MA 141-005 ①

Wednesday, March 6

today

- finish optimization problems (3.4)
- begin L'Hopital's Rule (3.5)

TICKET PRICE REDUCTION:

x = # of times the price is reduced

$$REV = \underbrace{(40 - 1\cdot x)}_{\text{ticket price}}(40{,}000 + 2000x)$$

$(40)(40{,}000) = 1{,}600{,}000$

$$R(x) = (40-x)(40{,}000 + 2{,}000x)$$

$$R(x) = 1{,}600{,}000 + 80{,}000x - 40{,}000x - 2{,}000x^2$$

$$\boxed{R(x) = 1{,}600{,}000 + 40{,}000x - 2{,}000x^2}$$

MAX REV

$$R'(x) = 0 \quad \boxed{+40{,}000 - 4000x = 0}$$

$$\frac{40{,}000}{4000} = \frac{\cancel{4000}x}{\cancel{4000}} \quad \boxed{x = 10}$$

$R''(x) = -4000$

∴ C. DOWN

∴ MAX

ticket price: $(40-x) = (40-10) = 30^{\underline{00}}$

MAX REV:

$(30)(60{,}000) = 1{,}800{,}000$

attending: $(40{,}000 + 2000x)$

$= (40{,}000 + 20{,}000)$

$= (60{,}000)$

3.5:

$$\lim_{x \to a} \frac{f(x)}{g(x)} \to \frac{0}{0} \text{ or } \frac{\infty}{\infty}$$

indeterminate form "standard"

$$\lim_{x \to \infty} \frac{3x}{x} = 3$$

$$\lim_{x \to a} \frac{f(x)}{g(x)} \xRightarrow{\text{L'HOP}} \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

L'HOPITAL'S RULE

$$\xRightarrow{\text{L'HOP}} \lim_{x \to a} \frac{f''(x)}{g''(x)}$$

$$\lim_{x \to 3} \frac{x-3}{x^2-9} \xrightarrow{\text{L'HOP}} \lim_{x \to 3} \frac{1}{2x} = \frac{1}{6}$$

$$\lim_{x \to 3} \frac{\cancel{(x-3)}^{1}}{\cancel{(x-3)}_{1}(x+3)} = \frac{1}{6}$$

$$\lim_{x \to 0} \frac{16x^2}{5\cos x - 5} \overset{\text{L'HOP}}{=} \lim_{x \to 0} \frac{32x}{-5\sin x}$$

$$\overset{\text{L'HOP}}{\Longrightarrow} \lim_{x \to 0} \frac{32}{-5\cos x} = \frac{32}{-5}$$

$$\lim_{x \to \infty} \frac{\ln(2x)}{5x^2} \overset{\text{L'HOP}}{\Longrightarrow} \lim_{x \to \infty} \frac{\frac{1}{\cancel{2}x} \cdot \cancel{2}}{10x}$$

$$= \lim_{x \to \infty} \frac{1}{x} \cdot \frac{1}{10x} = \lim_{x \to \infty} \frac{1}{10x^2} = 0$$

"other" indeterminate forms

($0 \cdot \infty$ and $\infty - \infty$) convert it to a "standard" indeterminate form

$$\lim_{x \to 0} \boxed{x \cdot \frac{1}{x}} = 1$$

$$\lim_{x \to 0} \frac{x}{x} = \frac{1}{1} = 1$$

$$\lim_{x \to 0^+} (x \ln x)$$

rearrange

$$\lim_{x \to 0^+} \frac{\ln x}{\frac{1}{x}} \xrightarrow{\text{L'HOP}} \lim_{x \to 0^+} \left(\frac{\frac{1}{x}}{\frac{-1}{x^2}} \right)$$

x^{-1}

$$\lim_{x \to 0^+} \left(\frac{1}{x} \cdot \frac{x^2}{-1} \right) = \lim_{x \to 0^+} -1 \cdot x = 0$$

$\infty - \infty = 1$

$$\lim_{x \to \infty} (x \cdot e^{1/x} - x) = \lim_{x \to \infty} x(e^{1/x} - 1)$$

$$= \lim_{x \to \infty} \frac{e^{1/x} - 1}{\frac{1}{x}} \overset{\text{L'HOP}}{=} \lim_{x \to \infty} \frac{e^{1/x} \left(\cancel{\frac{-1}{x^2}} \right)}{\left(\cancel{\frac{-1}{x^2}} \right)}$$

$$= \lim_{x \to \infty} e^{1/x} = 1$$

(1^∞ and ∞^0 and 0^0)

$$\lim_{x\to\infty} \boxed{\left(1+\frac{1}{x}\right)^x}$$

$$\boxed{e^{\ln u} = u}$$

$$\lim_{x\to\infty} \boxed{e^{\ln\left(1+\frac{1}{x}\right)^x}} = \lim_{x\to\infty} \boxed{e}^{x\cdot\ln\left(1+\frac{1}{x}\right)}$$

$$= \boxed{e}^{\lim_{x\to\infty}\left[x\cdot\ln\left(1+\frac{1}{x}\right)\right]}$$

$$\lim_{x\to\infty} x \ln\left(1+\frac{1}{x}\right) = \lim_{x\to\infty} \frac{\ln\left(1+\frac{1}{x}\right)}{\frac{1}{x}}$$

$$\xRightarrow{\text{L'HOP}} \lim_{x\to\infty} \frac{\boxed{\frac{1}{\left(1+\frac{1}{x}\right)}}\cdot\cancel{\frac{-1}{x^2}}}{\cancel{\frac{-1}{x^2}}} = 1$$

$$\therefore\ e^1 = e$$

$$\lim_{x\to 0^+} x^x = \lim_{x\to 0^+} e^{\ln x^x}$$

$$\lim_{x\to 0^+} e^{\boxed{x\cdot\ln x}} = e^{\boxed{\lim_{x\to 0^+} x\cdot\ln x}}$$

$$\lim_{x\to 0^+} x \cdot \ln x = \lim_{x\to 0^+} \frac{\ln x}{\frac{1}{x}} \xrightarrow{\text{L'HOP}}$$

$$\lim_{x\to 0^+} \left(\frac{\frac{1}{x}}{\frac{-1}{x^2}} \right) = \lim_{x\to 0^+} \left(\frac{1}{x} \cdot \frac{x^2}{-1} \right)$$

$$= \lim_{x\to 0^+} (-1 \cdot x) = 0$$

$$\therefore e^0 = \underline{1}$$