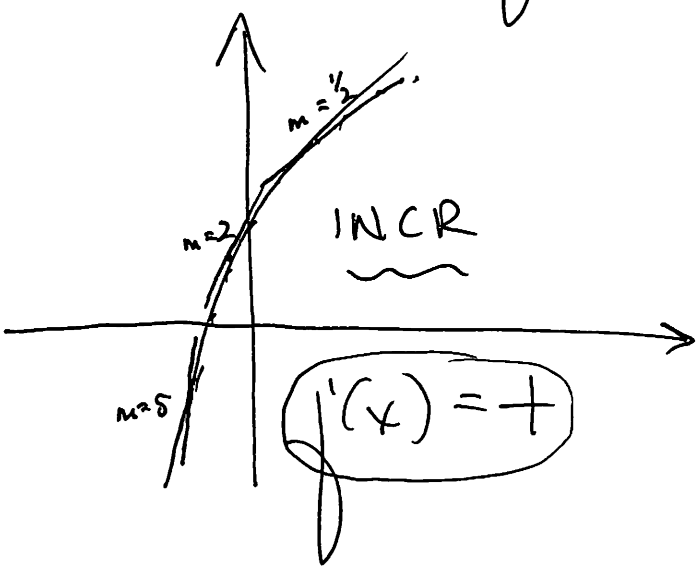
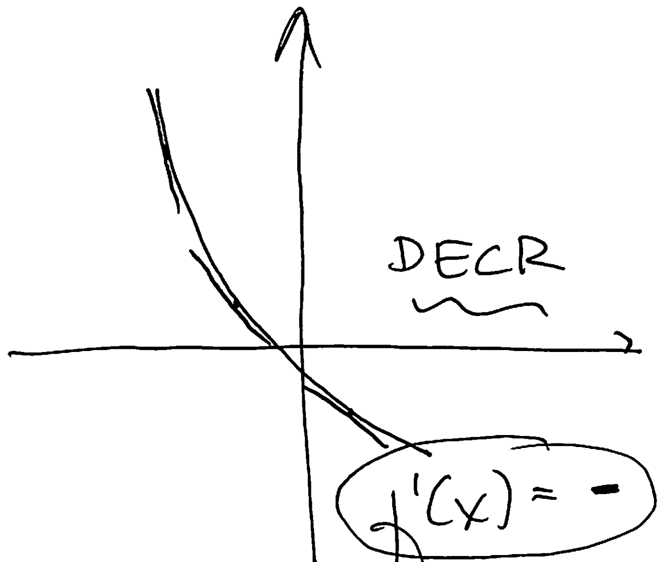
MA141-005

Monday, March 4

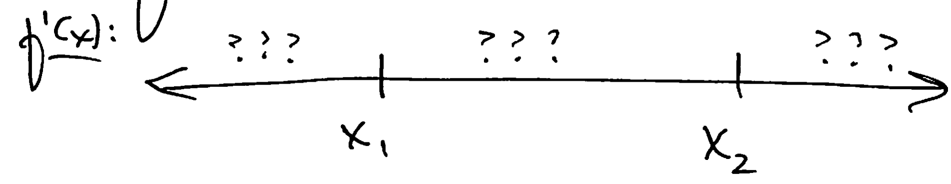
- this week (3.3: shape of curves)
  (3.4: optimization)
  (3.5: L'HOPITAL'S RULE)
- webassign due FRI:

3.3: using $f'(x)$ & $f''(x)$ to graph:



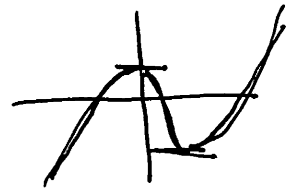


1. find the CRITICAL values for x
   - (a) $f'(x) = 0$ $\quad (m_{TAN} = 0)$
   - (b) $f'(x)$ undef
2. set up a number line (TABLE) — to determine the SIGN of $f'(x)$



$f(x) = x^3 + 3x^2 + 2$

$f(-2) = (-2)^3 + 3(-2)^2 + 2$



$f'(x) = 3x^2 + 6x + 0$

① $0 = 3x^2 + 6x$ ② $\cancel{3x^2 + 6x \text{ undef}}$?

$0 = \boxed{3x(x+2)}$

$x = 0$ or $x + 2 = 0$

$x = 0$ or $x = -2$

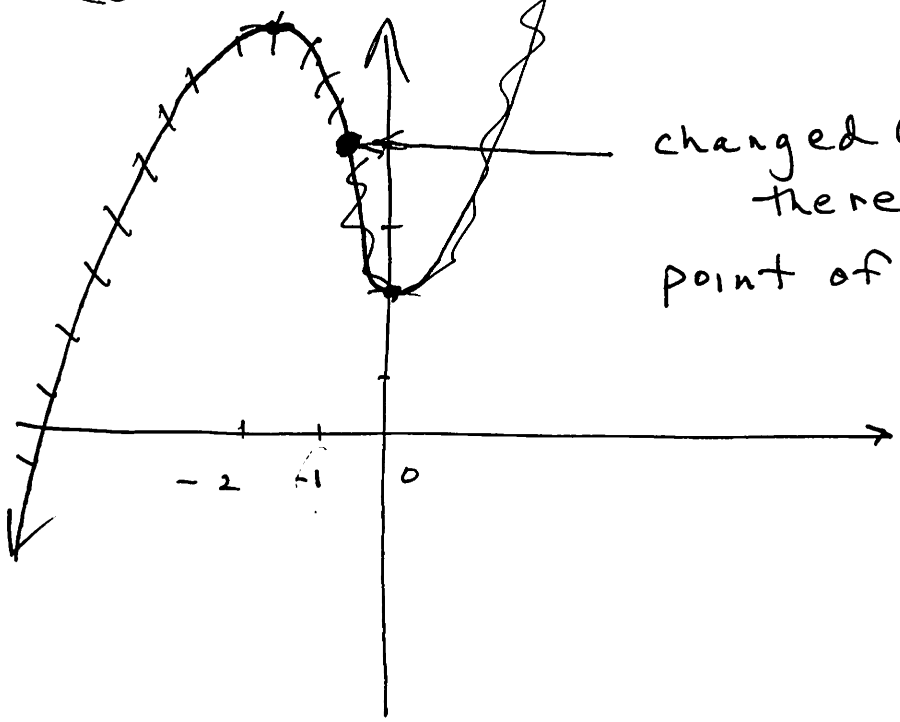
crit. pts:

$(0, 2)$ & $(-2, 6)$

$\uparrow$ $f(0)$ $\uparrow$ $f(-2)$

$f'(x)$: $f'(-3) = - \cdot - = +$ | $f'(-1) = - \cdot + = -$ | $f'(1) = + \cdot + = +$

$f(x)$ INCR. -2 $f(x)$ DECR. 0 $f(x)$ INCR.



changed CONCAVITY there: point of inflection

$f(x) = 1 + (x-3)^{2/3}$ $\qquad f(3) = 1 + (3-3)^{2/3} = 1$ $\qquad 3$

$f'(x) = 0 + \frac{2}{3}(x-3)^{-1/3}(1)$

$f'(x) = \frac{2}{3(x-3)^{1/3}} = \boxed{\frac{2}{3\sqrt[3]{x-3}}}$

① $\cancel{f'(x) = 0}$

$\cancel{\frac{2}{3(x-3)^{1/3}}} \overset{?}{\neq} 0$

never flat

② $\frac{2}{3\sqrt[3]{x-3}}$ undef?

when $\sqrt[3]{x-3} = 0$ $\qquad f(3)$

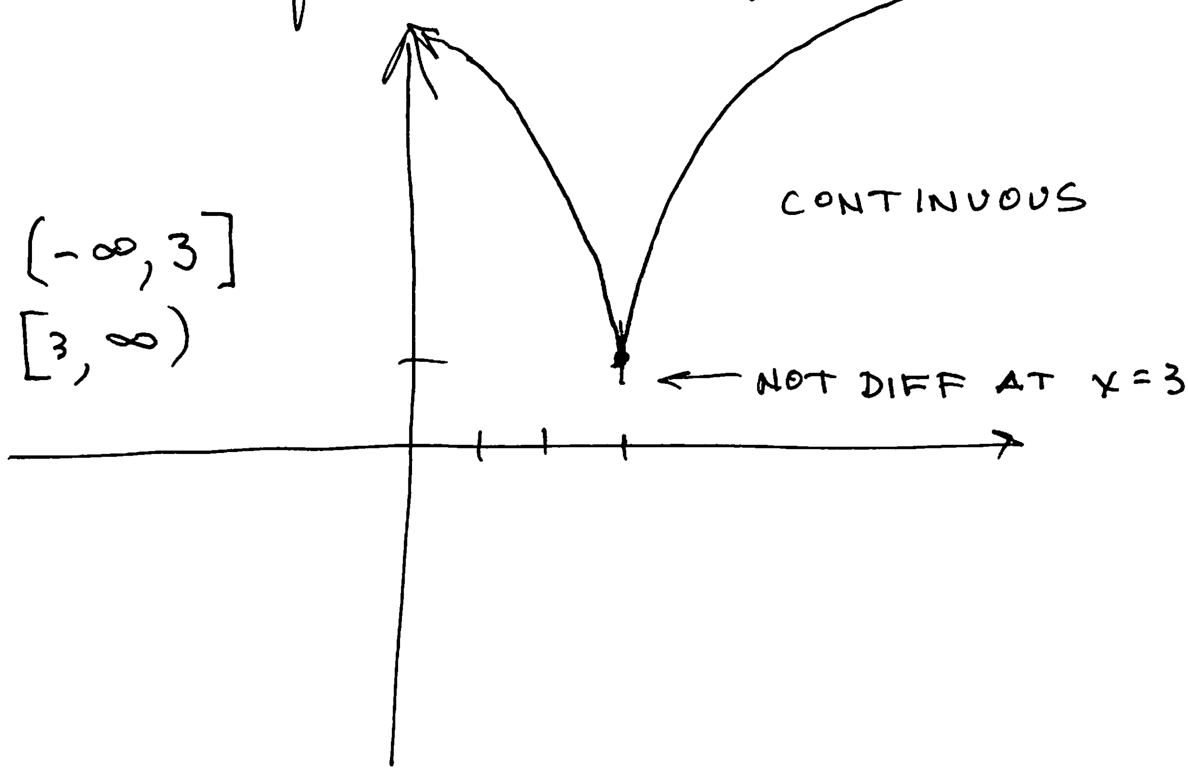
when $x = 3$

$(3, 1)$

$f'(x)$:

$f'(0) = \frac{+}{+\cdot -} = -$ $\qquad | \qquad$ $f'(4) = \frac{+}{+\cdot +} = +$

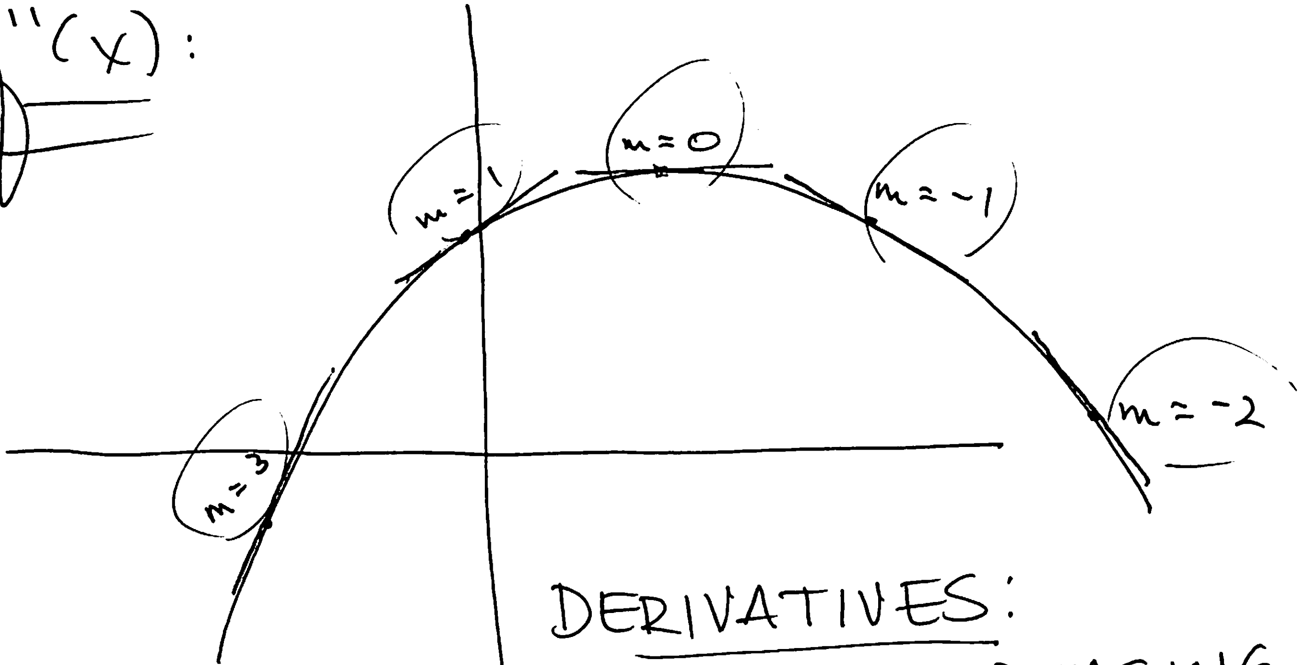
$f(x)$ = DECR 3 $\qquad f(x)$ INCR



DECR: $[-\infty, 3]$

INCR: $[3, \infty)$

$f''(x)$:



DERIVATIVES: ARE DECREASING $\Longrightarrow$ CONCAVE DOWN

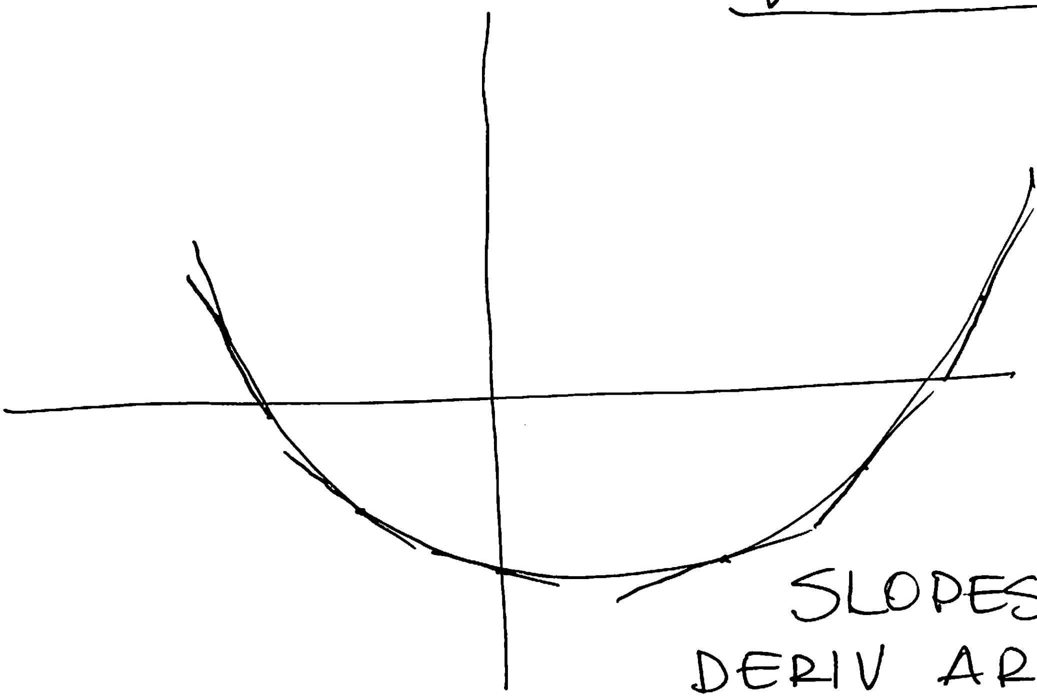
THEN $\boxed{f(x) \text{ is C.UP}}$

① IF $\boxed{f''(x) = +}$, then $f'(x)$ INCREASING

② IF $\boxed{f''(x) = -}$, then $f'(x)$ DECREASING

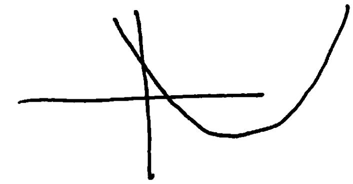
↓

THEN $\boxed{f(x) \text{ is CONC. DOWN}}$



SLOPES
DERIV ARE INCR.
CONCAVE UP

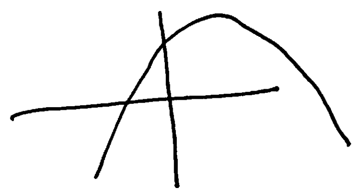
$f(x) = 8x^2 - 11x + 5$



$f'(x) = 16x - 11$

$f''(x) = 16$ always C.UP

$f(x) = -3x^2 + 5x - 8$



$f'(x) = -6x + 5$

$f''(x) = -6$

always C.DOWN

$f(x) = x^3 + 3x^2 + 2$

$f'(x) = 3x^2 + 6x$

$f''(x) = 6x + 6$? $\begin{cases} \text{where } f''(x) = + \\ \text{where } f''(x) = - \end{cases}$

① $f''(x) = 0$

② $f''(x)$ undef.

$f(-1) = (-1)^3 + 3(-1)^2 + 2$

$0 = \boxed{6x+6}$

$-6 = 6x$

$-1 = x$

$f(-1)$

$(-1, \underline{\ 4\ })$

$f''(x)$: $f''(-2) = -$ | $f''(0) = +$

$f(x)$ C.DOWN | -1 | $f(x)$ C.UP.

$$f(x) = 1 + (x-3)^{2/3}$$

$$f'(x) = \frac{2}{3}(x-3)^{-1/3}$$

$$f''(x) = \frac{2}{3} \cdot \frac{-1}{3}(x-3)^{-4/3} = \frac{-2\,(x-3)^{-4/3}}{9}$$

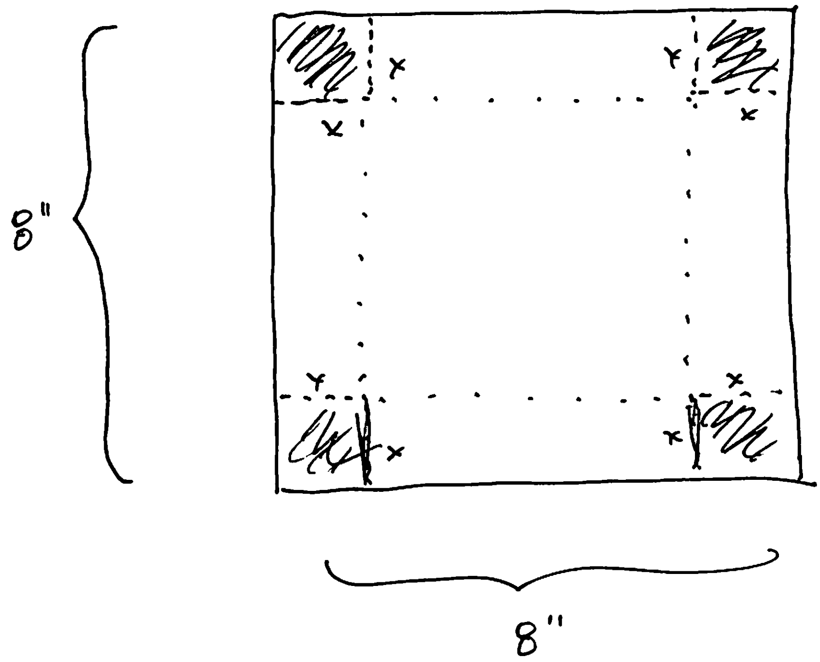
$$f''(x) = \frac{-2}{9\,\boxed{(x-3)^{4/3}}} = \frac{-}{+ \cdot +} = -$$

(always NEG)

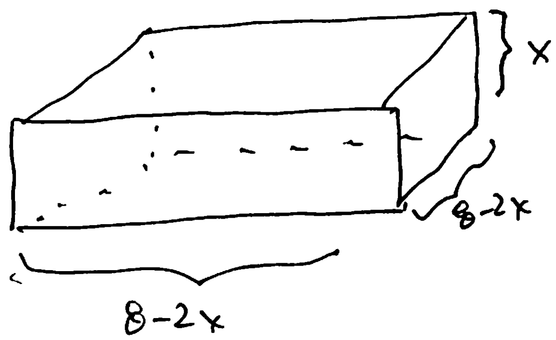
$f''(x)$:

$f''(2) = -$ | 3 | $f''(4) = -$

$f(x)$ C. DOWN | | $f(x)$ C. DOWN



form a box of MAX VOL



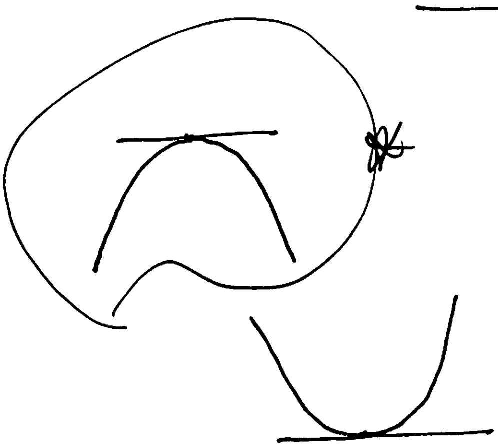
$$V = \underbrace{(8-2x)(8-2x)}\cdot x$$

$$V(x) = (64 - 32x + 4x^2)x$$

$$V(x) = 64x - 32x^2 + 4x^3$$

$(V'(x) = 0)$

$$V'(x) = \boxed{64 - 64x + 12x^2 = 0}$$



$$4(3x^2 - 16x + 16) = 0$$

$$4(3x-4)(x-4) = 0$$

$$3x - 4 = 0 \quad \text{or} \quad x - 4 = 0$$

$$x = 4/3 \quad \text{or} \quad \cancel{x = 4}$$

max or min??

$$V''(x) = -64 + 24x$$

$$V''(4/3) = -64 + 24(4/3) = \underline{\text{NEG}}$$

∴ C. DOWN

∴ MAX.

$V(x) = (8-2x)(8-2x)x$

$V(4/3) = \boxed{\left(\frac{16}{3}\right)\left(\frac{16}{3}\right)\left(4/3\right)}$

DIM: $4/3''$ by $16/3''$ by $16/3''$

VOL: $\frac{1024}{27}$ in^3

endpoints:
$x=0$ . . . . $x=4$

$\uparrow$
$x = 4/3$

let $x =$ # of price decreases

$REV = (40^{00} - 1 \cdot x)(40{,}000 + 2000x)$

Definition 6. Increasing and Decreasing

Let the function f be defined on an interval I.

f is **increasing on** *I (also called* **strictly increasing***) if, for $x_1, x_2 \in I$,*

$$f(x_1) < f(x_2) \text{ whenever } x_1 < x_2$$

Similarly, f is **decreasing on** *I (also called* **strictly decreasing***) if, for $x_1, x_2 \in I$,*

$$f(x_1) > f(x_2) \text{ whenever } x_1 < x_2$$

Theorem 1. Increasing and Decreasing on a Closed Interval

Let the function f be continuous on the closed, bounded interval $[a, b]$ and differentiable on the open interval (a, b). Then

1. *If $f'(x) > 0$ on (a, b), then f is increasing on $[a, b]$.*
2. *If $f'(x) < 0$ on (a, b), then f is decreasing on $[a, b]$.*

Theorem 2. Increasing and Decreasing on an Open Interval

Let the function f be differentiable on the open interval (a, b). Then

1. *If $f'(x) > 0$ on (a, b), then f is increasing on (a, b).*
2. *If $f'(x) < 0$ on (a, b), then f is decreasing on (a, b).*

Definition 8. Concavity

Let the function f be differentiable on the interval (a, b).

$(f''(x) = +)$

1. *If f' is increasing on (a, b), then the graph of f is* **concave up** *on (a, b).*
2. *If f' is decreasing on (a, b), then the graph of f is* **concave down** *on (a, b).*

$(f''(x) = -)$

Example 25. *When the ticket price is \$40, the average attendance at the football game is 40,000 people. It has been determined that for every \$1 decrease in the ticket price, an additional 2000 people will purchase tickets and attend the game. Under this arrangement, what price should be charged per ticket to maximize the revenue for the university? How many fans will attend the game at this price? What is the maximum revenue?*

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TEST #2 RESULTS:

A's	10	(27.8%)	55.6%
B's	10	(27.8%)	
C's	6	(16.7%)	
D's	2	(5.6%)	27.8%
F's	8	(22.2%)	

AVE: 75.91

grade went up from TEST #1: 21

(2 stayed the same)