MA 141-005

monday, april 8

this week:

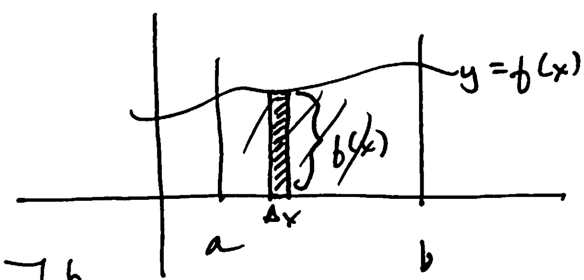
<u>5.1</u>: areas

<u>5.2</u>: volumes

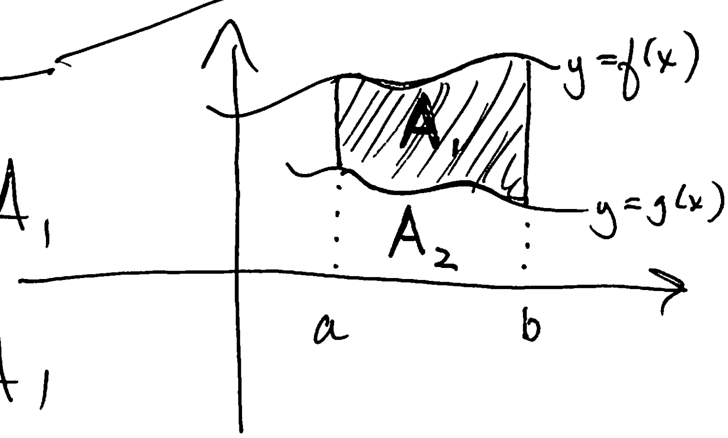
- <u>TEST #4</u>: WEDNESDAY, APRIL 17^{th}
- <u>FINAL EXAM</u>: MONDAY, APRIL 29^{th} 6:00-9:00 pm

<u>5.1</u>: AREA <u>BETWEEN</u> <u>TWO</u> <u>CURVES</u>:

area under a curve:



$$\int_a^b \underbrace{\underbrace{f(x)}_{HT}\,\underbrace{dx}_{WDTH}}_{\text{AREA OF REPR. RECT.}} = F(x)\Big]_a^b = F(b) - F(a)$$

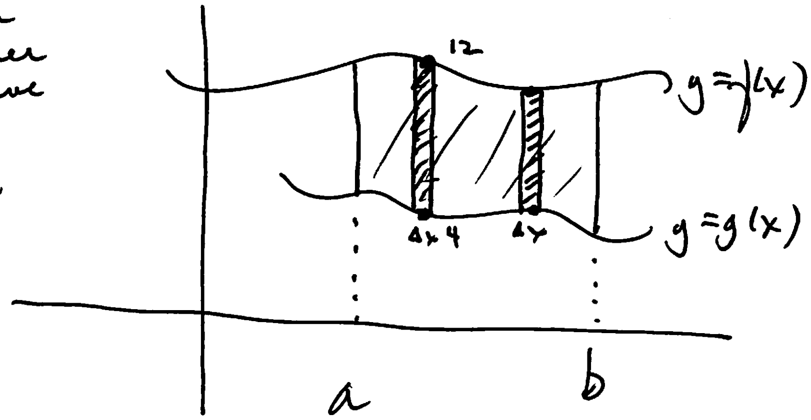


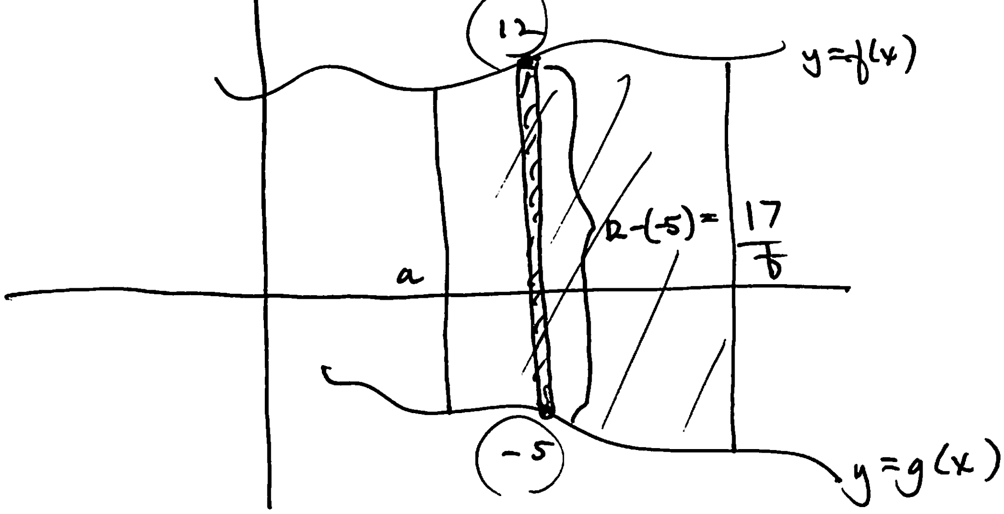
$$\int_a^b f(x)\,dx - \int_a^b g(x)\,dx = A_1$$

$$(A_1 + A_2) - (A_2) = A_1$$

$$\int_a^b [f(x) - g(x)]\,dx = \text{AREA BETWEEN } y = f(x) \text{ \& } y = g(x)$$

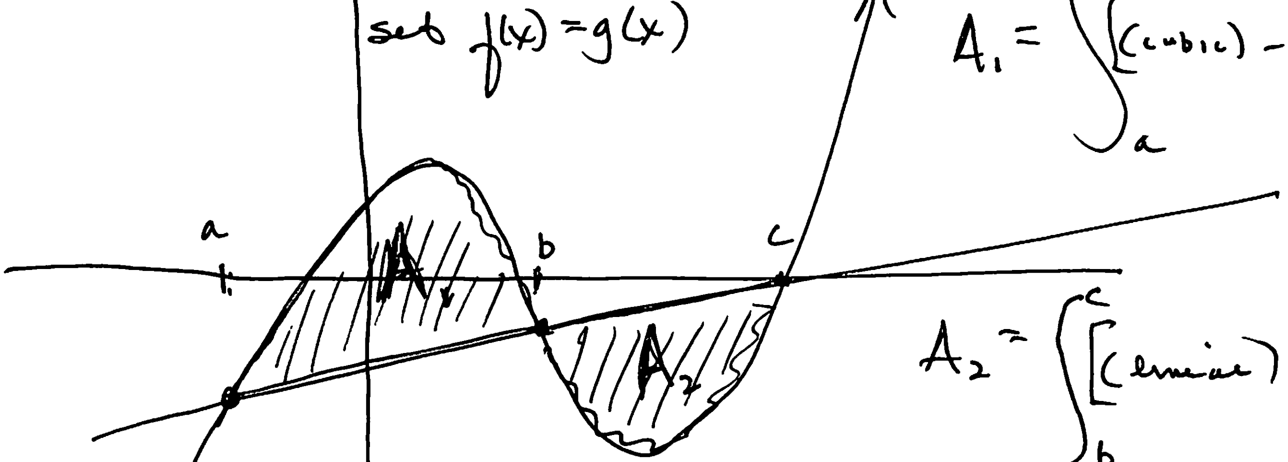
$$\int_a^b \Big[\underbrace{f(x)}_{\substack{\text{y-value}\\ \text{on the}\\ \text{upper curve}}} - \overbrace{g(x)}^{\substack{\text{y-value on}\\ \text{the lower}\\ \text{curve}}}\Big]\,dx$$





$$\int_a^b (f(x)-g(x))\,dx$$

set $f(x)=g(x)$



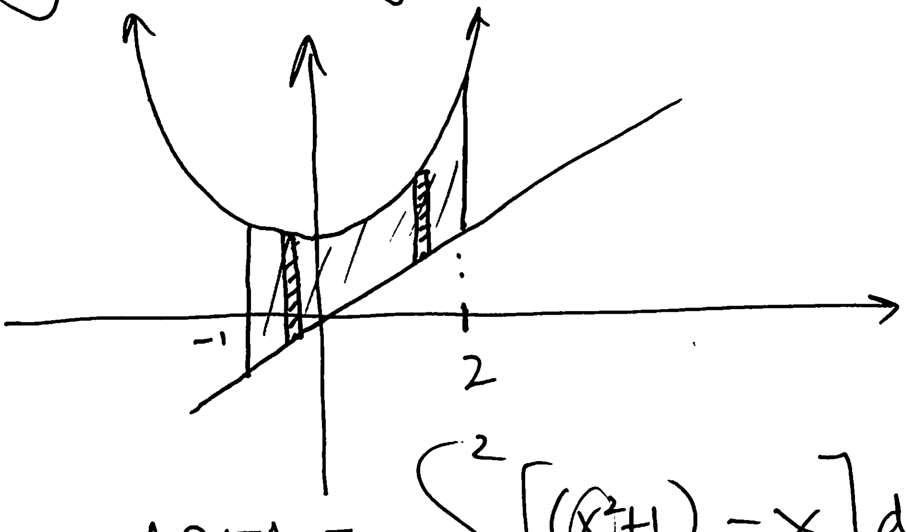
$$A_1=\int_a^b \Big[(\text{cubic})-(\text{linear})\Big]\,dx$$

$$A_2=\int_b^c \Big[(\text{linear})-(\text{cubic})\Big]\,dx$$

$$A=A_1+A_2$$

from $x=-1$ $x=2$

$y=x^2+1$ $y=x$



$$\text{AREA} = \int_{-1}^{2}\left[(x^2+1) - x\right]dx$$

$$= \left[\frac{x^3}{3} + x - \frac{x^2}{2}\right]_{-1}^{2}$$

$$= \left(\frac{2^3}{3} + 2 - \frac{2^2}{2}\right) - \left(\frac{(-1)^3}{3} + (-1) - \frac{(-1)^2}{2}\right)$$

$y = 6-x^2$ and $y = -2x+3$

find the area of the bounded region

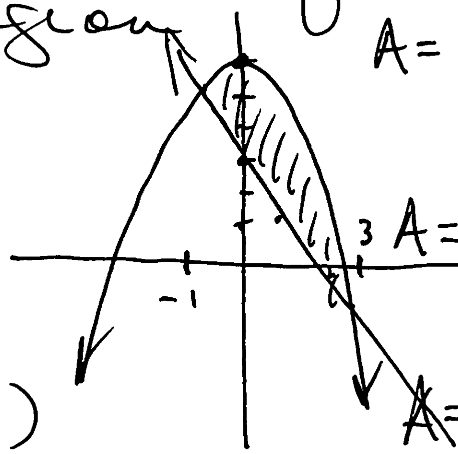
find pts of int:

$$6-x^2 = -2x+3$$

$$0 = x^2-2x-3$$

$$0 = (x-3)(x+1)$$

$x=3$ $x=-1$



$$A = \int_{-1}^{3}\left[(6-x^2) - (-2x+3)\right]dx$$

$$A = \int_{-1}^{3}(3-x^2+2x)\,dx$$

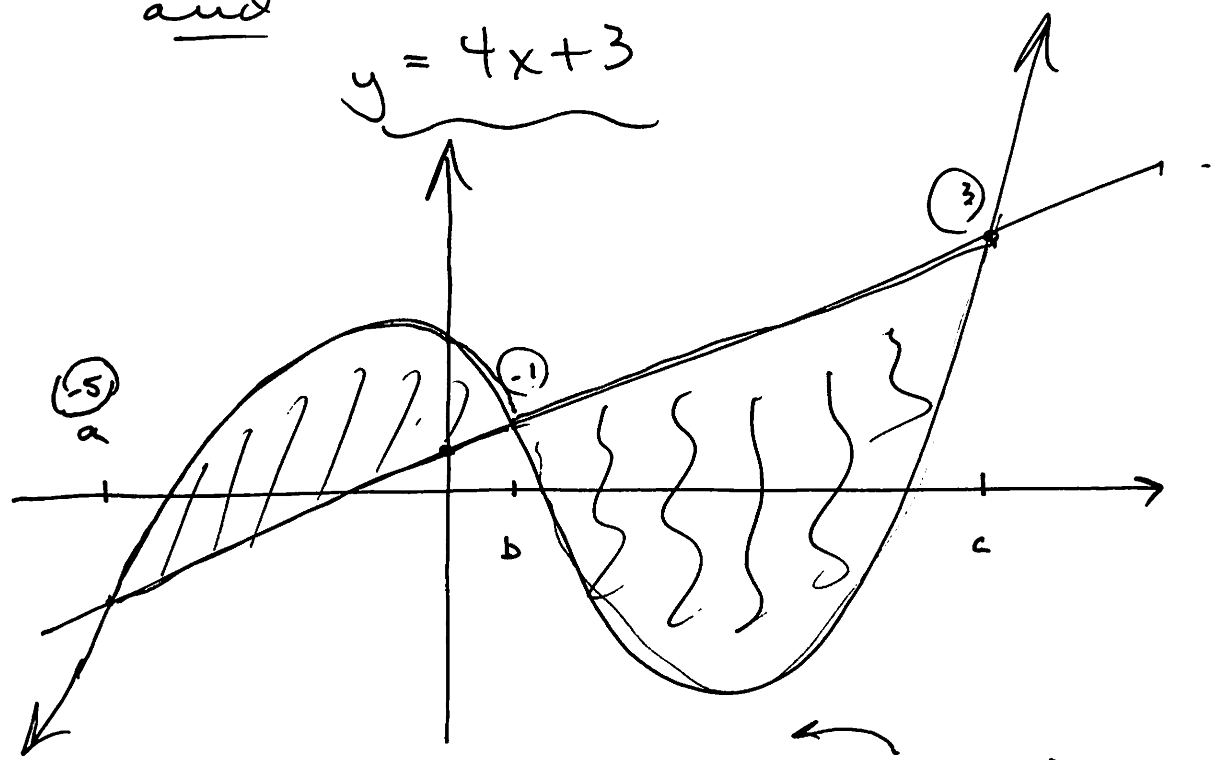
$$A = \left[3x - \frac{x^3}{3} + x^2\right]_{-1}^{3}$$

$A =$ ______

$y = x^3 + 3x^2 - 9x - 12$ "S" curve

and

$y = 4x + 3$



$x^3 + 3x^2 - 9x - 12 = 4x + 3$

$\rightarrow \boxed{x^3 + 3x^2 - 13x - 15 = 0}$

$(x + \underline{1})(x \quad \underline{\ \ })(x \quad \underline{\ \ }) = 0$

$\pm 1, \pm 3, \pm 5, \pm 15$

$f(1) = 1^3 + 3(1)^2 - \cancel{9}\,13(1) \cancel{-12}\,-15 \overset{??}{=} 0$

$f(1) = 1 + 3 - 13 - 15 \neq 0$

$f(-1) = (-1)^3 + 3(-1)^2 - \cancel{9}\,13(-1) \cancel{-12}\,-15$

$\cancel{f(-1) = -1 + 3 + 9 - 12}$

$f(-1) = -1 + 3 + 13 - 15 = 0$

$\boxed{(x+1)}$

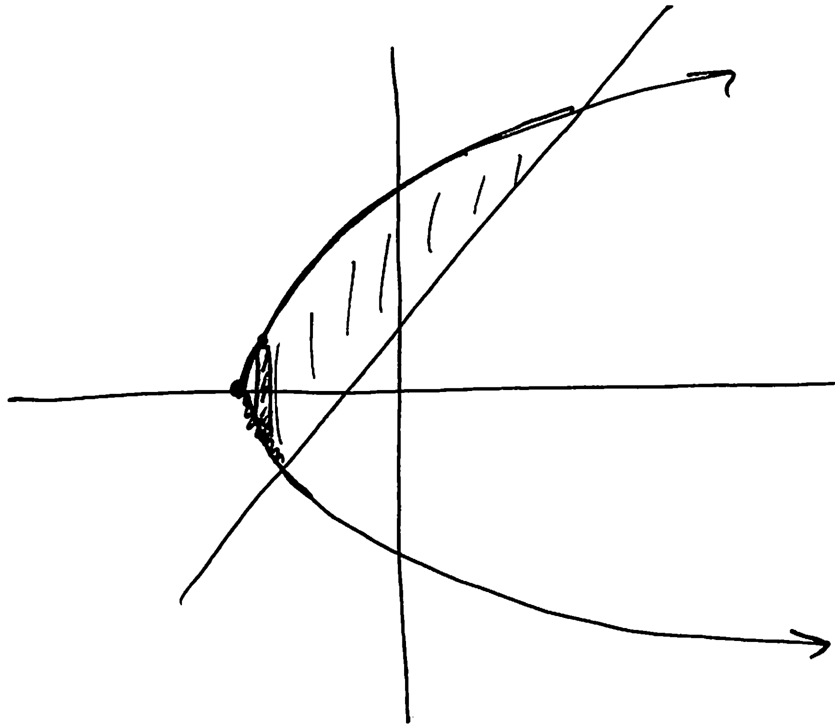
$$
\begin{array}{r}
x^2 + 2x - 15 \\
x+1 \,\big)\, \overline{x^3 + 3x^2 - 13x - 15} \\
-(x^3 + 1x^2) \\
\hline
2x^2 - 13x \\
-(2x^2 + 2x) \\
\hline
-15x - 15 \\
-(-15x - 15) \\
\hline
0
\end{array}
$$

$$(x+1)(x+5)(x-3) = 0$$

$$x = -1 \; ; \; x = -5 \; ; \; x = 3$$

$$A = \int_{-5}^{-1} \left[(x^3 + 3x^2 - 9x - 12) - (4x+3) \right] dx$$

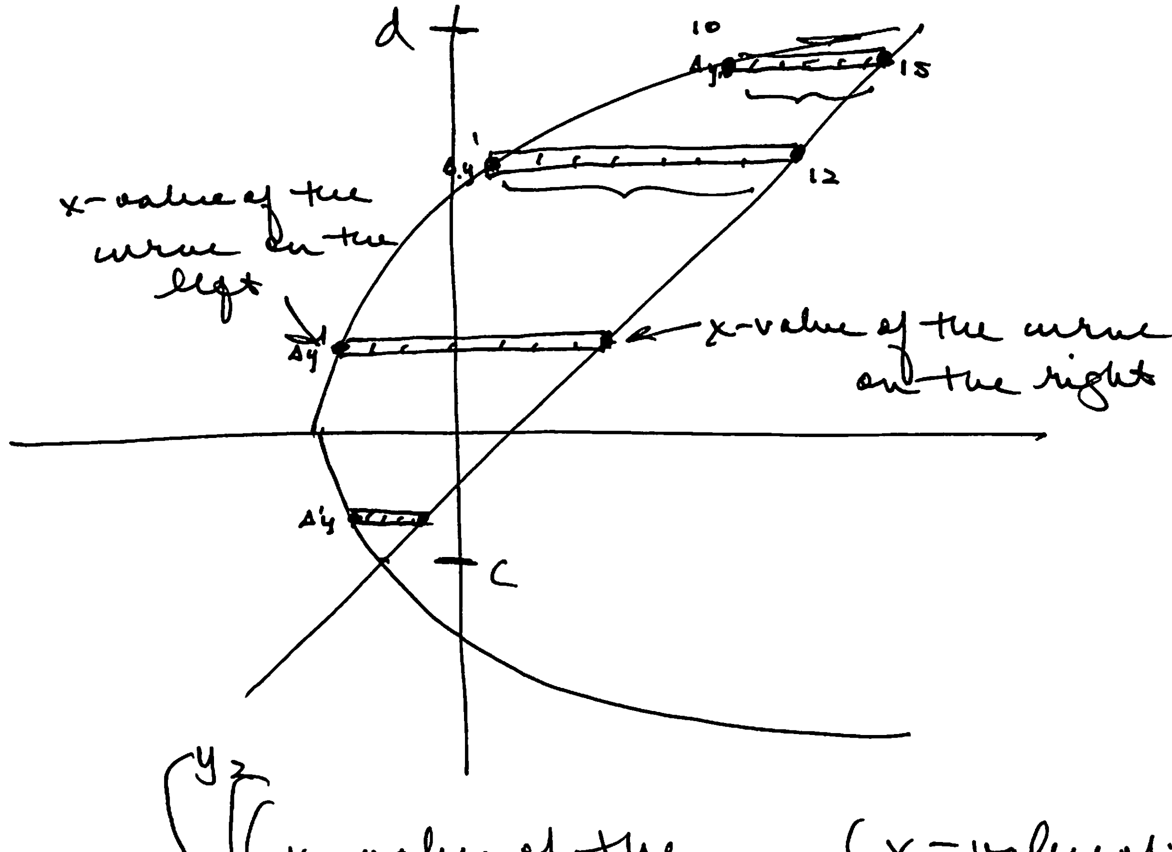
$$+ \int_{-1}^{3} \left[(4x+3) - (x^3 + 3x^2 - 9x - 12) \right] dx$$



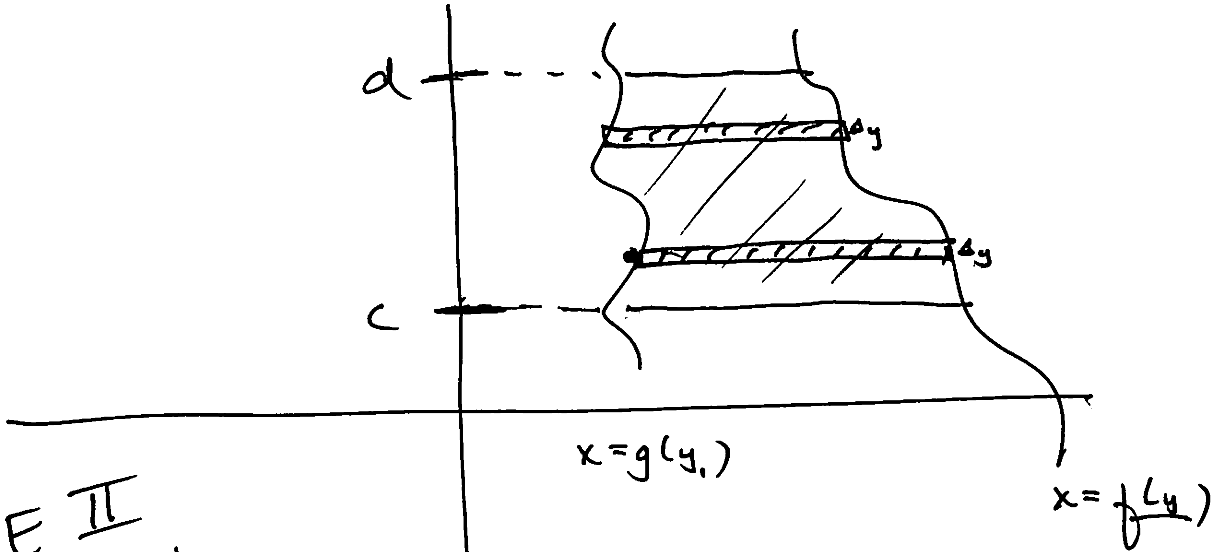
$y^2 = 2 - x$

$y = \sqrt{2-x}$

$y = -\sqrt{2-x}$



$$\int_{y_1}^{y_2} \underbrace{\left[\left(\text{x-value of the curve on the right} \right) - \left(\text{x-value of the curve on the left} \right) \right]} dy$$



TYPE II
REGION

$$A = \int_c^d [f(y) - g(y)]\,dy$$

$y^2 = 2x - 2 \Rightarrow \frac{y^2+2}{2} = x$ $\qquad y = x - 5 \Rightarrow x = y + 5$

find the area of the bounded region

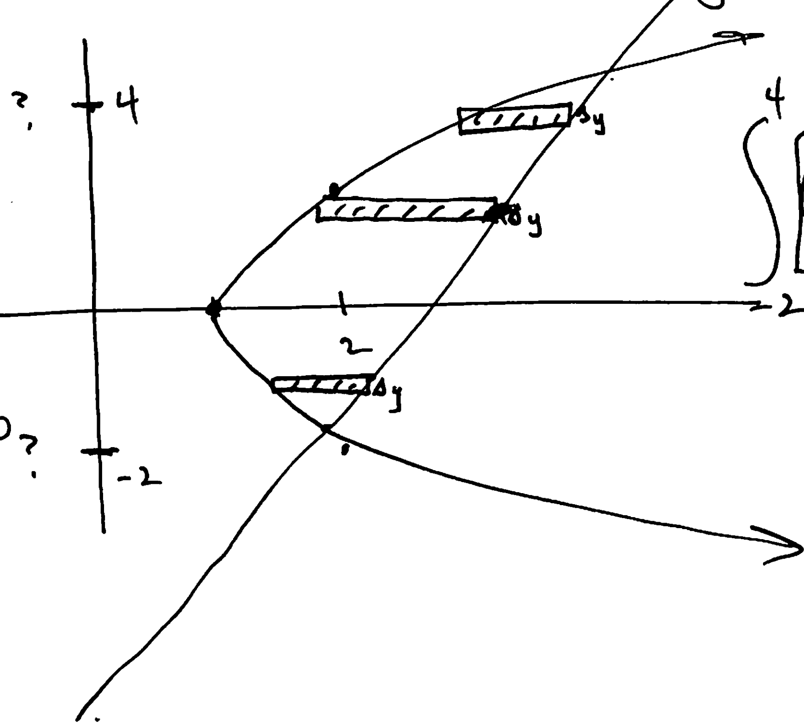
$\frac{y^2+2}{2} = y + 5$

$y^2 + 2 = 2y + 10$

$y^2 - 2y - 8 = 0$

$(y-4)\ (y+2) = 0$?

$y = 4 \qquad y = -2$



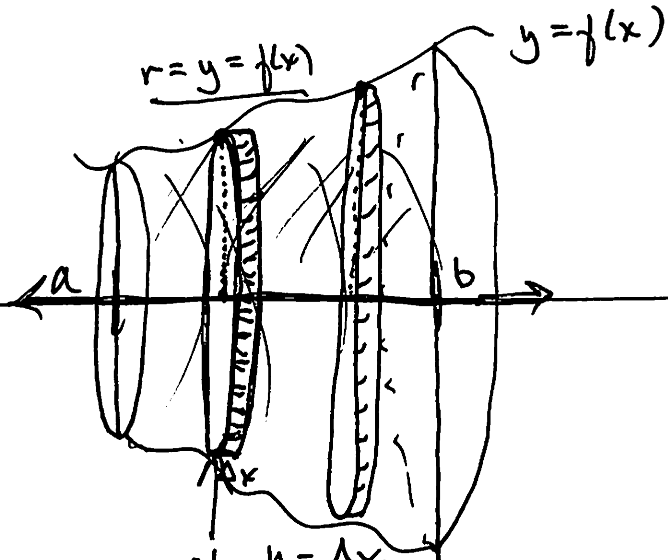
$$\int_{-2}^{4} \left[(y+5) - \left(\frac{y^2+2}{2}\right)\right] dy$$

$$A = \int_{-2}^{4} \left[(y+5) - \left(\frac{y^2+2}{2} \right) \right] dy$$

$$= \int_{-2}^{4} \left(-\frac{y^2}{2} + y + 4 \right) dy$$

5.1: VOLUMES OF SOLIDS OF REVOLUTION

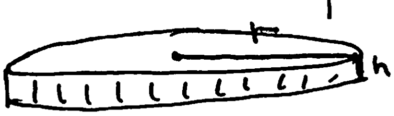
"slice it up" $\perp$ to the axis of rev.



x-axis is the AXIS OF REVOLUTION

revolve the bounded region ABOUT the x-axis

SOLID DISC

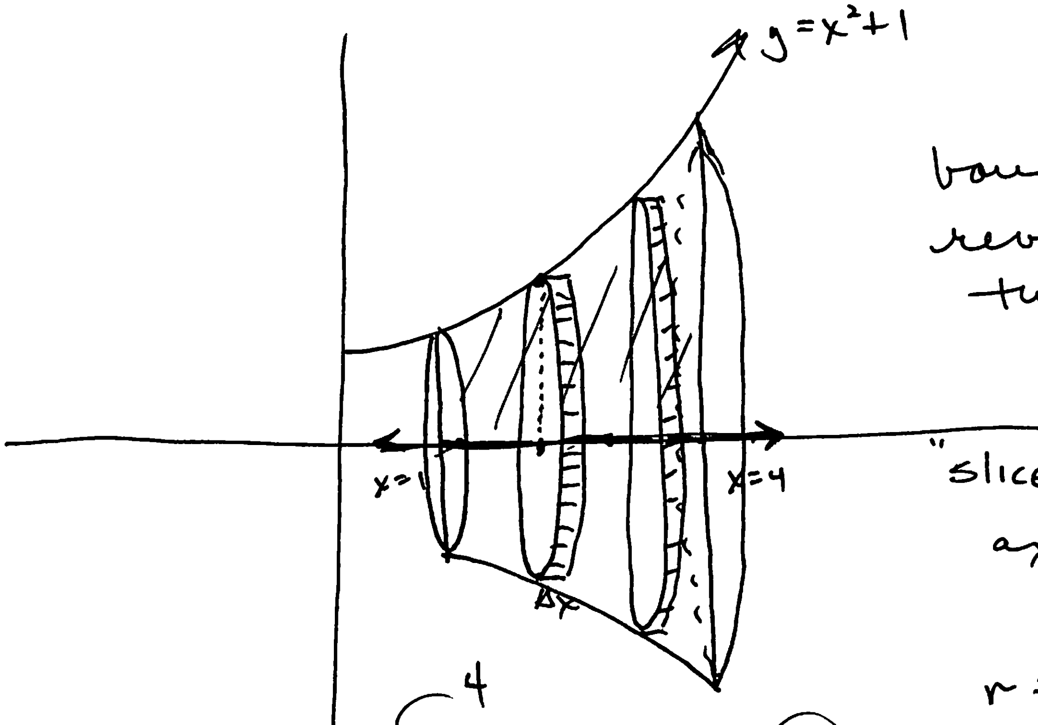


CYLINDER:

$V = \pi \cdot r^2 \cdot h$

VOL $h = \Delta x$

$$\int_a^b \underbrace{\pi \cdot r^2 \cdot h}_{\text{VOL OF ONE SLICE}} = \int_a^b \pi \cdot (f(x))^2 \cdot dx$$



bounded region revolved about the x-axis

"slice up" $\perp$ to axis of rev.

$r = f(x) = x^2 + 1$

$h = \cancel{\Delta y} \to dx$

$$V = \int_1^4 \pi \cdot r^2 \cdot h$$

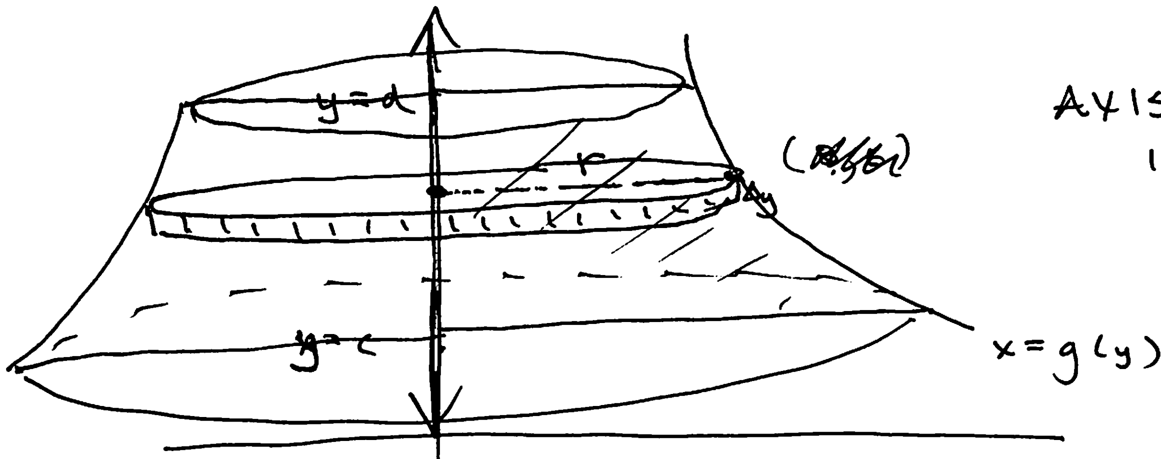
$$V = \pi \int_1^4 (x^2+1)^2 \cdot dx$$

$$V = \pi \int_1^4 (x^4 + 2x^2 + 1)\, dx$$

$$V = \pi \left[\frac{x^5}{5} + 2\frac{x^3}{3} + x \right]_1^4$$

$$V = \pi \left[\left(\frac{4^5}{5} + \frac{2}{3}(4)^3 + (4) \right) - \left(\frac{1^5}{5} + \frac{2}{3}(1)^3 + 1 \right) \right]$$

$$V =$$



AXIS OF REV. IS THE Y-AXIS

"SLICE UP" $\perp$ TO AXIS OF REV.

$$V = \int_c^d \pi r^2 h$$

$r = x = g(y)$

$h = \Delta y \to dy$

$$V = \pi \int_{y=c}^{y=d} [g(y)]^2 \cdot dy$$