MA141-005

Wednesday, March 27

- 4.2: (DEFINITE INTEGRAL) continued
- 4.3: (FUNDAMENTAL THEOREM OF CALCULUS)

5.) $\displaystyle\int_1^3 (2 - 3x + x^2)\,dx$ (part 2 of FTOC)

$= \left[2x - 3\cdot\frac{x^2}{2} + \frac{x^3}{3}\right]_1^3$

$= \left(2(3) - \frac{3}{2}(3)^2 + \frac{(3)^3}{3}\right) - \left(2(1) - \frac{3}{2}(1)^2 + \frac{(1)^3}{3}\right)$

$= \left(6 - \frac{27}{2} + 9\right) - \left(2 - \frac{3}{2} + \frac{1}{3}\right)$

$= \cancel{6} - \cancel{\frac{27}{2}} + \cancel{9} - \cancel{2} + \cancel{\frac{3}{2}} - \frac{1}{3}$

$= 13 - 12 - \frac{1}{3} = \frac{2}{3}$

11.) $\displaystyle\int_{\pi/2}^{\pi/2} \sqrt[3]{\sec^4(8\theta) - \tan^7(5\theta)}\,d\theta = 0$

6.) $\displaystyle\int_0^{\pi/2} (2\sin x + 3)\,dx$

$= \left[2\cdot(-\cos x) + 3x\right]_0^{\pi/2}$

$$= \left(-2\cos\tfrac{\pi}{2} + 3\cdot\tfrac{\pi}{2}\right) - \left(-2\cos 0 + 3\cdot 0\right)$$

$$= \left(-2(\cancel{0}) + \frac{3\pi}{2}\right) - \left(-2(1) + \cancel{0}\right)$$

$$= \frac{3\pi}{2} + 2 \approx \underline{\qquad\qquad}$$

9.)

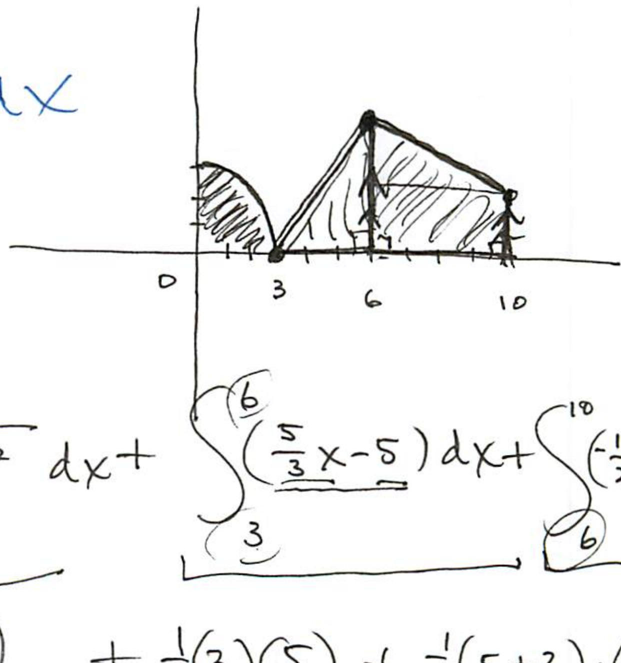
$$f(x) = \begin{cases} \sqrt{9-x^2} & 0 \le x \le 3 \\ \frac{5}{3}x - 5 & 3 < x \le 6 \\ -\frac{1}{2}x + 8 & 6 < x \le 10 \end{cases}$$

$y = \sqrt{9-x^2}$

$y^2 = 9 - x^2$

$x^2 + y^2 = 9$

find $\int_0^{10} f(x)\,dx$



$$\int_0^{10} f(x)\,dx = \int_0^3 \sqrt{9-x^2}\,dx + \int_3^6 \left(\frac{5}{3}x - 5\right)dx + \int_6^{10}\left(-\frac{1}{2}x + 8\right)dx$$

$$= \frac{1}{4}\left(\pi\cdot 3^2\right) + \frac{1}{2}(3)(5) + \frac{1}{2}(5+3)\cdot(4)$$

$$\int_{-1}^{2} \frac{1}{x^2}\, dx = +$$

HT WDTH

$$\int \frac{1}{x} \cdot dx = \ln|x| + C$$

x^{-1}

$$\int_{-1}^{2} x^{-2}\, dx = \left[\frac{x^{-1}}{-1}\right]_{-1}^{2}$$

$$= \left[\frac{-1}{x}\right]_{-1}^{2} = \left(-\frac{1}{2}\right) - \left(\frac{-1}{-1}\right)$$

$$= -\frac{1}{2} - 1 = \cancel{-\frac{3}{2}}$$

DISCONTINUOUS AT $x = 0$
(area cannot be found)

4.3 Fundamental Theorem of Calculus

Earlier in this course, we found it difficult to compute derivatives using only the formal definition

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Later we found several methods of finding derivatives that made the process more tractable. We now find ourselves at a similar place in the computation of the definite integral. To apply the definition we need to set up Riemann sums, simplify using various properties alongside summation equations proved by mathematical induction $\left(\sum_{i=1}^{n} i = \frac{n(n+1)}{2}; \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}; \ldots \right)$, and take a limit to find the area. Unfortunately, similar summation formulas are not available for many of the functions we encounter. As a result of this challenge, we introduce and validate more efficient methods of finding definite integrals and in the process make a fundamental connection between differential and integral calculus that is unrivaled in scientific importance.

The Fundamental Theorem of Calculus does just that – thanks to the ground breaking work of Isaac Newton (1642-1727) and Gottfried Leibniz (1646-1716). It establishes the inverse relationship between differentiation and integration – allowing the study of calculus to develop into a systematic mathematical method.

Theorem 4. The Fundamental Theorem of Calculus

Let f be continuous on $[a, b]$.

Part 1: The function G defined by

$$G(x) = \int_a^x f(t)\,dt \qquad \text{for all } x \text{ in } [a, b]$$

is an antiderivative of f on $[a, b]$.

Part 2: If F is any antiderivative of f on $[a, b]$, then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

Last update: December 7, 2017

$$G(x) = \int_a^x f(t)\,dt$$

part 1 of FTOC

G is antideriv of f

$$\frac{d(G(x))}{dx} = \frac{d\left[\int_a^x f(t)\,dt\right]}{dx}$$

$$\frac{d(G(x))}{dx} = f(x)$$

$$G(x) = \int_2^x (3t+5)\,dt$$

$$\frac{d(G(x))}{dx} = \frac{d\left(\int_2^x (3t+5)\,dt\right)}{dx}$$

$$\frac{d(G(x))}{dx} \stackrel{??}{=} 3x+5$$

$$= \left[\frac{3t^2}{2} + 5t\right]_2^x$$

$$= \frac{d\left[\left(\frac{3}{2}x^2+5x\right) - \left(\frac{3}{2}(2)^2+5(2)\right)\right]}{dx}$$

$$= \frac{3}{2}(2x) + 5$$

1.) find $G'(x)$:

$$G(x) = \int_{2}^{x} \frac{1}{t^3+1}\,dt$$

$$\frac{d(G(x))}{dx} = \frac{1}{x^3+1}$$

2.) find $G'(x)$:

$$\frac{d(G(x))}{dx} = \frac{d\int_{x}^{1} \cos(\sqrt{t})\,dt}{dx}$$

$\cancel{G'(x) =}$

$$G(x) = -\boxed{\int_{1}^{x} \cos(\sqrt{t})\,dt}$$

$$G'(x) = -(\cos\sqrt{x})$$

(3) find $G'(x)$:

$$G(x) = \boxed{\int_{1}^{x^3} 5 \cdot \ln t \cdot dt}$$

$(u = x^3)$

chain rule:

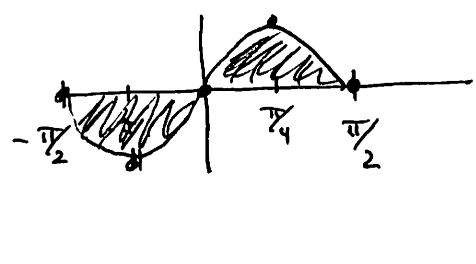
$$\frac{d(G(x))}{dx} = \frac{d\left[\int_{1}^{u} 5 \cdot \ln t \cdot dt\right]}{du} \cdot \frac{du}{dx}$$

$$\frac{d(G(x))}{dx} = (5 \cdot \ln u) \cdot (3x^2)$$

$$= \boxed{5 \ln x^3} \cdot 3x^2$$

$$= 15x^2 \ln(x^3)$$

18.) $\int_{-\pi/2}^{\pi/2} \sin^3(2\theta)\, d\theta = 0$ $\quad y = \sin^3(2\theta)$



4.3.1 Exercises

1. Find $\frac{dF}{dx}$ given $F(x) = \int_1^x (t^2 + t)\,dt$ $\longrightarrow$ $x^2 + x$

2. Find $\frac{dF}{dx}$ given $F(x) = \int_x^{\pi} 3\cos t\,dt$

3. Find $\frac{dF}{dx}$ given $F(x) = \int_0^{x^2} (2s - 1)\,ds$ $\longrightarrow$ $(2x^2 - 1)\cdot(2x)$

4. Find $\frac{dF}{dx}$ given $F(x) = \int_{\frac{\pi}{6}}^x (\tan 2\theta + 3\theta)\,d\theta$ $\longrightarrow$ $\tan 2x + 3x$

In **Exercises 5 - 7** find $G'(x)$.

5. $G(x) = \int_x^{\ln 3} (2e^t - 8t + 1)\,dt$

6. $G(x) = \int_2^{x^4} 5\sqrt{t}\,dt$

7. $G(x) = \int_{\pi/4}^x (1 + \cos 2\theta)\,d\theta$

In **Exercises 8 - 18** evaluate each definite integral. You may use the Fundamental Theorem of Calculus or a geometric understanding of the integral.

8. $\int_{-2}^4 (|3x - 2| + 1)\,dx$

9. $\int_0^3 \sqrt{9 - x^2}\,dx$

10. $\int_0^4 (x^2 + 1)\,dx$

11. $\int_0^{\frac{\pi}{6}} \sec\theta\tan\theta\,d\theta$ $\longrightarrow$ $\sec\theta \Big]_0^{\pi/6} = \sec \pi/6 - \sec 0$

$\downarrow$

$\left(\frac{2}{\sqrt{3}} - 1\right)$

12. $\int_1^8 6\sqrt[3]{x}\,dx$

13. $\int_{-1}^1 (2t - 3)^2\,dt$

14. $\int_1^{16} \frac{u + 1}{\sqrt[4]{u}}\,du$ $\longrightarrow$ $\int_1^{16} \left(\frac{u}{\sqrt[4]{u}} + \frac{1}{\sqrt[4]{u}}\right) du$

15. $\int_0^{\pi} (1 - 2\cos r)\,dr$

$\int_1^{16} \left(u^{3/4} + u^{-1/4}\right) du$

16. $\int_e^1 (\frac{1}{x} - 5x)\,dx$

17. $\int_0^3 (3^x + 4x)\,dx$

18. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^3 2\theta\,d\theta$

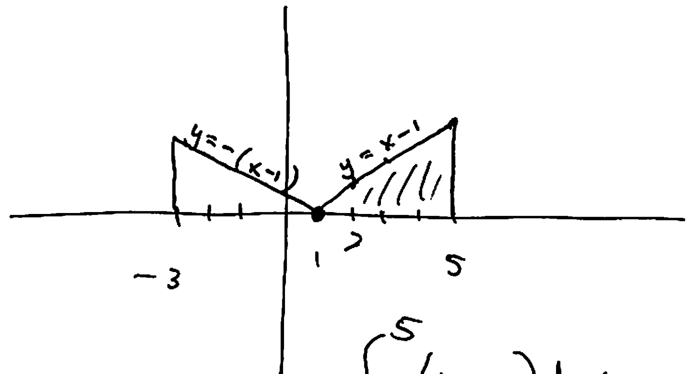
$\left[\frac{u^{7/4}}{\frac{7}{4}} + \frac{u^{3/4}}{3/4}\right]_1^{16}$

$\left[\frac{4}{7}u^{7/4} + \frac{4}{3}u^{3/4}\right]_1^{16}$

$\boxed{16^{7/4} = 2^7}$ $\boxed{16^{3/4} = 2^3}$

$$\int_{-3}^{5} |x-1|\,dx$$

$y = |x-1|$



$$\int_{-3}^{1} -(x-1)\,dx + \int_{1}^{5} (x-1)\,dx$$

$$\int_{-3}^{1} (1-x)\,dx + \int_{1}^{5} (x-1)\,dx$$

MA 141-005

TEST #3 RESULTS
(AFTER +9)

A's 8

B's 4

C's 5

D's 6

F's 10

average: 72.0