MA 121 - 002

Tuesday, January 22

1.2: LIMITS & CONTINUITY

p 107 (limit prime.)

$$\lim_{x \to 2} (8x^2 - 13x + 5) = 11$$

$(x \to 2^+, x \to 2^-)$

$$= \lim_{x \to 2} 8x^2 - \lim_{x \to 2} 13x + \lim_{x \to 2} 5$$

$$= 8 \cdot \lim_{x \to 2} x^2 - 13 \lim_{x \to 2} x + 5$$

$$= 8 \left[\lim_{x \to 2} x\right]^2 - 13(2) + 5$$

$$= 8[2]^2 - 13(2) + 5 = 11$$

We determined the limits of these functions by observing basic behaviors and making a reasonable generalization. The following list summarizes common limit properties that allow us to calculate limits much more efficiently.

Limit Properties

If $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} g(x) = M$, and c is any constant, then we have the following limit properties.

L1. The limit of a constant is the constant:

$$\lim_{x\to a} c = c.$$

L2. The limit of a power is the power of that limit, and the limit of a root is the root of that limit:

$$\lim_{x\to a} [f(x)]^m = [\lim_{x\to a} f(x)]^m = L^m, \text{ where } m \text{ is any integer;}$$

$$\lim_{x\to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x\to a} f(x)} = \sqrt[n]{L}, \text{ where } n \geq 2.$$

In the case of the power, we must have $L \neq 0$ if m is negative, and in the case of the root, we must have $L \geq 0$ if n is an even integer.

L3. The limit of a sum or difference is the sum or difference of the limits:

$$\lim_{x\to a} [f(x) \pm g(x)] = \lim_{x\to a} f(x) \pm \lim_{x\to a} g(x) = L \pm M.$$

L4. The limit of a product is the product of the limits:

$$\lim_{x\to a} [f(x) \cdot g(x)] = [\lim_{x\to a} f(x)] \cdot [\lim_{x\to a} g(x)] = L \cdot M.$$

L5. The limit of a quotient is the quotient of the limits:

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)} = \frac{L}{M}, \quad \text{assuming } M \neq 0.$$

L6. The limit of a constant times a function is the constant times the limit of the function:

$$\lim_{x\to a} c \cdot f(x) = c \cdot \lim_{x\to a} f(x) = c \cdot L.$$

Property L6 combines L1 and L4 but is stated separately because it is used so frequently.

EXAMPLE 1 Use the Limit Properties to find $\lim_{x\to 4} (x^2 - 3x + 7)$.

Solution We know that $\lim_{x\to 4} x$ is 4.

By Limit Property L2,

$$\lim_{x\to 4} x^2 = [\lim_{x\to 4} x]^2 = 4^2 = 16,$$

by Limit Property L6,

$$\lim_{x\to 4} (-3x) = -3 \cdot \lim_{x\to 4} x = -3 \cdot 4 = -12,$$

and by Limit Property L1,

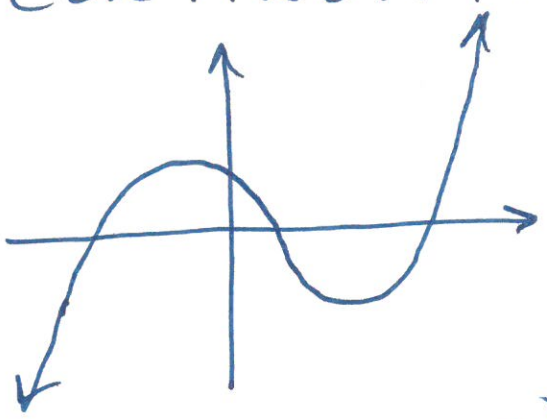
$$\lim_{x\to 4} 7 = 7.$$

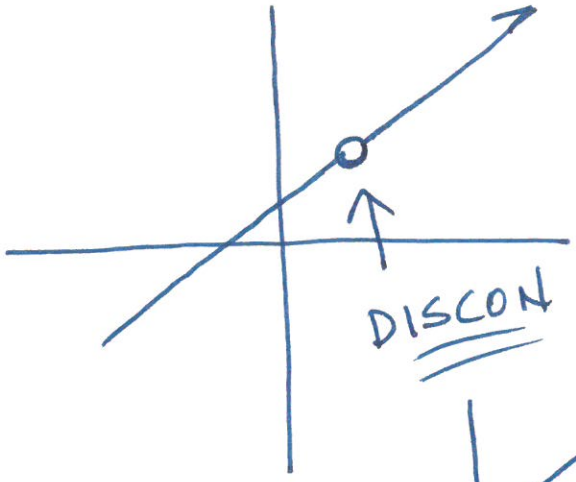
Using Limit Property L3 to combine these results, we have

$$\lim_{x\to 4} (x^2 - 3x + 7) = 16 - 12 + 7 = 11.$$

■

CONTINUITY:





(CONTIN. AT $x=a$)

3 parts:

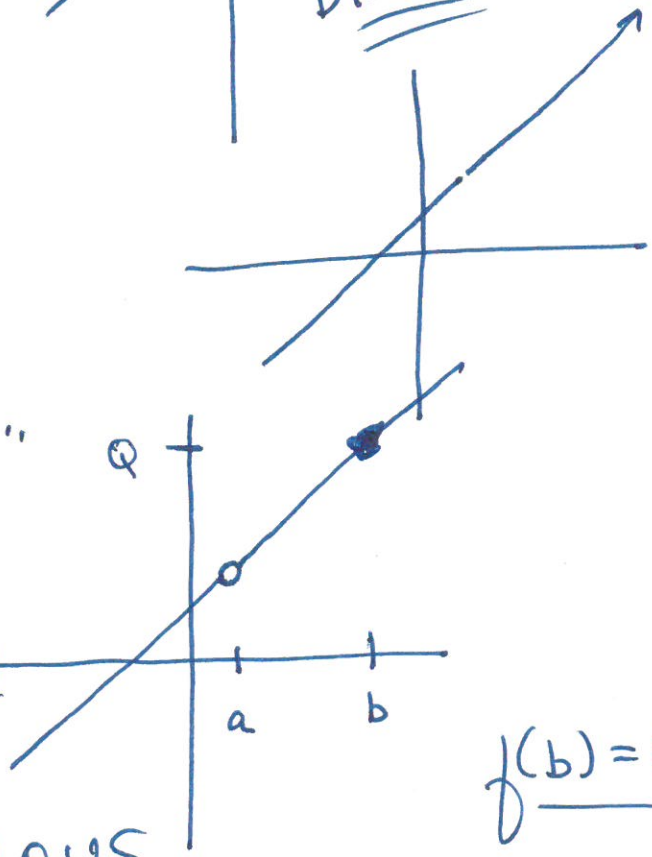
1.) $f(a)$ exists?

"is there a point plotted there?"

$f(a) = ??$

no point plotted there

$\therefore$ DISCONTINUOUS (at $x=a$)

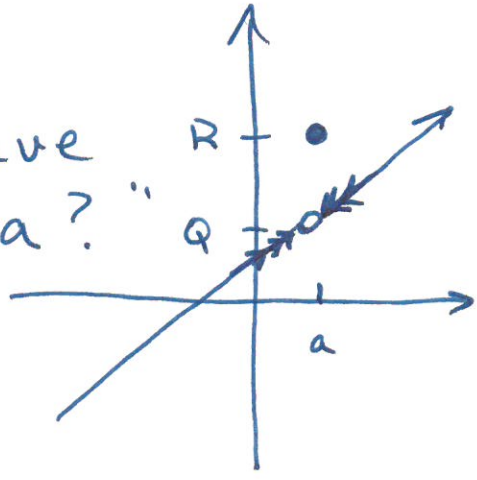


$f(b) = Q$

2.) $\lim_{x \to a} f(x)$ exists?

"does the function have a limit as $x \to a$?"

$\equiv$



① $f(a) = R$ ✓ (a, R)

② $\lim_{x \to a} f(x) = Q$ ✓

3.) $\lim_{x \to a} f(x) \overset{??}{=} f(a)$

Q $\overset{??}{\neq}$ R

$\therefore$ DISCON. (at $x = a$)

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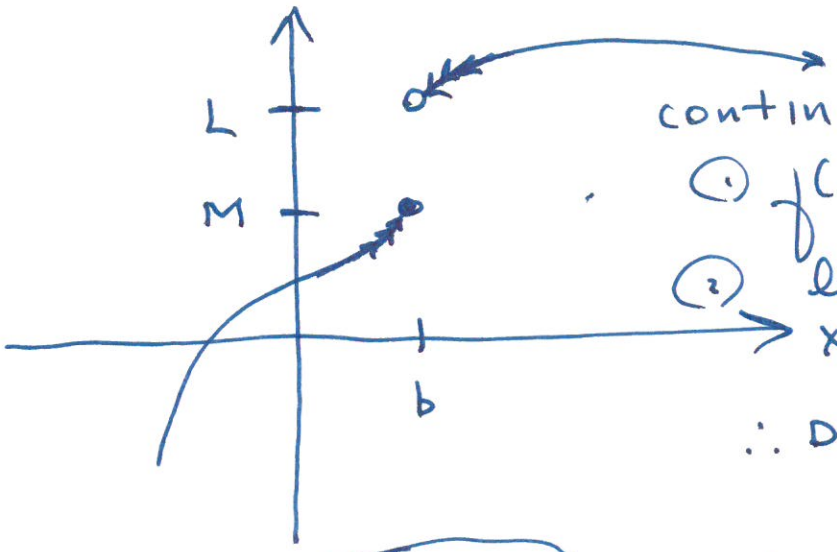
ex:



contin at $x = b$:

① $f(b) =$ M ✓

② $\lim_{x \to b} f(x) =$ D.N.E (no lim.)

$\therefore$ DISCON (at $x = b$)

ex:

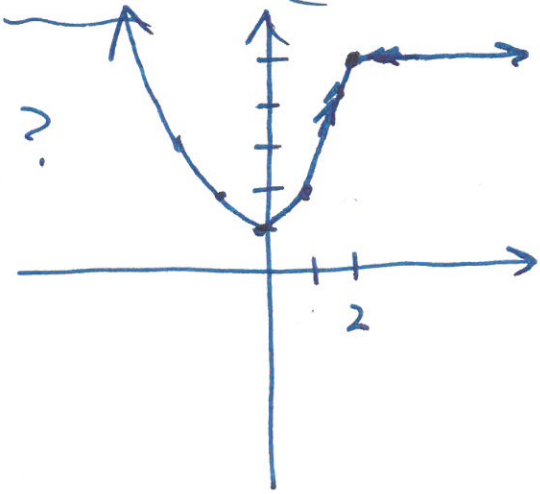
$$f(x) = \begin{cases} x^2+1, & x \leq 2 \\ 5, & x > 2 \end{cases}$$ (5)

$y = x^2+1$

x	y
2	5
1	2
0	1
-1	2
-2	5

$y = 5$

x	y
2	5
2.1	5
3	5



contin at $x = 2$?

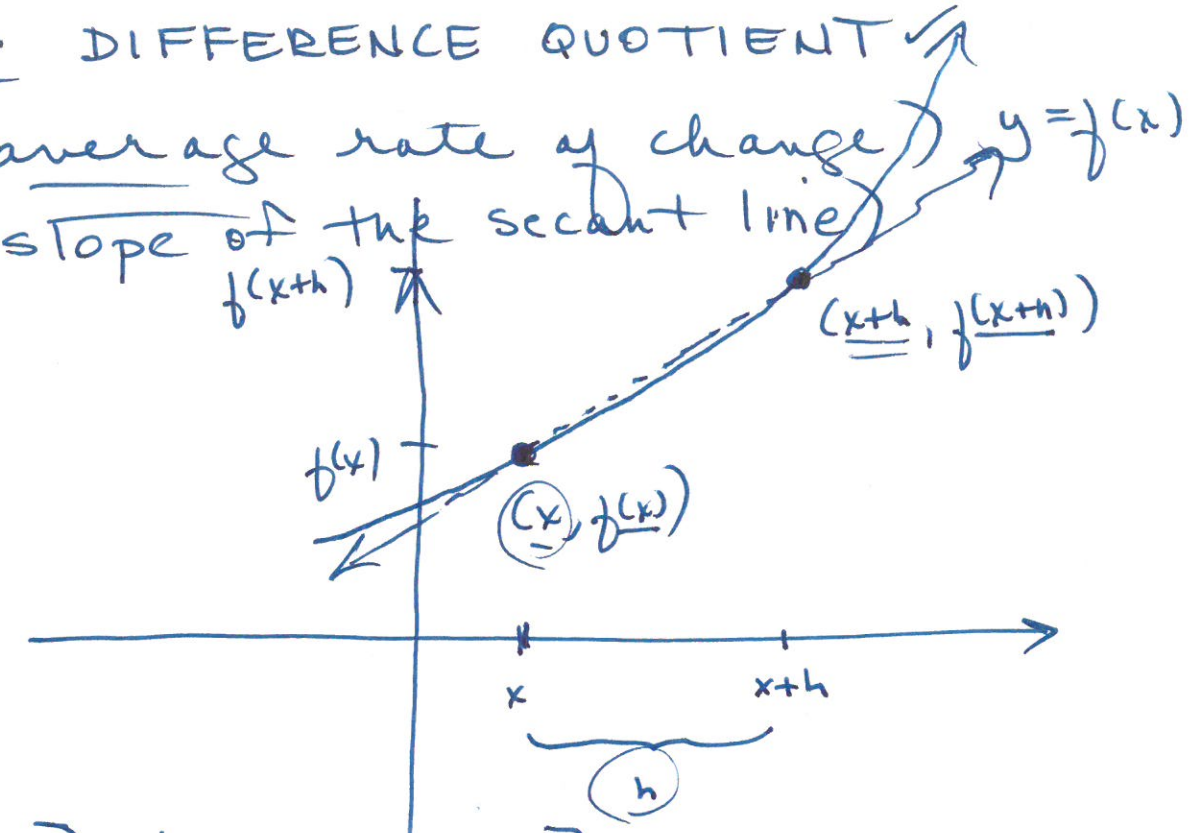
① $f(2) =$ 5

② $\lim_{x \to 2} f(x) =$ 5

③ $\lim_{x \to 2} f(x) \overset{??}{=} f(2)$ $\therefore$ contin @ $x = 2$

1.3: DIFFERENCE QUOTIENT

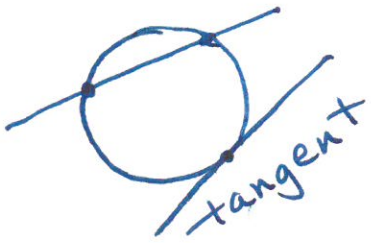
(average rate of change) (slope of the secant line)



$(x, f(x))$ & $(x+h, f(x+h))$ } 2 points

$$m_{SEC} = \frac{f(x+h) - f(x)}{(x+h) - x} = \frac{f(x+h) - f(x)}{h}$$

secant



$$m_{sec} = \frac{f(x+h) - f(x)}{h}$$

ex: $f(x) = 3x^2 + 5x - 1$

$$m_{SEC} = \frac{[3(x+h)^2 + 5(x+h) - 1] - [3x^2 + 5x - 1]}{h}$$

$(x^2 + 2xh + h^2)$

$$m_{SEC} = \frac{3x^2 + 6xh + 3h^2 + 5x + 5h - 1 - 3x^2 - 5x + 1}{h}$$

$$m_{SEC} = \frac{h(6x + 3h + 5)}{h} \qquad (h \neq 0)$$

$$\boxed{m_{SEC} = 6x + 3h + 5}$$

for ANY 2 points on this curve

x = initial x-value

h = change in x

* $\boxed{f(x) = 3x^2 + 5x - 1}$

$27 + 15 - 1$

$((1, 7) \text{ & } (3, 41))$

$x = 1$

$h = 2$

$$m = 6(1) + 3(2) + 5 = 17$$

$$m = \frac{41 - 7}{3 - 1} = \frac{34}{2} = 17$$

$$f(x) = \frac{2x+1}{5x+3}$$

$$\frac{f(x+h) - f(x)}{h} = m_{SEC}$$

$$= \frac{\dfrac{2(x+h)+1}{5(x+h)+3} - \dfrac{2x+1}{5x+3}}{h}$$

get a common denom. . . . .

(to be completed on THURS, 1/24)

TA OFFICE HOURS

Subject: Office hours
From: Jack Thompson <jtthomp4@ncsu.edu>
Date: 1/17/19, 1:28 PM
To: John Griggs <jrgriggs@ncsu.edu>

Hi,

I'll be holding office hours at 12:30-1:30 on Tuesdays and 10-11 on Thursdays in SAS room 4125

Also sorry, meant to send this Tuesday but it got stuck in drafts

Jack

Subject: [math-tap] Fwd: WebAssign Support table on Campus Tuesday
From: Carolyn Gunton <cbgunton@ncsu.edu>
Date: 1/22/19, 7:22 AM
To: all@math.ncsu.edu

FYI.....Carolyn

---------- Forwarded message ---------
From: **Steger, Scott A** <scott.steger@cengage.com>
Date: Mon, Jan 21, 2019 at 6:53 PM
Subject: WebAssign Support table on Campus Tuesday
To:

Hello Faculty and Staff,

I wanted to let you know that we will be hosting a WebAssign Activation Station in Fox on Tuesday and Thursday. We will be at a table on the second floor from 10-3.

Most WebAssign Grace Periods will be ending this week. Please encourage your students to purchase access now so that we can assist when questions arise.

If your students need support outside of this time, please direct them to our support line at 800.354.9706.

If you have any questions, please let me know.

Warmly,

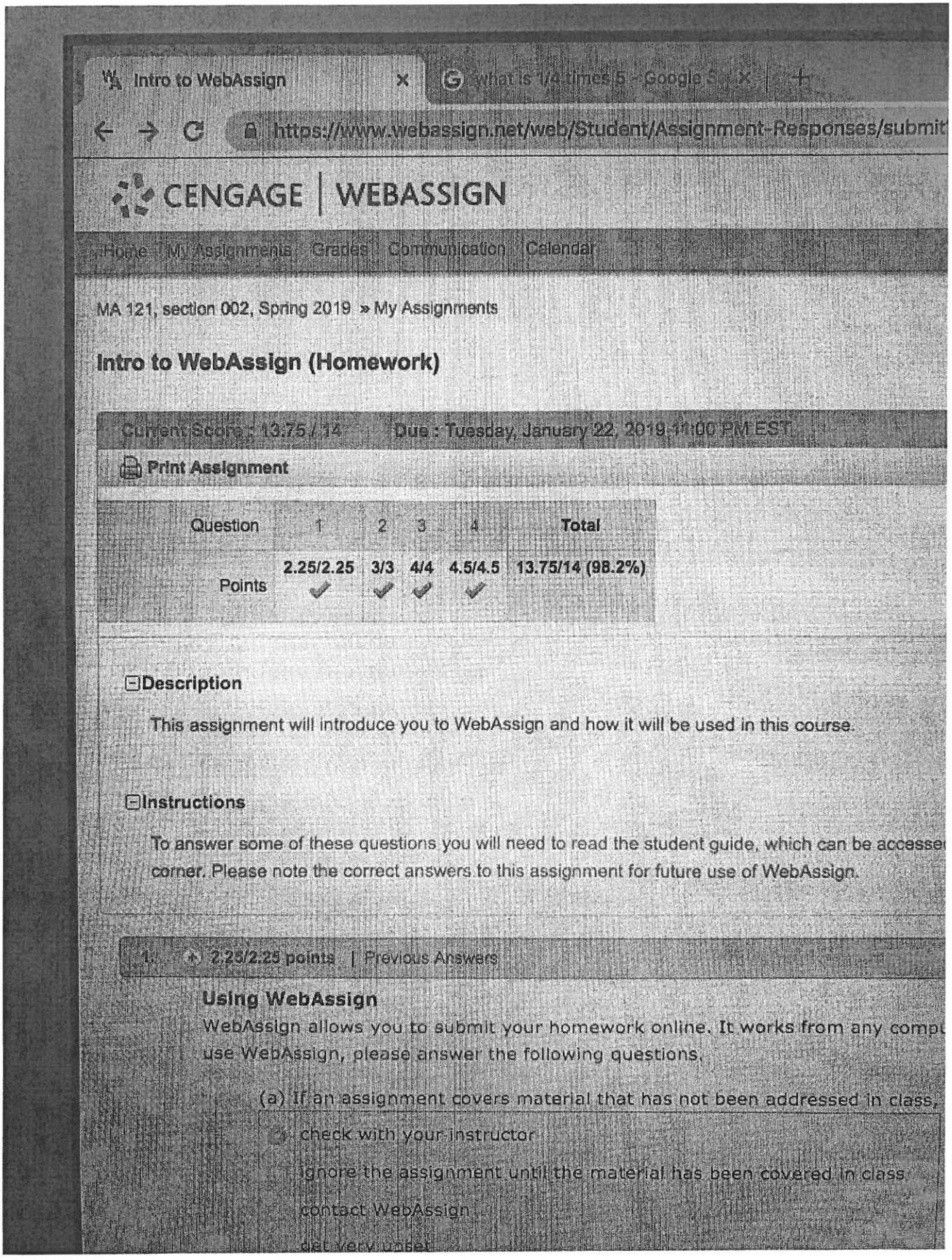
Scott Steger

Learning Consultant

Cengage Learning

T: 919.616.4954

Scott.Steger@Cengage.com



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