MA141-005

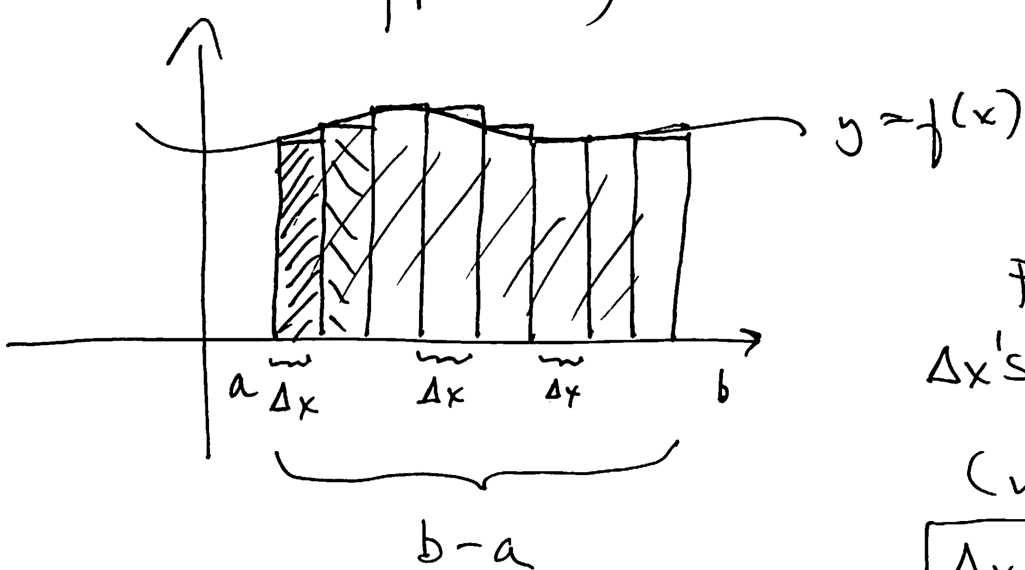
Wednesday, March 20

- TEST #3 (first hour)
- $\boxed{4.1}$ (2nd hour)
  (AREAS & RIEMANN SUMS)

$$\sum_{i=1}^{6} 2^i = 2^1 + 2^2 + 2^3 + 2^4 + 2^5 + 2^6$$

SIGMA
(summation)

AREAS: (approx.)



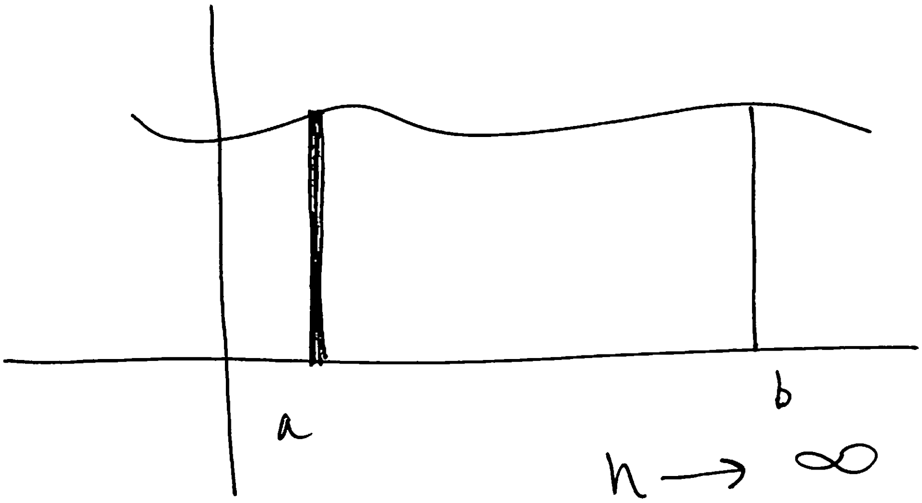
partition

Δx's are all the same (WIDTHS)

$$\boxed{\Delta x = \frac{b-a}{n}}$$

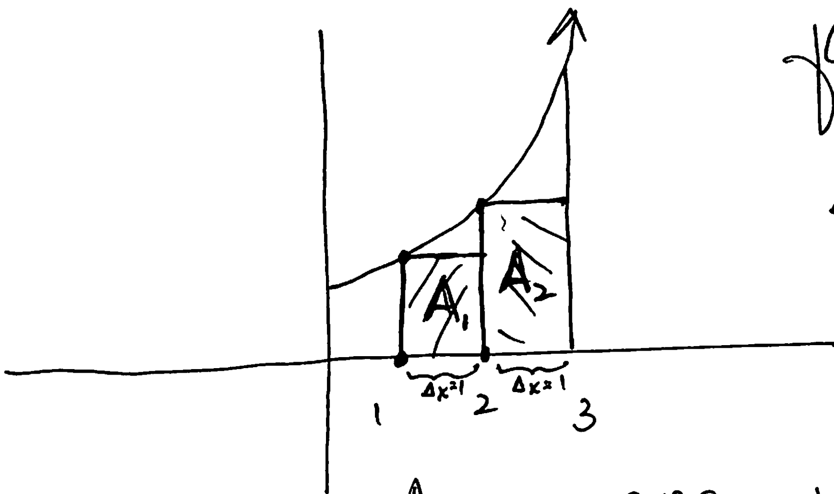
(n = # of rectangles)

more rectangles $\rightarrow$ higher accuracy



$n \to \infty$

EXACT AREA



$f(x) = x^2+1$

$$\Delta x = \frac{3-1}{2} = 1$$

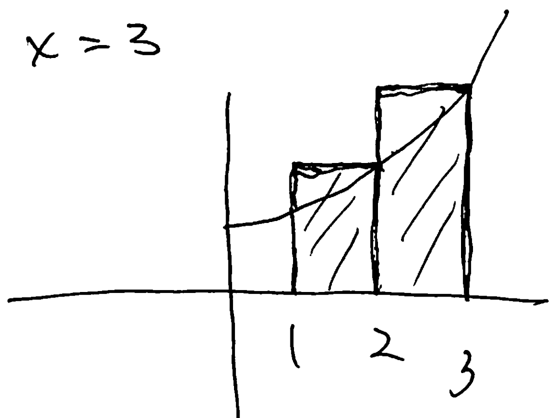
$$\uparrow$$

$$\frac{b-a}{n}$$

Approx area under $y = x^2+1$ from $x=1$ to $x=3$

$$\begin{aligned} A &\approx A_1 + A_2 \\ &\approx (f(1))(1) + (f(2))(1) \\ &\approx (2)(1) + (5)(1) \end{aligned}$$

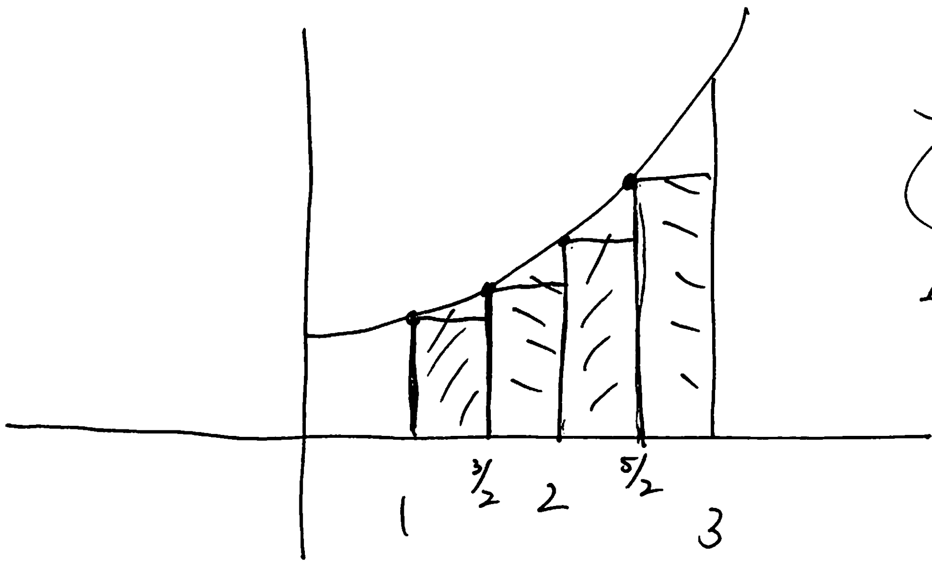
$A \approx 7$



$A \approx (5)(1) + (10)(1)$

$A \approx 5 + 10 = 15$

$$\underbrace{7 < A < 15}$$



$y = x^2 + 1$

4 rectangles

$$\Delta x = \frac{b-a}{n} = \frac{3-1}{4} = \frac{1}{2}$$

$$A \approx f(1)\cancel{f}(1/2) + f(3/2)(1/2) + f(2)(1/2) + f(5/2)(1/2)$$

$$A \approx (2)(1/2) + \left(\tfrac{13}{4}\right)(1/2) + (5)(1/2) + \left(\tfrac{29}{4}\right)(1/2)$$

$$\approx 1 + \frac{13}{8} + \frac{5}{2} + \frac{29}{8}$$

$$\approx \frac{8}{8} + \frac{13}{8} + \frac{20}{8} + \frac{29}{8} = \frac{70}{8} = \frac{35}{4}$$

$$A \approx 8\,3/4$$

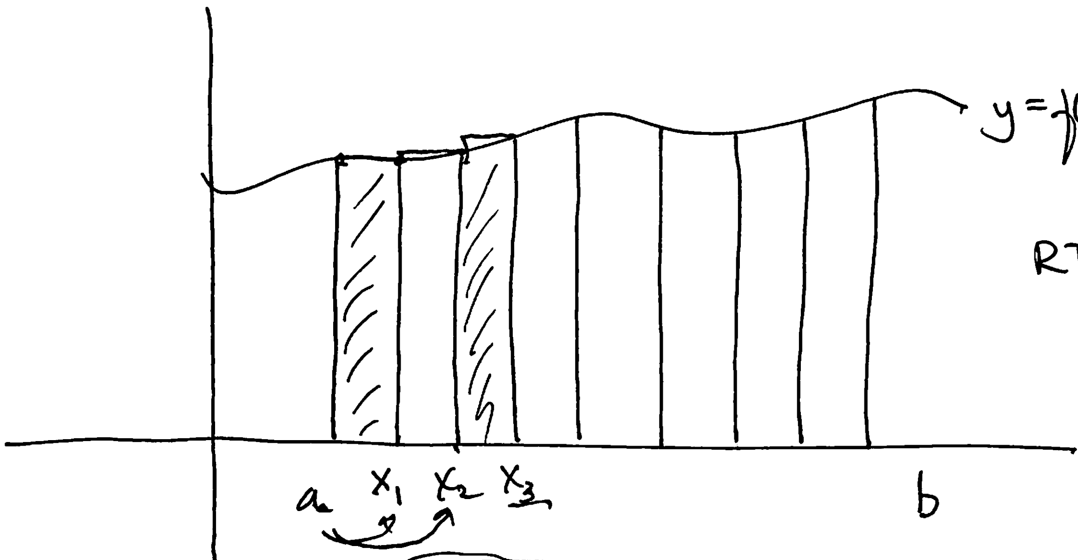
$$A \approx \sum_{i=1}^{n} \underbrace{f(x_i)}_{\text{HT.}} \cdot \underbrace{(\Delta x)}_{\text{WDTH}}$$

$$\approx \underbrace{f(x_1)\cdot\Delta x} + \underbrace{f(x_2)\cdot\Delta x} + \underbrace{f(x_3)\cdot\Delta x}$$

$$+ \ \cdots + \underbrace{f(x_{n-1})\,\Delta x} + \underbrace{f(x_n)\cdot\Delta x}$$

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\cdot\Delta x$$

$$\text{AREA} = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i) \cdot \Delta x$$



RT. ENDPOINT
(RT. SUM)

$$\boxed{A = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i) \cdot \Delta x}$$

$$f(x_1) = f(a + 1 \cdot \Delta x)$$

$$f(x_2) = f(a + 2 \cdot \Delta x)$$

$$f(x_3) = f(a + 3 \cdot \Delta x)$$

$$f(x_4) = f(a + 4 \cdot \Delta x)$$

$$\vdots$$

$$f(x_i) = f(a + i \Delta x)$$

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\cdot \Delta x$$

$$x_i = a + i \cdot \Delta x$$

$$\Delta x = \frac{b-a}{n}$$

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} f(a + i\Delta x) \cdot \Delta x$$

$f(x) = x^2 + 1$

$a = 1$

$b = 3$

$\Delta x = \frac{b-a}{n}$

$\Delta x = \frac{3-1}{n}$

$$\lim_{n\to\infty} \sum_{i=1}^{n} f\left(1 + i \cdot \frac{2}{n}\right) \left(\frac{2}{n}\right)$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} f\left(\boxed{1 + \frac{2i}{n}}\right) \left(\frac{2}{n}\right)$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \left[\left(1 + \frac{2i}{n}\right)^2 + 1\right] \left(\frac{2}{n}\right)$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \left[\left(1 + \frac{4i}{n} + \frac{4i^2}{n^2}\right) + 1\right] \left(\frac{2}{n}\right)$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \left(2 + \frac{4i}{n} + \frac{4i^2}{n^2}\right) \left(\frac{2}{n}\right)$$

$$\boxed{\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{4}{n} + \frac{8i}{n^2} + \frac{8i^2}{n^3}\right)}$$

pick up here on MON