NORTH CAROLINA STATE UNIVERSITY Department of Mathematics
MA141-005 Test #4 Form B Wednesday, April 17, 2019 Dr. J. Griggs

Please put all work and answers in the stamped blue book provided. Do <u>not</u> put work or answers on the test itself. Show all simplifications and substitutions. Do **not** use a calculator that does calculus; do not use a graphing calculator. Please include your **name, row and seat** on the front of the blue book. Put the test copy in the blue book.

1.) Integrate: $\int \frac{3\sec^2 x}{1+5\tan x} dx$

2.) Integrate and evaluate: $\int_0^6 (6x - x^2) dx$)

3.) Integrate using substitution and evaluate: $\int_0^1 5x^3(1+3x^4)^2 dx$

4.) Integrate by parts: $\int 7x^5 \ln x \, dx$

5.) Evaluate using the Fundamental Theorem of Calculus: $\frac{d \ (\int_0^{x^4} \sqrt{1+r^3}\, dr)}{dx}$

6.) Using a Riemann Sum and the sigma notation formulas (provided), find the exact area under the curve $f(x) = 6x - x^2$ *from* $x = 0$ *to* $x = 6$. (compare with #2 above)

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

7.) Find the area of the region bounded by the two curves:
$y = 4x - x^2$ *and* $y = x^2 - 6x + 8$

Bonus (5 points): Integrate $\int \sec^3 t \, dt$

MA 141 TEST #4 FORM B

TEST SOLUTIONS (14 pts each)

1.) $$\int \frac{3 \sec^2 x}{1+5\tan x}\,dx = 3\cdot\frac{1}{5}\int \frac{\sec^2 x \cdot 5}{1+5\tan x}\,dx$$

let $u = 1+5\tan x$

$du = 5\cdot\sec^2 x\,dx$

$$= \frac{3}{5}\int \frac{du}{u} = \frac{3}{5}\int \frac{1}{u}\,du = \frac{3}{5}\ln|u| + C$$

$$= \frac{3}{5}\ln|1+5\tan x| + C$$

2.) $$\int_0^6 (6x - x^2)\,dx = \left[6\cdot\frac{x^2}{2} - \frac{x^3}{3}\right]_0^6$$

$$= \left[3x^2 - \frac{x^3}{3}\right]_0^6 = \left(3\cdot 6^2 - \frac{6^3}{3}\right) - (0)$$

$$= 108 - 72 = 36$$

3.) $$\int_0^1 5x^3(1+3x^4)^2\,dx$$

let $u = 1+3x^4$

$du = 12x^3\,dx$

$x=0 \xrightarrow{u=1+3x^4} u=1$

$x=1 \longrightarrow u=4$

$$= 5\cdot\frac{1}{12}\int_0^1 (1+3x^4)^2\cdot \underbrace{x^3\cdot dx\cdot 12}$$

$$= \frac{5}{12}\int_1^4 u^2 \cdot du = \frac{5}{12}\cdot\frac{u^3}{3}\Big]_1^4 = \frac{5}{36}u^3\Big]_1^4$$

$$= \frac{5}{36}\left[4^3 - 1^3\right] = \frac{5}{36}\left[64-1\right] = \frac{5}{36}(63) = \frac{5(7)}{4}$$

$$= \frac{35}{4} = 8\tfrac{3}{4}$$

4.) $\int 7x^5 \cdot \ln x \, dx$

let $u = \ln x$ $\qquad v = 7\cdot\frac{x^6}{6}$

$du = \frac{1}{x}\cdot dx$ $\qquad dv = 7x^5$

$$= (u)(v) - \int v \cdot du$$

$$= (\ln x)\left(\tfrac{7}{6}x^6\right) - \int \left(\tfrac{7}{6}x^6\right)\left(\tfrac{1}{x}\cdot dx\right)$$

$$= \tfrac{7}{6}x^6 \cdot \ln x - \tfrac{7}{6}\int x^5 dx$$

$$= \tfrac{7}{6}x^6 \cdot \ln x - \tfrac{7}{6}\cdot\frac{x^6}{6} + C$$

$$= \tfrac{7}{6}x^6 \cdot \ln x - \tfrac{7}{36}x^6 + C$$

5.) $$\frac{d\left[\int_0^{x^4}\sqrt{1+r^3}\,dr\right]}{dx} = \sqrt{1+(x^4)^3}\cdot d(x^4)$$

$$= \sqrt{1+x^{12}} \cdot 4x^3$$

6.) AREA $= \lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} f(a + i\cdot\Delta x)\cdot\Delta x$

$a = 0 \qquad b = 6 \qquad\qquad f(x) = 6x - x^2$

$\Delta x = \frac{b-a}{n} = \frac{6-0}{n} = \frac{6}{n}$

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} f\left(0 + i\cdot\frac{6}{n}\right)\cdot\frac{6}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} f\left(\frac{6i}{n}\right)\cdot\frac{6}{n} = \lim_{n\to\infty} \sum_{i=1}^{n} \left(6\cdot\left(\frac{6i}{n}\right) - \left(\frac{6i}{n}\right)^2\right)\cdot\frac{6}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{36i}{n} - \frac{36i^2}{n^2}\right)\cdot\frac{6}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{216i}{n^2} - \frac{216i^2}{n^3}\right)$$

$$= \lim_{n\to\infty} \left[\frac{216}{n^2}\sum_{i=1}^{n} i - \frac{216}{n^3}\sum_{i=1}^{n} i^2\right]$$

$$= \lim_{n\to\infty} \left[\frac{216}{n^2}\cdot\frac{n(n+1)}{2} - \frac{216}{n^3}\cdot\frac{n(n+1)(2n+1)}{6}\right]$$

$$= \lim_{n\to\infty} \left[\frac{216}{2}\cdot\frac{n^2+n}{n^2} - \frac{216}{6}\,\frac{(2n^3+\cdots)}{n^3}\right]$$

$$= \frac{216}{2}(1) - \frac{216}{6}(2) = 108 - 72 = 36$$

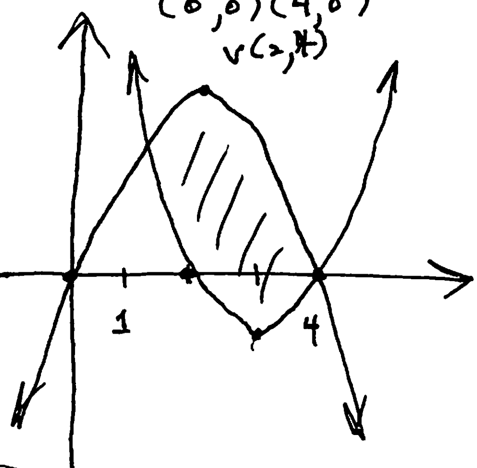
7.) area of the bounded region

14 pts $y = 4x - x^2$ & $y = x^2 - 6x + 8$

$(0,0)\ (4,0)$
$v(2,4)$

$y = (x-4)(x-2)$
$(4,0)\ (2,0)$
$v(3, 9-18+8) = (3,-1)$

graph not needed



$$\cancel{4x} - \cancel{x^2} = x^2 - 6x + 8$$
$$-\cancel{4x} + \cancel{x^2} \qquad +x^2 \quad -4x$$

$$0 = 2x^2 - 10x + 8$$
$$0 = 2(x^2 - 5x + 4)$$
$$0 = 2(x-4)(x-1)$$
$$x = 4, \quad x = 1$$
b↑ a↑

$$A = \int_1^4 \left[(4x - x^2) - (x^2 - 6x + 8)\right] dx$$

$$A = \int_1^4 (-2x^2 + 10x - 8)\,dx = \left[\frac{-2x^3}{3} + \frac{10x^2}{2} - 8x\right]_1^4$$

$$A = \left[\frac{-2(4)^3}{3} + 5(4)^2 - 8(4)\right] - \left[\frac{-2(1)^3}{3} + 5(1)^2 - 8(1)\right]$$

$$A = \frac{-128}{3} + 80 - 32 + \frac{2}{3} - 5 + 8$$

$$A = \frac{-126}{3} + 51 = -42 + 51 = 9$$

BONUS : 5pts

$$\int \sec^3 t\,dt = \int \sec t \cdot \sec^2 t\,dt$$

$u = \sec t \qquad v = \tan t$

$du = \sec t \tan t\,dt \qquad dv = \sec^2 t\,dt$

$$\int \sec^3 t\,dt = (\sec t)(\tan t) - \int \sec t \cdot \tan^2 t\,dt$$

$\tan^2 t = \sec^2 t - 1$

$$\int \sec^3 t\,dt = \sec t \tan t - \int \sec t(\sec^2 t - 1)\,dt$$

$$\int \sec^3 t\,dt = \sec t \tan t - \cancel{\int \sec^3 t\,dt} + \int \sec t\,dt$$

$$+\int \sec^3 t\,dt \qquad\qquad + \cancel{\int \sec^3 t\,dt}$$

$$2\int \sec^3 t\,dt = \sec t \tan t + \int \sec t\,dt$$

$$\int \sec^3 t\,dt = \frac{1}{2}\Big[\sec t \tan t + \ln|\sec t + \tan t|\Big] + C$$