MA141 – 005

Wednesday, April 10

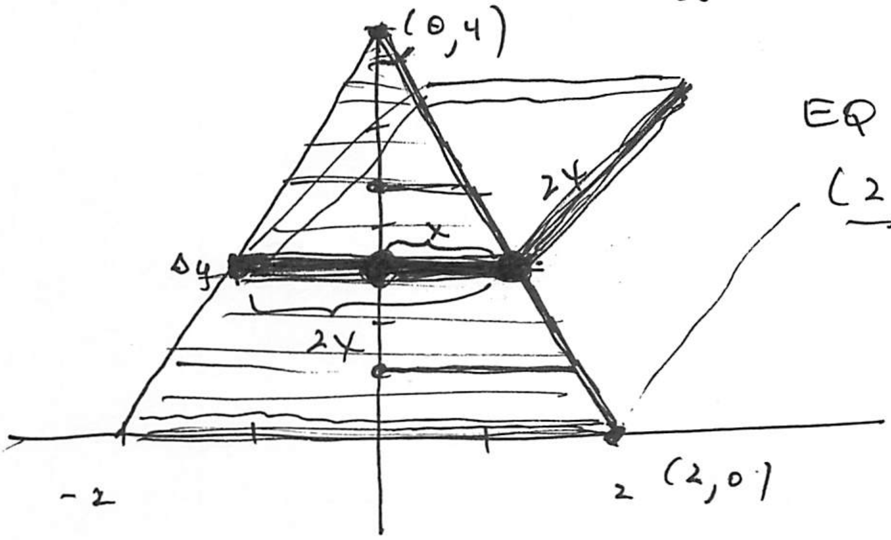
- today: 5.2 (washer method; cylindrical shell

next week:

MON., 4/15: finish course content; review

WED., 4/17: TEST #4 (4.1 → 4.5, 5.1)

VOLUMES "BY SLICING"



EQ OF LINE:

(2, 0) & (0, 4)

$m = \frac{4-0}{0-2} = \frac{4}{-2}$

$m = -2$

$y - y_1 = m(x - x_1)$

$y - 0 = -2(x-2)$

$y = -2x + 4$

$x = ??$

$\frac{y-4}{-2} = \frac{-2x}{-2}$

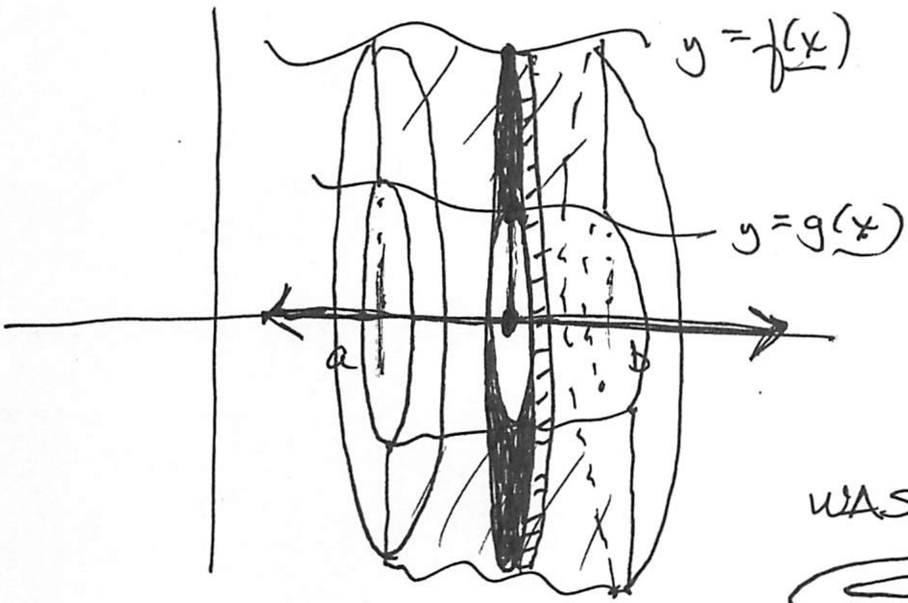
$\frac{4-y}{2} = x$

$$V = \int_0^4 [(2x)(2x)] \cdot dy$$

$$V = \int_0^4 \left(2\left(\frac{4-y}{2}\right)\right)\left(2\left(\frac{4-y}{2}\right)\right) dy = \int_0^4 (4-y)^2 dy$$

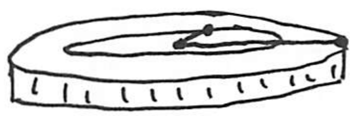
$$V = \int_0^4 (16 - 8y + y^2)\, dy$$

WASHER METHOD:



bounded region about the x-axis

WASHER:



outer radius: R
inner radius: r

$\pi \cdot R^2 \cdot h$ — $\pi \cdot r^2 \cdot h$

one slice

$$\text{VOL} = \pi \cdot R^2 \cdot h - \pi \cdot r^2 \cdot h$$

$$\text{VOL} = \pi \cdot h \left[R^2 - r^2 \right]$$

$$\underset{\text{(WASHER)}}{\text{VOL}} = \pi \left[R^2 - r^2 \right] \cdot h$$

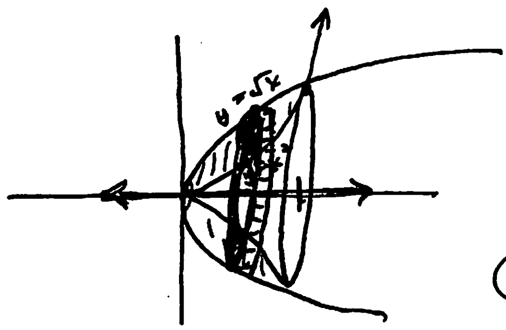
$$\text{VOL} = \int_a^b \pi \left[R^2 - r^2 \right] \cdot h$$

$R = f(x)$
$r = g(x)$
$h = \Delta x \to dx$

$$= \int_a^b \pi \left[[f(x)]^2 - [g(x)]^2 \right] \cdot dx$$

$f(x) = x^2 \qquad g(x) = \sqrt{x}$

bounded region
about the x-axis



$x^2 = \sqrt{x}$

$x^4 = x$

$x^4 - x = 0$

$x(x^3-1) = 0$

$x=0 \quad x=1$

WASHER:

$$\int_0^1 \pi\left[R^2 - r^2\right]\cdot h$$

$R = \sqrt{x}$

$r = x^2$

$h = dx$

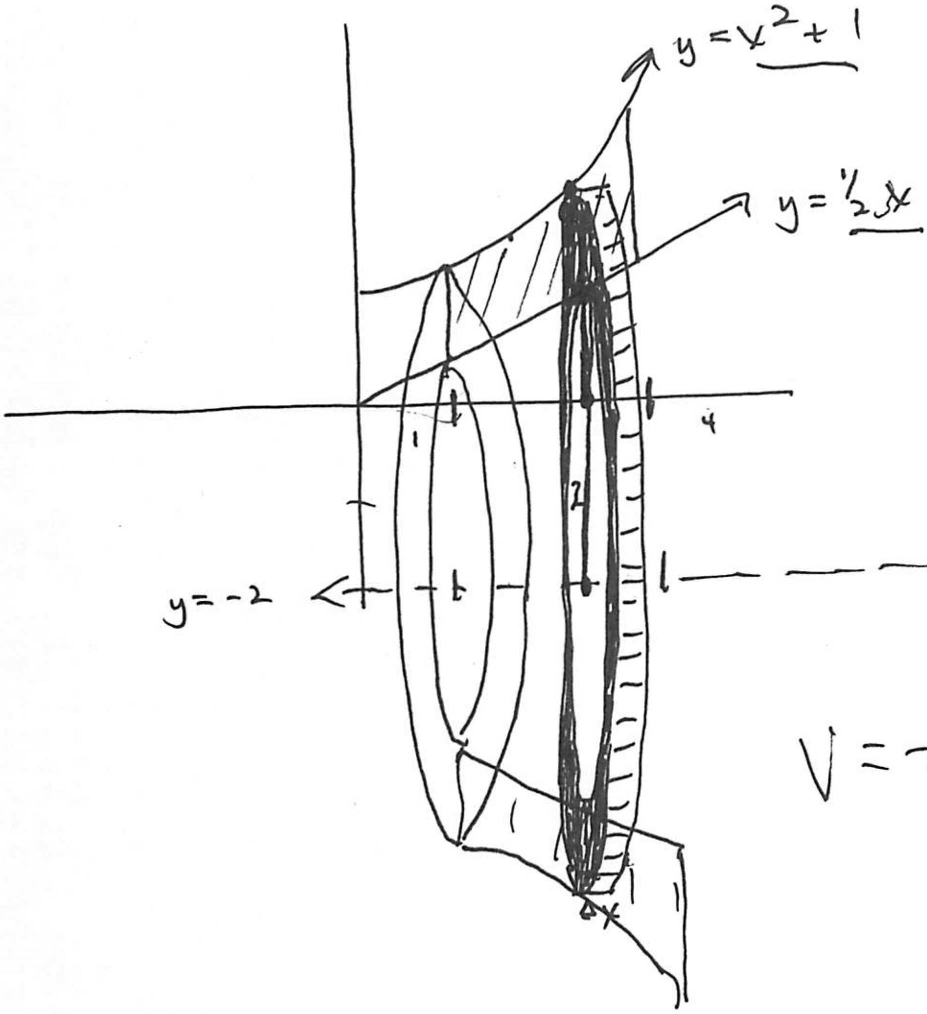
$$V = \pi \int_0^1 \left[(\sqrt{x})^2 - (x^2)^2\right] dx$$

$$= \pi \int_0^1 (x - x^4)\, dx$$

$$= \pi \left[\frac{x^2}{2} - \frac{x^5}{5}\right]_0^1$$

$$= \pi\left[\left(\frac{1}{2} - \frac{1}{5}\right) - 0\right]$$

$$= \pi\left[\frac{5}{10} - \frac{2}{10}\right] = \frac{3\pi}{10}$$



ABOUT $y=-2$:

$R = 2 + (x^2+1)$

$r = 2 + \frac{1}{2}x$

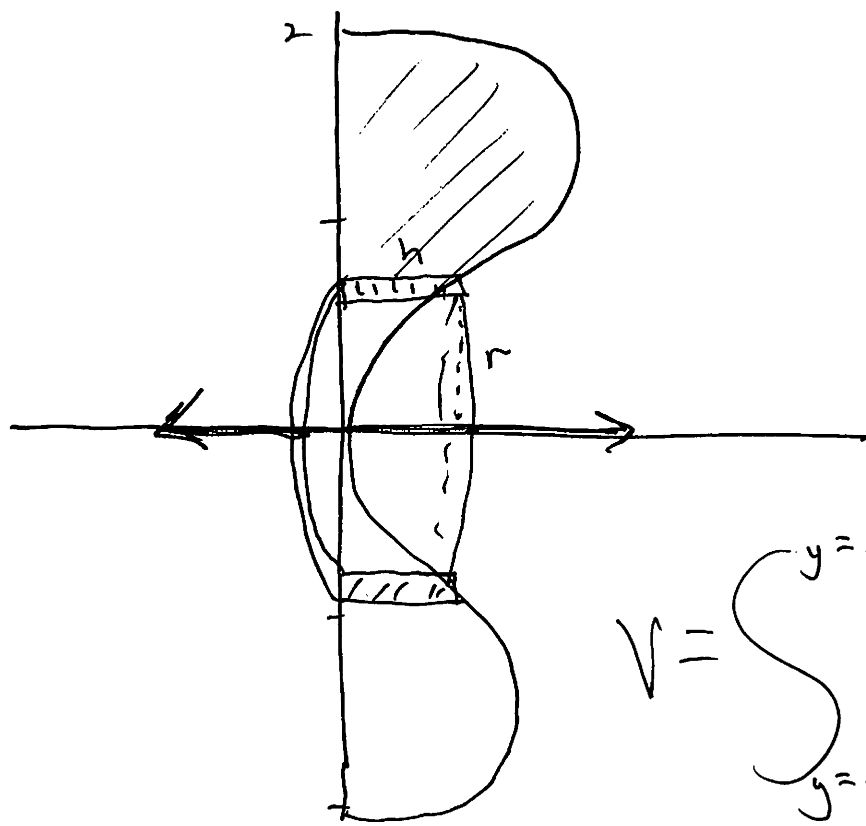
$h = dx$

AXIS OF REV.

$$V = \pi \int_1^4 \Big[\underbrace{(2+x^2+1)^2}_{R^2} - \underbrace{(2+\tfrac{1}{2}x)^2}_{r^2} \Big]$$

CYLINDRICAL SHELL:

$x = 2y^3 - y^4$



$$V = \int_{y=0}^{y=2} 2\pi r h \, (th)$$

$r = y \quad h = x$

$(th) = \Delta y$

$$V = \int_0^2 2\pi (y)(x)\, dy$$

$$V = \int_0^2 2\pi \cdot y \, (2y^3 - y^4)\, dy$$

$$V = 2\pi \int_0^2 (2y^4 - y^5)\, dy$$

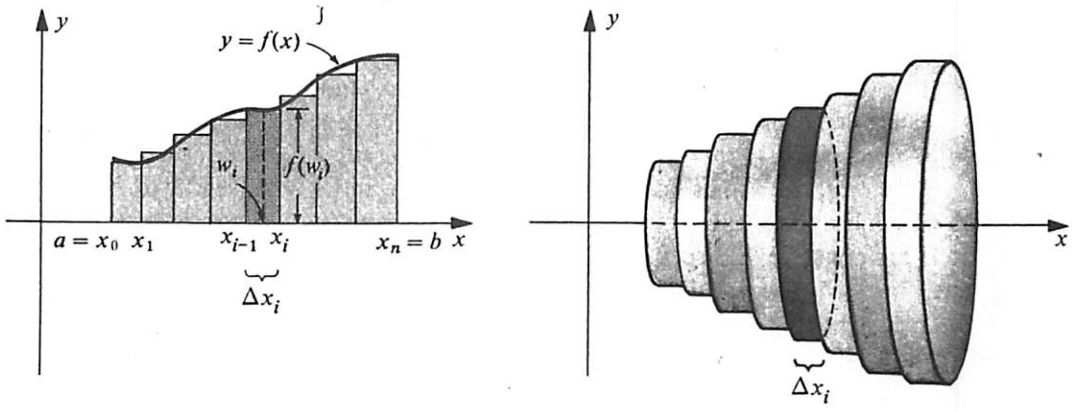
$$V = 2\pi \left[\frac{2y^5}{5} - \frac{y^6}{6} \right]_0^2$$

$$V = 2\pi\left[\left(\frac{2(2)^5}{5} - \frac{(2)^6}{6}\right) - (0)\right]$$

$$V = 2\pi\left[\frac{64}{5} - \frac{64}{6}\right]$$

$$V = 2\pi\left[\frac{64\cdot 6}{5\cdot 6} - \frac{64\cdot 5}{6\cdot 5}\right]$$

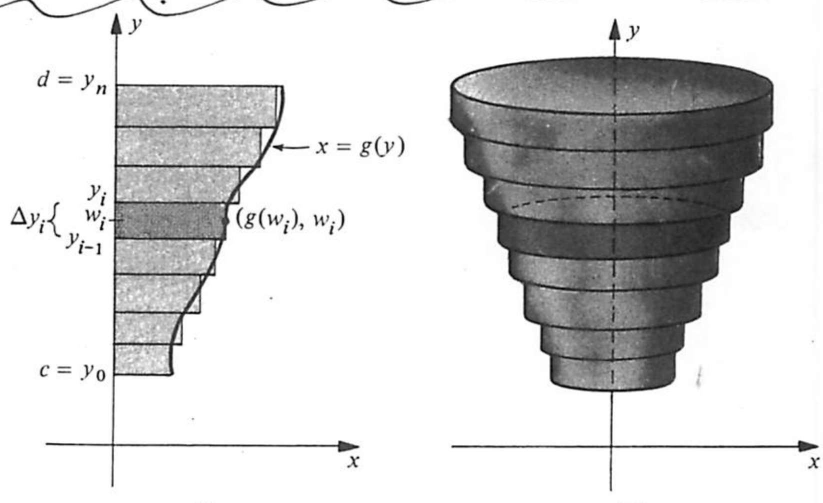
$$V = 2\pi\left[\frac{64}{30}\right] \cdots$$



(i) *Figure 6.12* (ii)

solid disk:

$$V = \int_a^b \pi r^2 h \qquad r = f(x) \qquad h = \Delta x \to dx$$



(i) (ii)

Figure 6.15

solid disk:

$$V = \int_c^d \pi r^2 h \qquad r = g(y) \qquad h = \Delta y \to dy$$

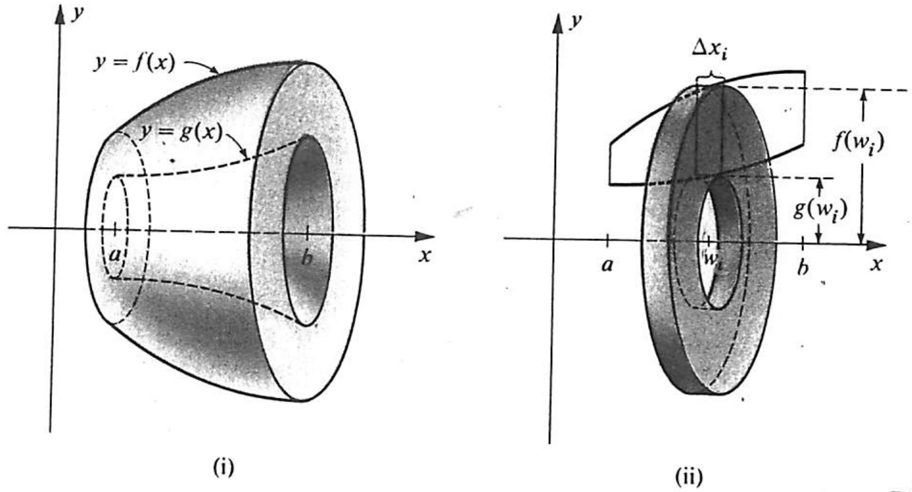


Figure 6.17

WASHER:

$$V = \int_a^b \pi(R^2 - r^2)\,h$$

$R = f(x)$
$r = g(x)$
$h = dx$

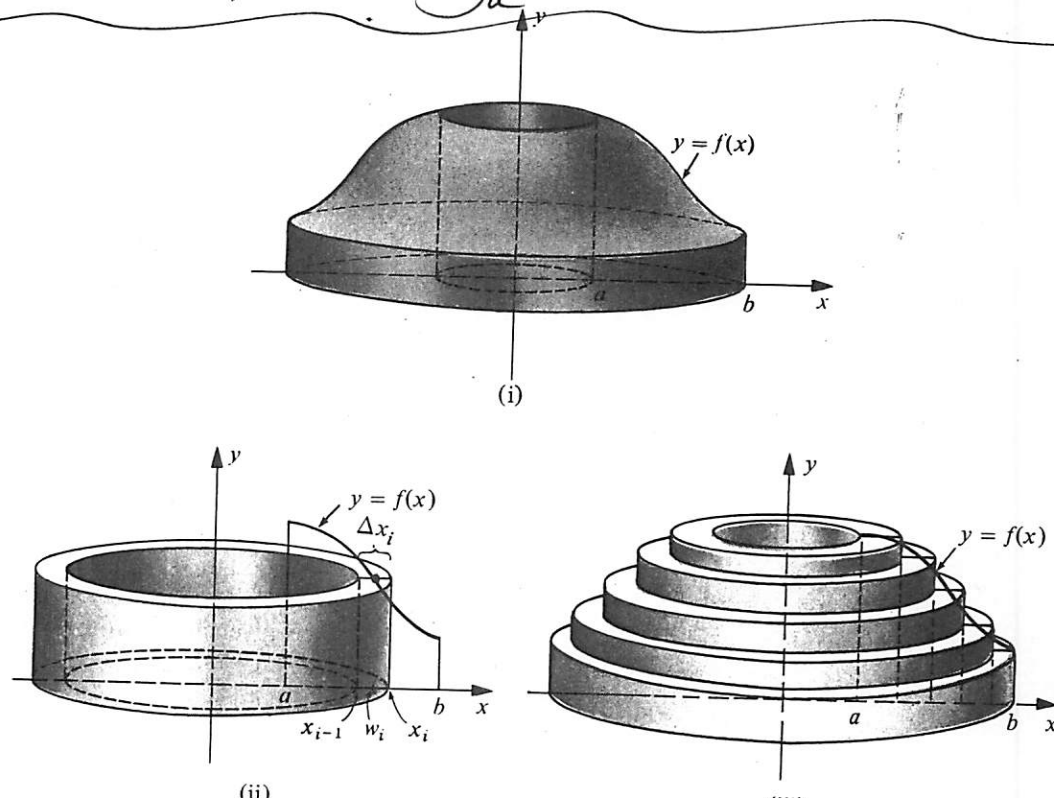


Figure 6.22

CYLINDRICAL SHELL:

$$V = \int_a^b 2\pi r h \,(th)$$