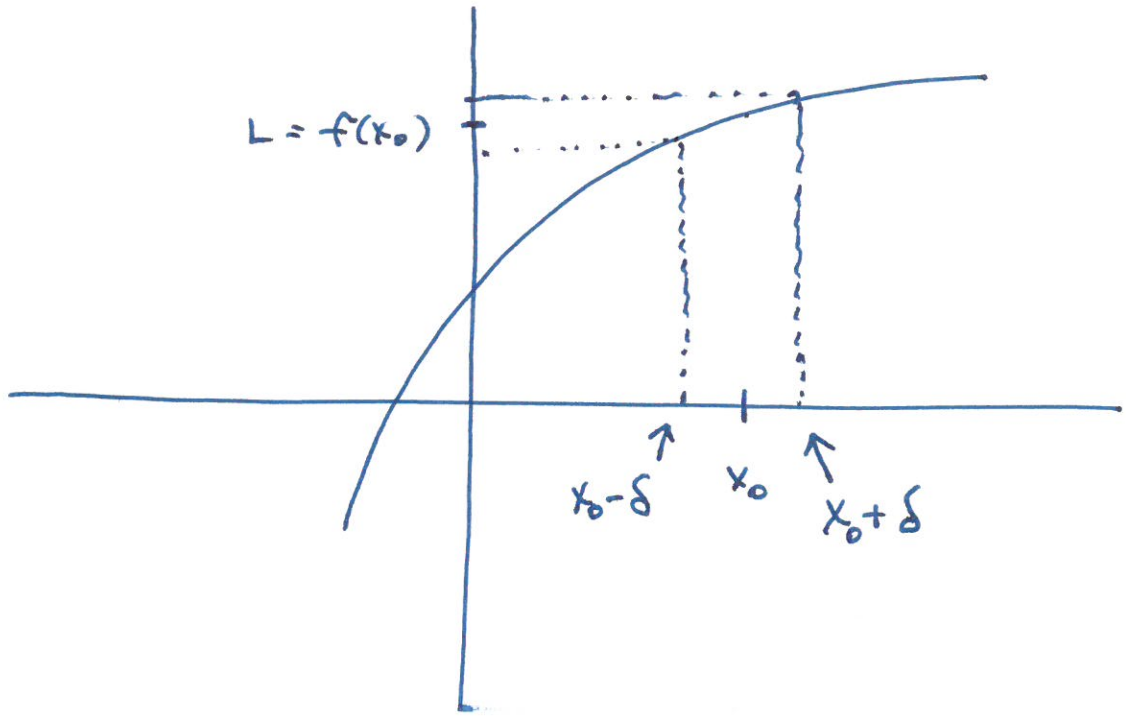
(1) $\lim_{x\to 0^+}\left(\frac{1}{x}\right) = \text{D.N.E.}$ ✓

(2) $\lim_{x\to 0^-}\left(\frac{1}{x}\right) = \text{D.N.E.}$ ✓

$$\lim_{x\to 0^-}\left(\frac{1}{x}\right) = -\infty$$



"closer & closer"




$$\lim_{x \to x_0} f(x) = L$$

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$$f(x) = 2x + 4$$

$$\lim_{x \to \underline{3}} (2x+4) = \underline{10}$$

if $|x-3| < \delta$, .... then

$(|x - x_0| < \delta)$

$$|(2x+4) - 10| < \epsilon$$

$|f(x) - L| < \epsilon$

if $|x-3| < \delta$ then
$|f(x) - L| < \epsilon$

working backwards . . . .

$|f(x) - L| < \epsilon$

$|(2x+4) - 10| < \epsilon$

$|2x - 6| < \epsilon$

$|2(x-3)| < \epsilon$

$2\,|x-3| < \epsilon$

$|x-3| < \epsilon/2$

choose $\delta = \epsilon/2$

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proof:

$|x-3| < \delta$

choose $\delta = \epsilon/2$

$|x-3| < \epsilon/2$

$|2(x-3)| < \epsilon$

$$|2x - 6| < \epsilon \qquad\qquad |\underline{f(x)} - \underline{L}|$$

$$|\underline{2x + 4} - \underline{10}| < \epsilon$$

$$|(2x+4) - 10| < \epsilon$$

$$\downarrow \qquad\quad \downarrow$$

$$|f(x) - L| < \epsilon$$

1. Find the equation of the circle whose center is $(3, 4)$ and contains the point $(11, -1)$.

2. Consider the parabola given by the equation $x^2+2x-12y+37 = 0$. Identify the vertex of the parabola by rewriting the equation in the standard form $(y-k) = \frac{1}{4c}(x-h)^2$. Use the value $c$, the distance between the focus and vertex, to identify the focus and the directrix. Sketch the parabola.

3. Sketch the hyperbola given by the equation $\frac{(x+2)^2}{4} - \frac{(y-1)^2}{9} = 1$ by identifying the following features. Where is the center? Is transverse axis, the axis containing the foci and vertices, parallel to the $x$-axis or the $y$-axis? Identify the vertices, foci, and points on the conjugate axis. Identify the asymptotes.

Recall: A **function** $f$ is a map or rule that assigns to each element $x$ of a set $\mathcal{D}$, called the domain, one and only one element $f(x)$ in a set $\mathcal{R}$, called the range. We refer to $f(x)$ as "the value of the function $f$ at the point $x$."

4. Identify the natural domain of the following real valued functions

   a) $g(x) = \sqrt{2x-1}$

   b) $h(x) = \frac{1}{(x+5)(x-3)}$

5. Let $f(x) = \frac{2}{x-9}$ and $g(x) = \frac{x}{2-x}$. Find $(f \circ g)(x)$ and determine the domain of $(f \circ g)(x)$ using interval notation.

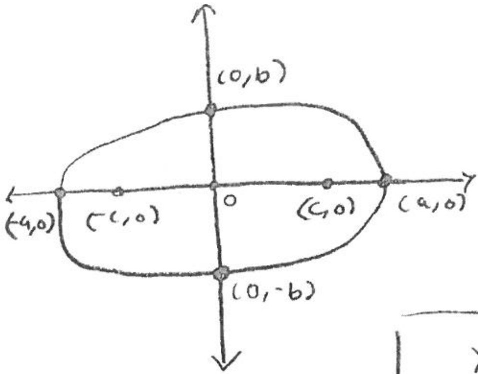
6. Sketch the curve represented by the parametric equations $x = t+1, \quad y = \frac{t}{t+1}$. Eliminate the parameter to determine the Cartesian equation.

7. Determine the Cartesian equation of the parametrized curve $x = 2\sec(\theta), y = 6\tan^2(\theta)$. (Hint: Recall the trig identity $1+\tan^2\theta = \sec^2\theta$)

## Ellipse

vertices $(\pm a, 0)$

foci $(\pm c, 0)$

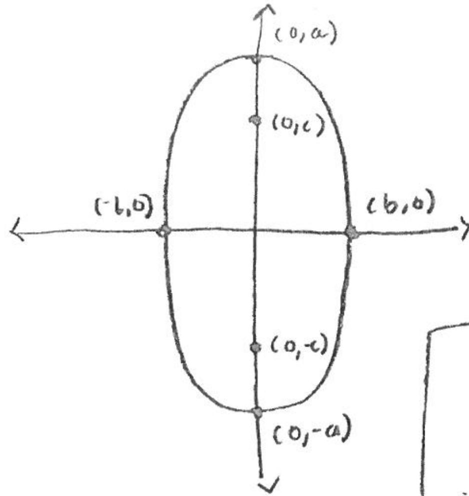



$$\boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1}$$

with $a^2 = b^2 + c^2$

vertices $(0, \pm a)$

foci $(0, \pm c)$




$$\boxed{\frac{y^2}{a^2} + \frac{x^2}{b^2} = 1}$$

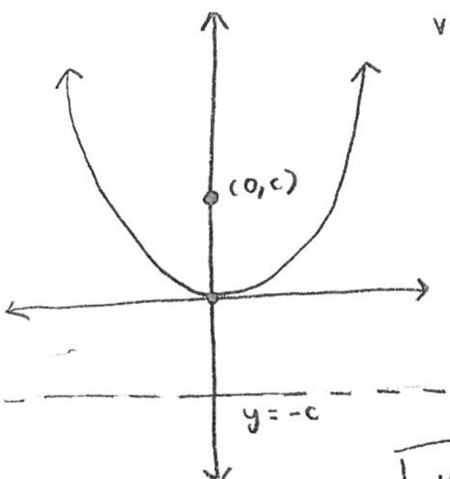
with $a^2 = b^2 + c^2$

## Parabola

focus $(0, c)$

directrix $y = -c$

vertex $= (0,0)$

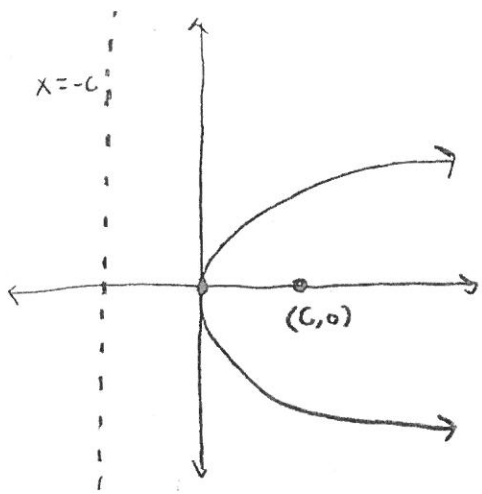



$$\boxed{y = \frac{1}{4c} x^2}$$

focus $(c, 0)$

directrix $x = -$

vertex $= (0, 0$

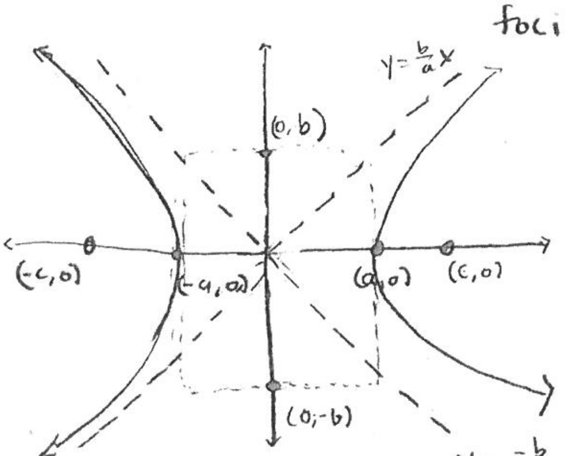



$$\boxed{x = \frac{1}{4c} y^2}$$

## Hyperbola

vertices $(\pm a, 0)$

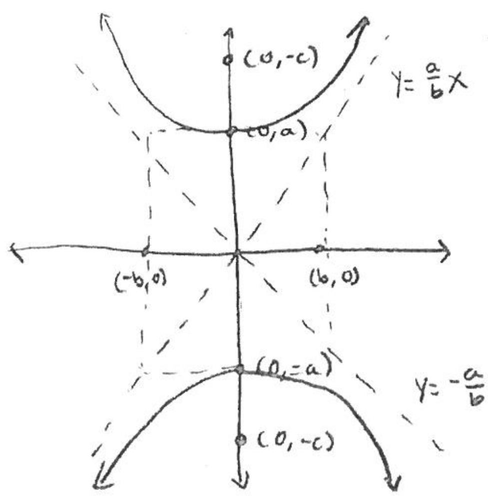
foci $(\pm c, 0)$




$x^2 - y^2 = 1$ with $c^2 = a^2 + b^2$

vertices $(0, \pm a$

foci $(0, \pm c$




$y^2 - x^2 = 1$ with $c^2 = a^2 + b^2$