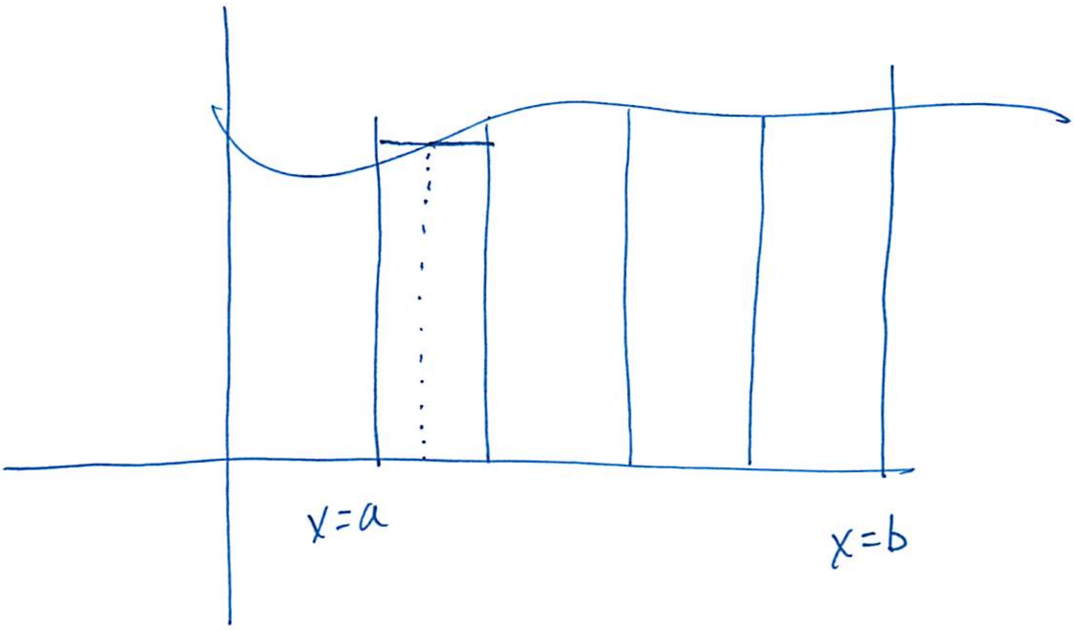
MA 141-005

Monday, March 25

- today: finish 4.1 (areas & Riemann sums)
  4.2 (definite integrals; properties)



$y = x^2+1$
from $x=1$ to $x=3$

from WED....

$$\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{4}{n} + \frac{8i}{n^2} + \frac{8i^2}{n^3} \right) = \text{AREA}$$

$$\lim_{n\to\infty} \left[\sum_{i=1}^{n} \frac{4}{n} + \sum_{i=1}^{n} \frac{8i}{n^2} + \sum_{i=1}^{n} \frac{8i^2}{n^3} \right]$$

$$\lim_{n\to\infty} \frac{4}{n} \sum_{i=1}^{n} 1 + \frac{8}{n^2} \sum_{i=1}^{n} i + \frac{8}{n^3} \sum_{i=1}^{n} i^2$$

$$\lim_{n\to\infty} \left(\frac{4}{n} \left[n \right] + \frac{8}{n^2} \left[\frac{n(n+1)}{2} \right] + \frac{8}{n^3} \left[\frac{n(n+1)(2n+1)}{6} \right] \right)$$

(1) $\sum_{i=1}^{n} 1 = \underbrace{1+1+1+\dots+1}_{\text{"n" times}} = n$

$$\boxed{\sum_{i=1}^{n} 1 = n}$$

(2) $\sum_{i=1}^{n} i = 1+2+3+\dots+n$

$$\boxed{\sum_{i=1}^{n} i = \frac{n(n+1)}{2}}$$

$\sum_{i=1}^{4} i = 1+2+3+4 \stackrel{?}{=} \frac{4(4+1)}{2} = \frac{4(5)}{2} = 10$

$\sum_{i=1}^{100} i = 1+2+3+\dots+98+99+100$

$= \frac{100(101)}{2} = 5050$

(3) $\sum_{i=1}^{n} i^2 = 1+4+9+16+\dots+n^2$

$$\boxed{\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}}$$

$\sum_{i=1}^{5} i^2 = \underbrace{1+4+9+16+25} \stackrel{?}{=} \frac{5(5+1)(2\cdot 5+1)}{6}$

$= \frac{(5)(\cancel{6})(11)}{\cancel{6}}$

(4) $\sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$

$$\lim_{n \to \infty} \left(4 + \frac{8}{2}\left[\frac{n(n+1)}{n^2}\right] + \frac{8}{6}\left[\frac{n(n+1)(2n+1)}{n^3}\right] \right)$$

$$\lim_{n \to \infty} \left(4 + 4\left[\frac{n^2+n}{n^2}\right] + \frac{4}{3}\left[\frac{2n^3 + \cdots\cdots}{n^3}\right] \right)$$

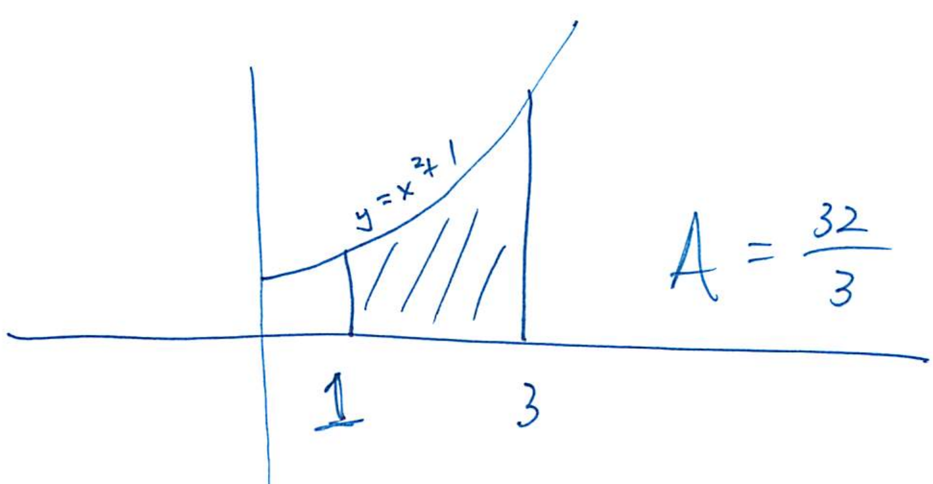
$$\lim_{n \to \infty} \frac{1 \cdot n^2 + n}{1 \cdot n^2} = 1 \qquad \lim_{n \to \infty} \frac{2n^3 + \cdots\cdots}{1 \cdot n^3} = 2$$

$$\left(\lim_{x \to \infty} \frac{1x^2 + x}{1 \cdot x^2} = 1 \right)$$

$$\left(4 + 4 \cdot (1) + \frac{4}{3}(2) \right) = A$$

$$8 + \frac{8}{3} = A$$

$$\frac{24}{3} + \frac{8}{3} = \frac{32}{3} = A$$



$$A = \frac{32}{3}$$

Definition 1. Definite Integral of a Continuous Function on $[a, b]$

Let $f(x)$ be defined and continuous on the interval $[a, b]$. For each partition

$$a = x_0 < x_1 < x_2 < \cdots < x_{n-1} < x_n = b$$

of the interval $[a, b]$ into n equal parts, the length of each subinterval is $\boxed{\Delta x = \frac{b-a}{n}}$ *and each* $\boxed{x_i = a + i\Delta x}$ $i = 1, 2, \cdots, n$. *The* **definite integral** *of f on $[a, b]$, denoted $\int_a^b f(x)\,dx$, is defined by*

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

provided this limit exists.

Properties of the Definite Integral

Let a, b and c be real numbers with $a < b$ and f and g be continuous functions. Then

1. $\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$
2. $\int_a^a f(x)\,dx = 0$
3. $\int_a^b c\,dx = c(b-a)$
4. $\int_a^b (f(x) \pm g(x))\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$
5. $\int_a^b cf(x)\,dx = c\int_a^b f(x)\,dx$
6. $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$
7. If $f(x) \geq 0$ for all $x \in [a, b]$, then $\int_a^b f(x)\,dx \geq 0$
8. If $f(x) \geq g(x)$ for all $x \in [a, b]$, then $\int_a^b f(x)\,dx \geq \int_a^b g(x)\,dx$
9. If $m \leq f(x) \leq M$ for all $x \in [a, b]$, then $m(b-a) \leq \int_a^b f(x)\,dx \leq M(b-a)$

Theorem 4. The Fundamental Theorem of Calculus

Let f be continuous on $[a, b]$.

Part 1: The function G defined by

$$G(x) = \int_a^x f(t)\,dt \qquad \text{for all } x \text{ in } [a, b]$$

is an antiderivative of f on $[a, b]$.

Part 2: If F is any antiderivative of f on $[a, b]$, then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$