NORTH CAROLINA STATE UNIVERSITY Department of Mathematics

MA141-005 Test #4 Form A Wednesday, April 17, 2019 Dr. J. Griggs

Please put all work and answers in the stamped blue book provided. Do <u>not</u> put work or answers on the test itself. Show all simplifications and substitutions. Do **not** use a calculator that does calculus; do not use a graphing calculator. Please include your **name, row and seat** on the front of the blue book. Put the test copy in the blue book.

1.) Integrate: $\int \frac{5\sec^2 x}{1+3\tan x}dx$

2.) Integrate and evaluate: $\int_0^6 (6x - x^2)\,dx$)

3.) Integrate using substitution and evaluate: $\int_0^1 7x^3(1+4x^4)^2\,dx$

4.) Integrate by parts: $\int 3x^4 \ln x\,dx$

5.) Evaluate using the Fundamental Theorem of Calculus: $\dfrac{d\ \left(\int_0^{x^3} \sqrt{1+r^2}\,dr\right)}{dx}$

6.) Using a Riemann Sum and the sigma notation formulas (provided), find the exact area under the curve $f(x) = 6x - x^2$ *from* $x = 0$ *to* $x = 6$. (compare with #2 above)

$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

7.) Find the area of the region bounded by the two curves:
$y = 4x - x^2$ *and* $y = x^2 - 6x + 8$

Bonus (5 points): Integrate $\int \sec^3 t\,dt$

MA141 TEST #4 FORM A

TEST SOLUTIONS (14 pts each)

1.) $$\int \frac{5\sec^2 x}{1+3\tan x}\,dx = \frac{1}{3}\cdot 5\int \frac{\sec^2 x}{1+3\tan x}\,dx\cdot 3$$

let $u = 1+3\tan x$

$du = 3\sec^2 x\,dx$

$$= \frac{5}{3}\int \frac{du}{u} = \frac{5}{3}\int \frac{1}{u}\,du = \frac{5}{3}\ln|u| + C$$

$$= \frac{5}{3}\ln|1+3\tan x| + C$$

2.) $$\int_0^6 (6x-x^2)\,dx = \left[6\cdot\frac{x^2}{2} - \frac{x^3}{3}\right]_0^6$$

$$= \left[3x^2 - \frac{x^3}{3}\right]_0^6 = \left(3\cdot 6^2 - \frac{6^3}{3}\right) - (0)$$

$$= 108 - 72 = 36$$

3.) $$\int_0^1 7x^3(1+4x^4)^2\,dx$$

let $u = 1+4x^4$

$du = 16x^3\,dx$

$x=0 \xrightarrow{u=1+4x^4} u=1$

$x=1 \longrightarrow u=5$

$$= 7\cdot\frac{1}{16}\int_0^1 (1+4x^4)^2\cdot \underbrace{x^3\cdot dx\cdot 16}$$

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$$= \frac{7}{16}\int_1^5 u^2 \cdot du = \frac{7}{16}\cdot\frac{u^3}{3}\Big]_1^5 = \frac{7}{48}u^3\Big]_1^5$$

$$= \frac{7}{48}\left[5^3 - 1^3\right] = \frac{7}{48}\left[125-1\right] = \frac{7}{48}(124) = \frac{7(31)}{12}$$

$$= \frac{217}{12} \approx 18.083$$

4.) $\int 3x^4 \cdot \ln x \, dx$

let $u = \ln x$ $\qquad v = 3\cdot\frac{x^5}{5}$

$du = \frac{1}{x}dx$ $\qquad dv = 3x^4 \cdot dx$

$$= (u)(v) - \int v \cdot du$$

$$= (\ln x)\left(\frac{3}{5}\cdot x^5\right) - \int\left(\frac{3}{5}x^5\right)\left(\frac{1}{x}\right)dx$$

$$= \frac{3}{5}x^5 \cdot \ln x - \frac{3}{5}\int x^4 dx$$

$$= \frac{3}{5}x^5 \cdot \ln x - \frac{3}{5}\cdot\frac{x^5}{5} + C$$

$$= \frac{3}{5}x^5 \cdot \ln x - \frac{3}{25}x^5 + C$$

5.) $$\frac{d\left[\int_0^{x^3}\sqrt{1+r^2}\,dr\right]}{dx} = \sqrt{1+(x^3)^2}\cdot d(x^3)$$

$$= 3x^2\sqrt{1+x^6}$$

6.) $\text{AREA} = \lim_{n\to\infty} \sum_{i=1}^{n} f(a+i\cdot\Delta x)\cdot\Delta x$

$a=0 \qquad b=6 \qquad\qquad f(x) = 6x - x^2$

$\Delta x = \frac{b-a}{n} = \frac{6-0}{n} = \frac{6}{n}$

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} f\left(0 + i\cdot\frac{6}{n}\right)\cdot\frac{6}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} f\left(\frac{6i}{n}\right)\cdot\frac{6}{n} = \lim_{n\to\infty} \sum_{i=1}^{n} \left(6\cdot\left(\frac{6i}{n}\right) - \left(\frac{6i}{n}\right)^2\right)\cdot\frac{6}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{36i}{n} - \frac{36i^2}{n^2}\right)\cdot\frac{6}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{216i}{n^2} - \frac{216i^2}{n^3}\right)$$

$$= \lim_{n\to\infty} \left[\frac{216}{n^2}\sum_{i=1}^{n} i - \frac{216}{n^3}\sum_{i=1}^{n} i^2\right]$$

$$= \lim_{n\to\infty} \left[\frac{216}{n^2}\cdot\frac{n(n+1)}{2} - \frac{216}{n^3}\cdot\frac{n(n+1)(2n+1)}{6}\right]$$

$$= \lim_{n\to\infty} \left[\frac{216}{2}\cdot\frac{n^2+n}{n^2} - \frac{216}{6}\,\frac{(2n^3+\dots)}{n^3}\right]$$

$$= \frac{216}{2}(1) - \frac{216}{6}(2) = 108 - 72 = 36$$

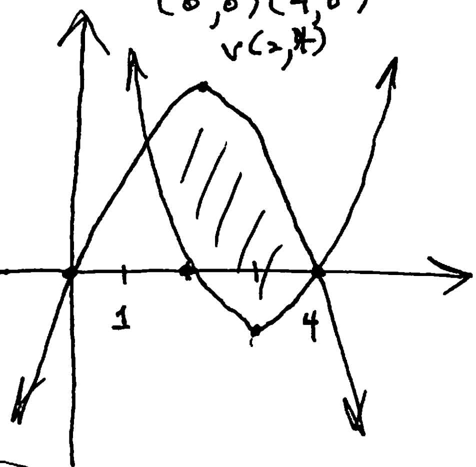
7.) area of the bounded region

14 pts $y = 4x - x^2$ & $y = x^2 - 6x + 8$

$(0,0)(4,0)$
$v(2,4)$

$y = (x-4)(x-2)$
$(4,0)$ $(2,0)$
$v(3, 9-18+8) = (3,-1)$

graph not needed



$$\cancel{4x} - \cancel{x^2} = x^2 - 6x + 8$$
$$-\cancel{4x} + \cancel{x^2} \quad +x^2 \quad -4x$$

$$0 = 2x^2 - 10x + 8$$
$$0 = 2(x^2 - 5x + 4)$$
$$0 = 2(x-4)(x-1)$$
$$x = 4, \quad x = 1$$
b↑ a↑

$$A = \int_1^4 \left[(4x - x^2) - (x^2 - 6x + 8)\right] dx$$

$$A = \int_1^4 (-2x^2 + 10x - 8)\, dx = \left[\frac{-2x^3}{3} + \frac{10x^2}{2} - 8x\right]_1^4$$

$$A = \left[\frac{-2(4)^3}{3} + 5(4)^2 - 8(4)\right] - \left[\frac{-2(1)^3}{3} + 5(1)^2 - 8(1)\right]$$

$$A = \frac{-128}{3} + 80 - 32 + \frac{2}{3} - 5 + 8$$

$$A = \frac{-126}{3} + 51 = -42 + 51 = 9$$

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BONUS : 5 pts

$$\int \sec^3 t\,dt = \int \sec t \cdot \sec^2 t\,dt$$

$u = \sec t$ $\quad v = \tan t$

$du = \sec t \tan t\,dt$ $\quad dv = \sec^2 t\,dt$

$$\int \sec^3 t\,dt = (\sec t)(\tan t) - \int \sec t \cdot \tan^2 t\,dt$$

$\tan^2 t = \sec^2 t - 1$

$$\int \sec^3 t\,dt = \sec t \tan t - \int \sec t(\sec^2 t - 1)\,dt$$

$$\int \sec^3 t\,dt = \sec t \tan t - \cancel{\int \sec^3 t\,dt} + \int \sec t\,dt$$

$$+\int \sec^3 t\,dt \qquad + \cancel{\int \sec^3 t\,dt}$$

$$2\int \sec^3 t\,dt = \sec t \tan t + \int \sec t\,dt$$

$$\int \sec^3 t\,dt = \frac{1}{2}\Big[\sec t \tan t + \ln|\sec t + \tan t|\Big] + C$$