NORTH CAROLINA STATE UNIVERSITY Department of Mathematics

MA141-005 Test #3 Form A Wednesday, March 20, 2019 Dr. J. Griggs

Please put all work and answers in the blue book provided. Nothing written on the test itself will be graded. Scientific calculators OK; do **not** use a graphing calculator. Validate all graphs with appropriate calculus techniques. Put your name, row number and seat number on the front of the blue book.

1.) Evaluate: a.) $\lim_{x\to 0} \frac{e^x - 1 - x}{x^2}$ b.) $\lim_{x\to 0^+} x^{x^2}$

2.) For the function $f(x) = x^3 - x^2 - x + 4$ on [-2, 3], using the first and second derivative - find the critical points and points of inflection; where the function is increasing, decreasing; where it is concave up, concave down; all relative and absolute maximum and minimum values; graph the function.

3.) Use Newton's Method to find the positive root of the equation $f(x) = 2x - e^{-x} = 0$ using $x_1 = .3$, find x_2 *and* x_3 .

4.) A closed wooden box (top included) is to be constructed to have a volume of 45 cubic feet. The length of the box must be 5 times the height. Once it is built, it will be coated with a very expensive acrylic – so its surface area needs to be minimized. What dimensions should the box have in order to minimize the surface area?

5.) Use differentials to approximate the increase in volume of a cube if the length of each edge of the cube is changed from 10 cm to 10.1 cm. What is the exact change in volume?

6.) Integrate: a.) $\int (x^3 - 5\sin x + \frac{4}{\sqrt[3]{x}})\,dx$ b.) $\int (5e^{3x} - \frac{2}{x} + \frac{3}{1+x^2})\,dx$

7.) Using $f(x) = x^2 - 4x + 2$ on the interval $[1,4]$, verify the hypotheses of the Mean Value Theorem and find c in $[1,4]$ guaranteed by the conclusion. Construct a graph of this scenario.

MA141 TEST #3A SOLUTIONS:

(7 questions; 14 points each)

1.) a.) $\lim_{x\to 0} \frac{e^x - 1 - x}{x^2} \xrightarrow{\text{L'HOP}} \lim_{x\to 0} \frac{e^x - 1}{2x} \xrightarrow{\text{L'HOP}}$

7pts

$= \lim_{x\to 0} \frac{e^x}{2} = \frac{1}{2}$

b.) $\lim_{x\to} x^{x^2}$ (rewrite) $\to \lim_{x\to 0^+} e^{\ln x^{x^2}}$

7pts

$= \lim_{x\to 0^+} e^{(x^2)(\ln x)} = e^{\boxed{\lim_{x\to 0^+} (x^2)(\ln x)}}$

exponent only... $\lim_{x\to 0^+} (x^2)(\ln x)$

$= \lim_{x\to 0^+} \frac{(\ln x)}{\frac{1}{x^2}} \xrightarrow{\text{L'HOP}} \lim_{x\to 0^+} \frac{\frac{1}{x}}{\frac{-2}{x^3}}$

$= \lim_{x\to 0^+} \frac{1}{x} \cdot \frac{x^3}{-2} = \lim_{x\to 0^+} \frac{x^2}{-2} = 0$

$\therefore\ e^0 = 1$

2.) $f(x) = x^3 - x^2 - x + 4$ on $[-2, 3]$

14 pts

endpoints:

$f(-2) = (-2)^3 - (-2)^2 - (-2) + 4 = -6 \longrightarrow (-2, -6)$

$f(3) = (3)^3 - (3)^2 - (3) + 4 = 19 \longrightarrow (3, 19)$

(page 2)

$f'(x) = 3x^2 - 2x - 1 = 0$ ($f'(x)$ ~~never undef~~)

$(3x+1)(x-1) = 0$

$x = -\frac{1}{3} \quad x = 1$

$f(-\frac{1}{3}) = (-\frac{1}{3})^3 - (-\frac{1}{3})^2 - (-\frac{1}{3}) + 4 = \frac{113}{27} \approx 4.185$

$f(1) = 1^3 - 1^2 - 1 + 4 = 3$

critical points: $(-\frac{1}{3}, 4.185)$ & $(1, 3)$ "FLAT"

$f'(x)$:

$f'(-1) = +$ | $-\frac{1}{3}$ | $f'(0) = -$ | 1 | $f'(2) = +$

$f(x)$ INCR | $-\frac{1}{3}$ | $f(x)$ DECR | 1 | $f(x)$ INCR

$f''(x) = 6x - 2 = 0$ ($f''(x)$ ~~never undef~~)

$6x = 2 \quad x = \frac{1}{3}$

$f(\frac{1}{3}) = (\frac{1}{3})^3 - (\frac{1}{3})^2 - (\frac{1}{3}) + 4 = \frac{97}{27} \approx 3.59$

$(\frac{1}{3}, 3.59)$ pt. of. inflection

$f''(x)$:

$f''(0) = -$ | $\frac{1}{3}$ | $f''(1) = +$

$f(x)$ CONC. DOWN | $\frac{1}{3}$ | $f(x)$ CONC. UP

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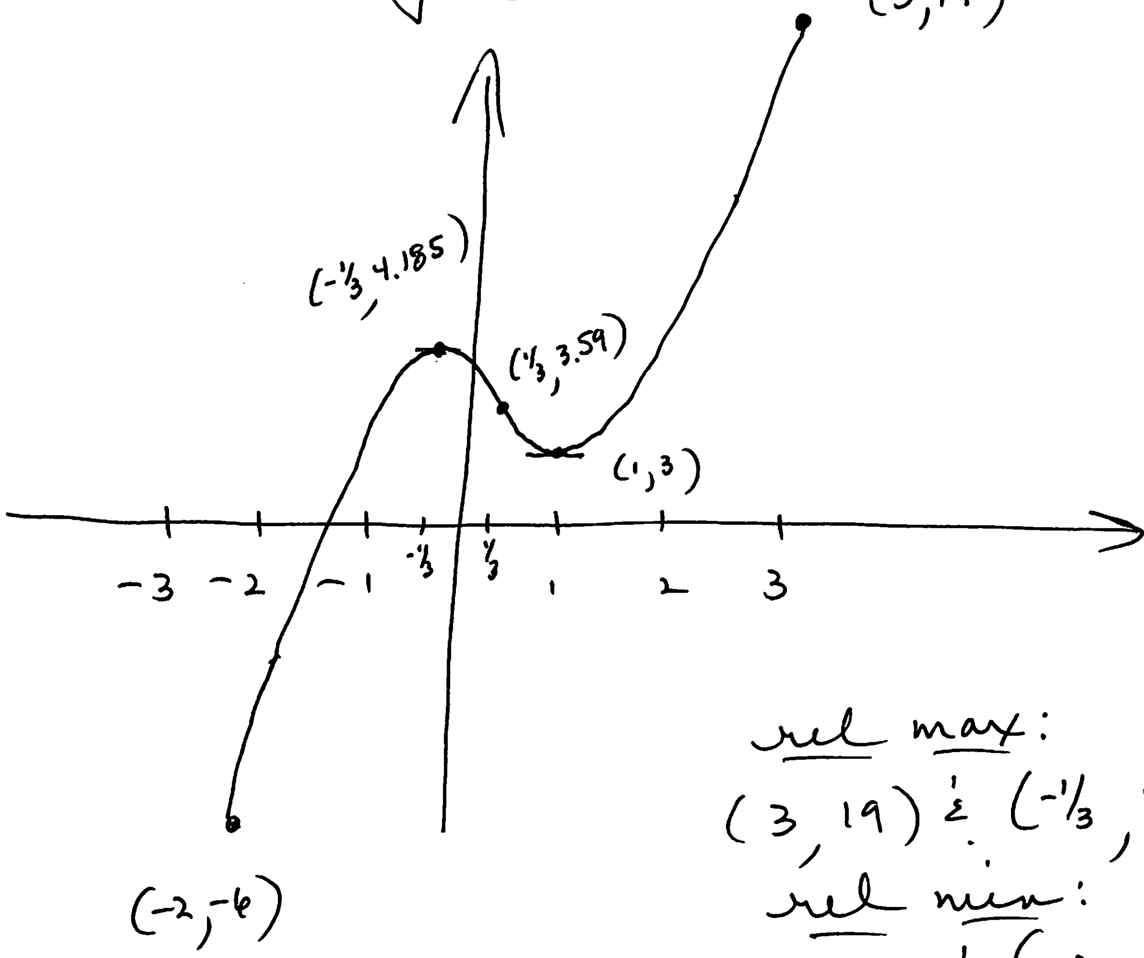

rel max:
(3, 19) & (-1/3, 4.185)
rel min:
(1, 3) & (-2, -6)

ABS MAX: 19 (3, 19)
ABS MIN: -6 (-2, -6)

3.) $f(x) = 2x - e^{-x} \quad x_1 = .3$

14 pts

$$f'(x) = 2 + e^{-x}$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = .3 - \frac{f(.3)}{f'(.3)}$$

$$= .3 - \frac{[.6 - .7408]}{[2 + .7408]} = .3 + .0514$$

$$= .3514$$

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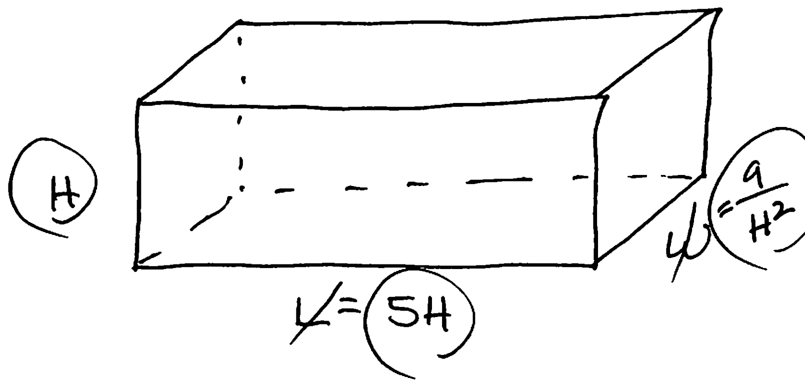
$$X_3 = X_2 - \frac{f(X_2)}{f'(X_2)} = .3514 - \frac{[.7028 - .7037]}{[2 + .7037]}$$

$$X_3 = .3514 + .00033$$

$$X_3 = .35173$$

4.)

14 pts



$$V = 45$$

$$(H)(w)(5H) = 45$$

$$5H^2 w = 45$$

$$w = \frac{45}{5H^2} = \frac{9}{H^2}$$

surface area

$$S = 2[(H)(5H)] + 2[(w)(H)] + 2[(5H)(w)]$$

$$S = 10H^2 + 2wH + 10wH = 10H^2 + 12wH$$

$$S = 10H^2 + 12\left(\frac{9}{H^2}\right)(H) = 10H^2 + \frac{108}{H} = S$$

$$S' = 20H - \frac{108}{H^2} = 0$$

$$20H = \frac{108}{H^2} \qquad \frac{20H}{1} = \frac{108}{H^2}$$

$$20H^3 = 108 \qquad H^3 = \frac{108}{20} = \frac{54}{10} = 5.4$$

$$H = \sqrt[3]{5.4} \approx 1.7544 \approx 1.75 \text{ ft}$$

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$$5H \approx 5(1.7544) \approx 8.77 \text{ ft}$$

$$W \approx \frac{9}{H^2} \approx 2.94 \text{ ft}$$

max or min?

$$S'' = 20 + \frac{216}{H^3} = +$$

$\therefore$ CONC. UP $\therefore$ MIN

5.) CUBE $\quad V = x^3 \quad x = 10 \quad \Delta x = .1$

14pts
$$dV = V'(x) \cdot \Delta x$$
$$= (3x^2) \cdot \Delta x = (3 \cdot 10^2)(.1) = 30 \text{ cm}^3$$

APPROX

$$\Delta V = (10.1)^3 - (10)^3 = 1030.301 - 1000$$
$$= 30.301 \text{ cm}^3$$

EXACT

6.) a.) $\int \left(x^3 - 5\sin x + 4x^{-1/3}\right) dx$

7pts
$$= \frac{x^4}{4} - 5(-\cos x) + 4\left(\frac{x^{2/3}}{2/3}\right) + C$$

$$= \frac{x^4}{4} + 5\cos x + 6x^{2/3} + C$$

b.) $\int \left(5 \cdot e^{3x} - \frac{2}{x} + \frac{3}{1+x^2}\right) dx$

7pts

$= 5\left(\frac{1}{3} \cdot e^{3x}\right) - 2\ln|x| + 3\tan^{-1}x + C$

$= \frac{5}{3} e^{3x} - 2\ln|x| + 3\tan^{-1}x + C$

7.) $f(x) = x^2 - 4x + 2$ on $[1, 4]$

hypotheses: ① $f(x)$ contin on $[1,4]$??

yes, it is a polynomial

② $f(x)$ differentiable on $(1,4)$

yes, deriv. exists for all values in $(1,4)$

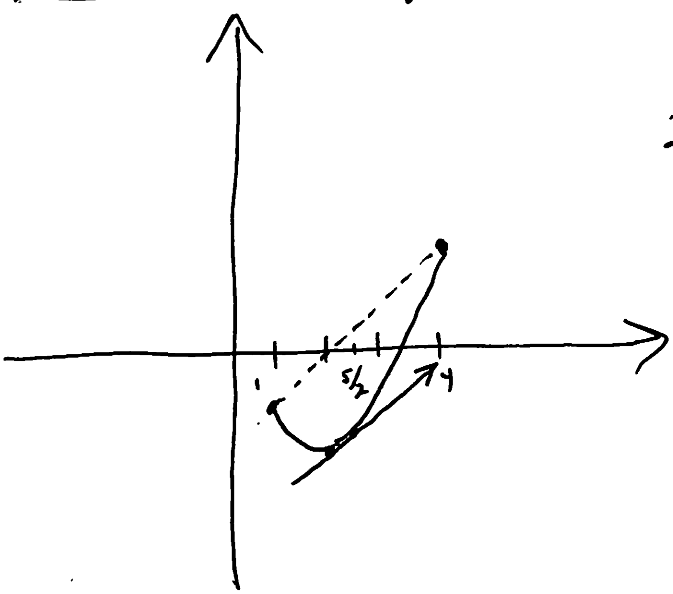
$f'(x) = 2x - 4 = 0$

$2x = 4$

$x = 2$

Vertex: $(2, -2)$

$f(1) = -1 \quad f(4) = 2$



find c such that...

$f'(c) = \dfrac{f(b) - f(a)}{b - a}$

$2c - 4 = \dfrac{f(4) - f(1)}{4 - 1}$

$2c - 4 = \dfrac{2 - (-1)}{4 - 1} = \dfrac{3}{3} = 1$

$2c - 4 = 1 \quad 2c = 5 \quad \boxed{c = 5/2}$

$5/2 \in (1, 4)$