MA 141 – 005

Wednesday, March 3

## 4.5: INTEGRATION BY PARTS

$$\int \frac{3}{5y^2+4}\,dy$$

$\cancel{\text{let } u = 5y^2+4}$

$\cancel{du = 10y\,dy}$

won't work

$$\int \frac{3}{5y^2+4}\,dy$$

$$3\int \frac{1}{5y^2+4}\,dy$$

$$\int \frac{1}{x^2+1}\,dx$$

$$\frac{1}{10}\cdot 3\int \frac{\cancel{3}y\cdot 10}{5y^2+4}\,dy$$

let $u = 5y^2+4$

$du = 10y\,dy$

$$\frac{3}{10}\int \frac{du}{u} =$$

$$\frac{3}{10}\ln|u| + C$$

$$\int \frac{1}{\cancel{u}^2 + a^2}\, du$$

$$\int \frac{1/a^2}{\frac{u^2}{a^2} + \frac{a^2}{a^2}}\, du$$

$$\frac{1}{a^2} \int \frac{1}{\left(\frac{u}{a}\right)^2 + 1}\, du$$

let $w = \frac{u}{a}$

$dw = \frac{1}{a} \cdot du$

$$a \cdot \frac{1}{a^2} \int \frac{1}{w^2 + 1}\, du \cdot \frac{1}{a}$$

$$\frac{1}{a} \boxed{\int \frac{1}{w^2+1}\, dw}$$

$$\int \frac{1}{x^2+1}\, dx = \tan^{-1} x + C$$

(2)

$$\frac{1}{a}\left[\tan^{-1} w\right] + C$$

$$\frac{1}{a}\left[\tan^{-1} \frac{u}{a}\right] + C = \int \frac{1}{u^2+a^2}\, du$$

$$\int \frac{1}{u^2+a^2}\,du = \left[\frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right)\right]+C$$

$$\int \frac{3}{5y^2+4}\,dy = 3\int \frac{1}{5y^2+4}\,dy$$

$$= \frac{3}{\sqrt{5}}\int \frac{1}{[\sqrt{5}y]^2+(2)^2}\,dy\sqrt{5}$$

$$= \frac{3}{\sqrt{5}}\left[\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{5}y}{2}\right)\right]+C$$

4.5: Integration "by parts"
(reverse product rule)

$$d(\underline{u}\cdot\underline{v}) = u\cdot dv + \cancel{v\cdot du}$$

$$-v\cdot du \qquad\qquad -\cancel{v\cdot du}$$

$$\int[d(u\cdot v)-v\cdot du] = \int u\cdot dv$$

$$\int d(u\cdot v) - \int v\cdot du = \int u\cdot dv$$

$$\boxed{u \cdot v - \int v \cdot du = \int u \cdot dv}$$

$$\int u \cdot dv = \underline{u \cdot v} - \underline{\int v \cdot du}$$

let $u =$ ______

$dv =$ ______

ex: $\int x \cdot \boxed{\cos x \, dx}$

$d(???) = x \cdot \cos x$

[ integrate by parts

$\int \underline{u} \cdot \underline{dv} = \underline{u \cdot v} - \int v \, du$ ]

let $\underline{u = x}$ $\qquad$ $v = \sin x$

$du = 1 \cdot dx$ $\qquad$ $\underline{dv = \cos x \, dx}$

$$(u \cdot v) \quad - \quad \int v \cdot du$$

$$\int x \cdot \cos x \, dx = \boxed{x \cdot \sin x - \int \sin x \cdot dx}$$ (5)

let $u = x$ $\qquad v = \sin x$

$du = 1 \cdot dx$ $\qquad dv = \cos x \, dx$

$$\int x \cos x \, dx = x \cdot \sin x - \int \sin x dx$$

$$= x \cdot \sin x - (-\cos x) + C$$

$$= x \cdot \sin x + \cos x + C$$

check:

$$d(x \cdot \sin x + \cos x + C) \overset{??}{=} x \cdot \cos x$$

$$= x \cdot \cos x + \cancel{(\sin x)(1)} \cancel{- \sin x} + 0$$

$$=$$

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ex:

$$\int x^3 \ln x \, dx = (\ln x)\left(\frac{x^4}{4}\right) - \int \frac{x^4}{4} \cdot \frac{1}{x} dx$$

let $u = \ln x$ $\qquad v = \frac{x^4}{4}$

$du = \frac{1}{x} dx$ $\qquad dv = x^3 dx$

$$= \frac{x^4}{4} \cdot \ln x - \int \frac{1}{4} \cdot x^3 \cdot dx$$

$$= \frac{x^4}{4} \cdot \ln x - \frac{1}{4} \frac{x^4}{4} + C$$

$$\int x^3 \cdot \ln x \, dx$$

let $u = x^3$ $\quad v = ???$

$du = 3x^2 dx$ $\quad dv = \ln x \cdot dx$

$$\boxed{\int u \, dv = u \cdot v - \int v \cdot du}$$

$$\int 5x^2 \cdot e^x \cdot dx$$

let $u = 5x^2$ $\qquad v = e^x$

$du = 10x\,dx$ $\qquad dv = e^x \cdot dx$

$$\int 5x^2 e^x \cdot dx = (5x^2)(e^x) - \left[\int e^x \cdot 10x\,dx\right]$$

integration by parts AGAIN ....

let $U = 10x$ $\qquad V = e^x$

$dU = 10 \cdot dx$ $\qquad dV = e^x \cdot dx$

$U \cdot V$

$$= 5x^2 \cdot e^x - \left[(10x)(e^x) - \int e^x \cdot 10\,dx\right]$$

$\int V \cdot dU$

$$= 5x^2 e^x - 10x e^x + 10\boxed{\int e^x \cdot dx}$$

$$= 5x^2 e^x - 10x e^x + 10 e^x + C$$

$$\int u \cdot dv = \underbrace{u \cdot v} - \int \boxed{v \cdot du}$$

integrable or better off

let $u =$ ____

$dv =$ ____

$$\rightarrow \int \underbrace{\tan^{-1} x}\ \underbrace{dx} = (x)(\tan^{-1} x) - \int (x)\left(\frac{1}{1+x^2}\right) dx$$

let $u = \tan^{-1} x$ $\qquad v = x$

$du = \frac{1}{1+x^2}\, dx$ $\qquad dv = dx$

$$= x \cdot \tan^{-1} x - \frac{1}{2} \int \frac{2x\, dx}{\underbrace{1+x^2}_{u}}$$

$u = 1+x^2$

$du = 2x\, dx$

$$= x \cdot \tan^{-1} x - \frac{1}{2} \int \frac{du}{u}$$

$$= x \cdot \tan^{-1} x - \frac{1}{2} \ln|u| + C$$

$$= x \cdot \tan^{-1} x - \frac{1}{2} \ln|1+x^2| + C$$

$$\int e^x \cdot \sin x \, dx = (e^x)(-\cos x) - \int (-\cos x) e^x \, dx$$

let $u = e^x$ $\qquad$ $v = -\cos x$

$du = e^x \, dx$ $\qquad$ $dv = \sin x \, dx$

$$\int e^x \sin x \, dx = -e^x \cos x + \left[ \int \cos x \cdot e^x \, dx \right]$$

$U = e^x$ $\qquad$ $V = \sin x$

$dU = e^x \, dx$ $\qquad$ $dV = \cos x \, dx$

$$\int e^x \cdot \sin x \, dx = -e^x \cos x + \left[ e^x \cdot \sin x - \cancel{\int \sin x \cdot e^x \, dx} \right]$$

$$+ \int \sin x \cdot e^x \, dx \qquad + \cancel{\int \sin x \cdot e^x \, dx}$$

$$2 \int e^x \cdot \sin x \, dx = -e^x \cos x + e^x \sin x$$

$$\int e^x \sin x \, dx = \frac{1}{2} \left[ -e^x \cos x + e^x \sin x \right] + C$$

$$\int_2^3 \frac{\ln x}{3x}\,dx$$

subst. or
integ. by parts

$$\int_2^3 \frac{1}{3}\cdot \ln x \cdot \frac{1}{x}\,dx$$

$$= \frac{1}{3}\int_2^3 \underbrace{\ln x}_{u} \cdot \underbrace{\frac{1}{x}\,dx}_{du}$$

let $u = \underline{\ln x}$

$du = \frac{1}{x}\cdot dx$

$\int x^1 dx = \frac{x^2}{2}$

$\int u^1 du = \frac{u^2}{2}$

$$= \frac{1}{3}\int u^1 \cdot du = \frac{1}{3}\left[\frac{u^2}{2}\right]$$

$$= \frac{1}{6}(\ln x)^2 \Big]_2^3$$

$$= \frac{1}{6}\left[(\ln 3)^2 - (\ln 2)^2\right] \approx \underline{\hspace{4cm}}$$

$$\int \sec^3 x\,dx = \int \sec x \cdot \underline{\sec^2 x\,dx}$$

let $u = \sec x$ $\quad v = \tan x$

$du = \sec x \tan x\,dx$ $\quad dv = \sec^2 x\,dx$

$$\int \sec^3 x\,dx = (\sec x)(\tan x) - \int \tan x \cdot \sec x \tan x\,dx$$

$$\int \sec^3 x\,dx = (\sec x)(\tan x) - \int \sec x \cdot \underline{\tan^2 x}\,dx$$

$$\int \sec^3 x\,dx = (\sec x)(\tan x) - \int \sec x \cdot (\sec^2 x - 1)\,dx$$

$$\int \sec^3 x\,dx = \sec x \cdot \tan x - \cancel{\int \sec^3 x\,dx} + \int \sec x\,dx$$

$$+\int \sec^3 x\,dx \qquad\qquad + \cancel{\int \sec^3 x\,dx}$$

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$$2\int \sec^3 x\,dx = \sec x \cdot \tan x + \int \sec x\,dx$$

$$\int \sec^3 x\,dx = \frac{\sec x \cdot \tan x}{2} + \frac{\ln|\sec x + \tan x|}{2} + C$$

(11)

$$\int \sec x\, dx = \int \frac{\sec x\,(\sec x + \tan x)}{(\sec x + \tan x)}\, dx$$

$$= \int \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x}\, dx$$

let $u = \sec x + \tan x$

$du = (\sec x \tan x + \sec^2 x)\, dx$

$$= \int \frac{du}{u} = \ln|u| + C$$

$$= \ln|\sec x + \tan x| + C$$