MA141-005

Monday, April 1

4.4 (integration using SUBSTITUTION)
("change of variable" method)

4.4:

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C$$

$(n \neq -1)$

subst ...

let $u =$ ~~~

$$\int u^n \cdot du = \frac{u^{n+1}}{n+1} + C$$

$$\int x^{-1} dx = \int \frac{1}{x} dx = \ln|x| + C$$

subst ...

let $u =$ ~~~

$$\int \frac{1}{u} du = \ln|u| + C$$

$$\int e^x dx = e^x + C$$

subst ...

let $u =$ ~~~

$$\int e^u \cdot du = e^u + C$$

DERIV.:

"reverse" chain rule: $F' = f$

$$\frac{d\left[F(g(x))\right]}{dx} = \underbrace{F'(g(x))}\cdot g'(x)$$

$$\int \frac{d\left[F(g(x))\right]}{dx} = \int f(g(x)) \cdot g'(x)$$

$$* \quad F(g(x)) = \int \underline{f(g(x))} \cdot \underline{g'(x)} \quad *$$

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ex:

$\int f(g(x)) \cdot g'(x)$

$$\int x \cdot \sqrt{6-8x^2}\, dx$$

let $\boxed{u = 6-8x^2}$

$dx \left(\frac{du}{\cancel{dx}}\right) = 0 - 16x \cdot dx$

$\boxed{du = -16x\, dx}$

$$\frac{-1}{16} \int (6-8x^2)^{1/2}\, \boxed{x \cdot dx\, (-16)}$$

$$\frac{-1}{16} \boxed{\int u^{1/2}\, du}$$

$$= \frac{-1}{16} \cdot \frac{u^{3/2}}{3/2} + C$$

$$= \frac{-1}{\cancel{16}_8} \cdot \frac{\cancel{2}^1}{3} u^{3/2} + C$$

$$= \frac{-1}{24} u^{3/2} + C$$

$$= \frac{-1}{24} (6-8x^2)^{3/2} + C$$

ex: $\frac{1}{8}\int \sec^2(4x^2+1)\,\boxed{x \cdot dx \cdot 8}$

let $\boxed{u = 4x^2+1}$

$dx\,\frac{du}{dx} = 8x \cdot dx$

$\boxed{du = 8x\,dx}$

$$\frac{1}{8}\int \sec^2 u\ du = \frac{1}{8}\tan u + C$$

$$= \frac{1}{8}\tan(4x^2+1) + C$$

$$\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$$

$\int e^x dx = e^x + C$

$\int e^u \cdot du$

let $u = \sqrt{x} = x^{1/2}$

$du = \frac{1}{2}x^{-1/2} \cdot dx = \frac{1}{2} \cdot \frac{1}{\sqrt{x}}\,dx = du$

$$2\int e^{\sqrt{x}} \cdot \boxed{\frac{1}{\sqrt{x}} \cdot dx\,\frac{1}{2}} = 2\int e^u \cdot du$$

$$= 2e^u + C$$

$$= 2e^{\sqrt{x}} + C$$

$$\int \tan x \, dx$$

$$\int \frac{1}{u} \cdot du$$

rewrite ....

$$-1 \int \frac{\sin x}{\cos x} \, dx \, (-1)$$

let $u = \cos x$

$du = -\sin x \, dx$

$$-1 \int \frac{du}{u} = -1 \int \frac{1}{u} \, du$$

$$= -1 \cdot \ln|u| + C$$

$$= \boxed{-1} \cdot \ln|\cos x| + C \quad ✓$$

$$= \ln|\cos x|^{-1} + C$$

$$= \ln|\sec x| + C \quad *$$

DEF. INTEGRAL .... ~~SUBST~~ SUBST.

$$\int_{x=1}^{x=4} \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx = 2\int_1^4 e^{\sqrt{x}} \cdot \boxed{\frac{1}{\sqrt{x}} \cdot dx \cdot \frac{1}{2}}$$

let $u = \sqrt{x}$ $\quad du = \frac{1}{2} \cdot x^{-1/2}\,dx$

$du = \frac{1}{2} \cdot \frac{1}{\sqrt{x}} \cdot dx$

$$2\int_1^4 e^u\,du$$

change the "x" limits to "u" limits

$x=1$ $(u=\sqrt{x}) \longrightarrow u=1$

$x=4$ $(u=\sqrt{x}) \longrightarrow u=2$

$$2\int_1^2 e^u\,du = 2\Big[e^u\Big]_{u=1}^{u=2}$$

$$= 2\left[e^2 - e^1\right]$$

≈ 9.34

ALT.

$u=\sqrt{x}$ $\quad du = \frac{1}{2} \cdot \frac{1}{\sqrt{x}} \cdot dx$

$$2\int e^u\,du = 2\,e^u\Big] = 2\,e^{\sqrt{x}}\Big]_1^4$$

$$= 2\left[e^{\sqrt{4}} - e^{\sqrt{1}}\right] = 2\left(e^2 - e^1\right)$$

$$\int x \sqrt{x+3}\, dx$$

let $u = x+3 \longrightarrow u-3 = x$

$du = 1 \cdot dx$

$$\int (u-3)\, u^{1/2} \cdot du$$

$$\int \left(u^{3/2} - 3u^{1/2}\right) du = \frac{u^{5/2}}{5/2} - 3 \cdot \frac{u^{3/2}}{3/2} + C$$

$$= \frac{2}{5} \cdot u^{5/2} - \cancel{3} \cdot \frac{2}{\cancel{3}} u^{3/2} + C$$

$$= \frac{2}{5}(x+3)^{5/2} - 2(x+3)^{3/2} + C$$

$$\int x \cdot \sqrt{x+3}\, dx$$

let $\boxed{u = \sqrt{x+3}}$ $\quad u^2 = x+3$

$x = u^2 - 3$

~~$du = \frac{1}{2}(x+3)^{-1/2} \cdot dx$~~

DERIV.

$\boxed{1\,dx = 2u \cdot du}$

$$\int (u^2-3)\, u\, 2u \cdot du$$

$$= 2\int u^2 (u^2-3)\, du$$

$$= 2\int (u^4 - 3u^2)\,du$$

$$= 2\left[\frac{u^5}{5} - \cancel{3}\frac{u^3}{\cancel{3}}\right] + C$$

$$= \frac{2}{5}\left(\sqrt{x+3}\right)^5 - 2\left(\sqrt{x+3}\right)^3 + C$$

$$\int e^{\tan\theta}\cdot \sec^2\theta \cdot d\theta$$

let $u = \tan\theta$

$du = \sec^2\theta \, d\theta$

$$\int e^u \cdot du = e^u + C$$

$$= e^{\tan\theta} + C$$

$$-\frac{1}{5}\int \cos^3(5\theta)\,\sin(5\theta)\cdot d\theta\,(-5)$$

let $u = \cos(5\theta)$

$du = -\sin(5\theta)\cdot 5\cdot d\theta$

$$-\frac{1}{5}\int u^3\cdot du = -\frac{1}{5}\cdot\frac{u^4}{4} + C$$

$$= -\frac{1}{20}(\cos 5\theta)^4 + C$$

let $u = \cos^3(5\theta)$

$u = (\cos 5\theta)^3$

$du = 3(\cos 5\theta)^2(-\sin 5\theta)5\,d\theta$

$$\frac{1}{6}\int x^2\sqrt{2x^3-1}\,dx\cdot 6$$

let $u = 2x^3-1$

$du = 6x^2\,dx$

$$\frac{1}{6}\int u^{1/2}\,du = \frac{1}{\cancel{6}3}\cdot\frac{\cancel{2}1}{3}u^{3/2} + C$$

$$= \frac{1}{9}(2x^3-1)^{3/2} + C$$

$$\frac{1}{2}\int_{t=0}^{t=1} \frac{4t+1}{4t^2+2t+3}\,dt\,(2) \qquad \int \frac{du}{u}$$

let $u = 4t^2+2t+3$

$du = (8t+2)\,dt$

$du = 2(4t+1)\,dt$

$$\frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\int \frac{1}{u}\cdot du$$

$$= \frac{1}{2}\ln|u|\Big]$$

$$= \frac{1}{2}\Big[\ln|4t^2+2t+3|\Big]_0^1$$

$$= \frac{1}{2}\Big[\ln|9| - \ln|3|\Big] = \frac{1}{2}\ln\frac{9}{3}$$

$$= \frac{1}{2}\ln 3 \approx \underline{\qquad\qquad}$$