NORTH CAROLINA STATE UNIVERSITY — Department of Mathematics

MA141-005 Test #1 Form B Monday, February 4, 2019 Dr. J. Griggs

Put all work and answers in the stamped blue book provided; one problem per page please. (the back of a sheet can serve as a new page) Nothing written on the test itself will be graded. Calculators may be used (not a graphing calculator nor any calculator that actually does calculus). Simplify completely. Turn in this test copy inside of your blue book. (Seven questions; 14 points each; 2 points for following directions)

1.) a.) For $f(x)=\frac{5}{x^2}$, find $\frac{f(x)-f(a)}{x-a}$ and simplify completely.

b.) Put in standard form; identify; find all relevant points (center, ends of axes, foci, ...) and **graph:**

$$x^2+y^2-8x+2y+8=0$$

2.) Sketch the curve represented by the following parametric equations; eliminate the parameter and find the Cartesian (rectangular) equation:

$$x=1+3t \quad and \quad y=2-t^2 \qquad -1\le t\le 3$$

3.) Use the formal definition of a limit (ε....δ) to show that $\lim_{x\to 2}(3x+5)=11$.

4.) Find the derivative using the DEFINITION OF DERIVATIVE: $f(x)=\frac{4}{5x+7}$

5.) Evaluate the following limits:

a.) $\lim_{x\to\infty}(\tan^{-1}x)$ b.) $\lim_{x\to 5}\frac{2x^2-10x}{x^2-4x-5}$ c.) $\lim_{x\to 0^+}\ln x$

6.) Graph the following: $f(x)=\begin{cases} 3 & if\ x\le -2 \\ 2x+1 & if\ -2<x<0 \\ 4-x^2 & if\ x\ge 0\end{cases}$

For this function, find: $\lim_{x\to 0^-}f(x)=$ $\lim_{x\to 0^+}f(x)=$ $\lim_{x\to 0}f(x)=$

Is this function continuous at x=-2? (verify; 3 possible steps)

7.) Find the average rate of change of $f(x)$ as x changes from x = 1 to x = 4; find the instantaneous rate of change of $f(x)$ at x = 3: $f(x)=5x^2-6x+2$ **(use the definition of derivative to find $f'(x)$)**

Bonus (5 pt): Write out the words to NC State's Alma Mater

MA 141 TEST #1

(7 questions; 14 points each)

1\. a.) 7 pts

$f(x)=\frac{5}{x^2}$ $\qquad \frac{f(x)-f(a)}{x-a}=\frac{\frac{5}{x^2}-\frac{5}{a^2}}{x-a}$

$=\frac{\frac{5}{x^2}\cdot\frac{a^2}{a^2}-\frac{5}{a^2}\cdot\frac{x^2}{x^2}}{(x-a)}=\frac{5a^2-5x^2}{x^2\cdot a^2}\cdot\frac{1}{(x-a)}$

$=\frac{5(a^2-x^2)}{x^2\cdot a^2}\cdot\frac{1}{(x-a)}=\frac{5\cancel{(a-x)}^{-1}(a+x)}{x^2a^2\cancel{(x-a)}_{1}}=\frac{-5(a+x)}{a^2x^2}$

$\cdot(x+a)$

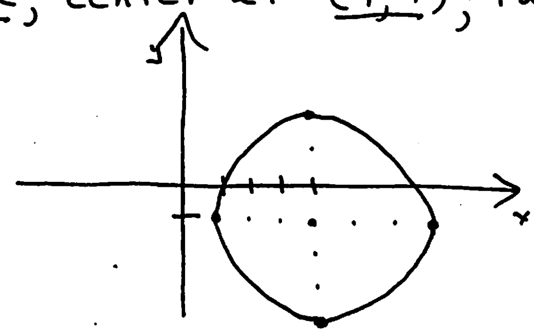
b.) 7 pts $x^2+y^2-8x+2y+8=0$

$x^2-8x \qquad +y^2+2y \qquad =-8$

$\underline{x^2-8x+16}+\underline{y^2+2y+1}=-8+16+1$

$(x-4)^2+(y+1)^2=9$

circle; center at $(4,-1)$; radius $=3$



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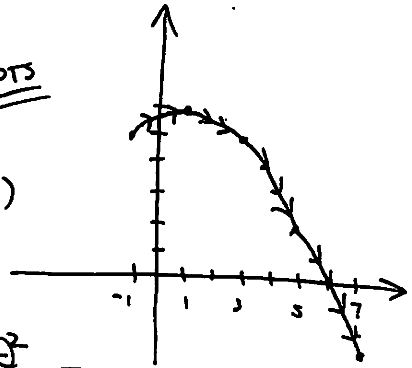
2.) $x = 1 + 2t \quad y = 6 - t^2 \quad -1 \le t \le 3$

t	-1	0	1	2	3
x	-1	1	3	5	7
y	5	6	5	2	-3

$(-1,5)\ldots(1,6)\ldots(3,5)\ldots(5,2)\ldots(7,-3)$

7 PTS



Cartesian Equation:

$x = 1 + 2t$

$x - 1 = 2t$

$\frac{x-1}{2} = t$

subst. for t

$y = 6 - t^2$

$y = 6 - \left[\frac{(x-1)}{2}\right]^2$

or

$y = 6 - \frac{(x^2 - 2x + 1)}{4}$

7 PTS

③ $\lim_{x \to 2} (8x - 10) = 6$

14 PTS

we want..... (so we can work backwards)

$|(8x-10) - 6| < \epsilon$

$|8x - 16| < \epsilon$

$8|x-2| < \epsilon$

$|x-2| < \epsilon/8$

choose $\delta = \epsilon/8$ (or smaller)

* begin with: *

$0 < |x-2| < \delta$

choose $\delta = \epsilon/8$

$|x-2| < \epsilon/8$

$8|x-2| < \epsilon/8\,(8)$

$|8x-16| < \epsilon$

$|(8x-10)-6| < \epsilon$

$\therefore |f(x) - L| < \epsilon$

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4.) $f(x) = \frac{5}{4x+9}$

14 PTS

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \frac{\frac{5}{4(x+h)+9} - \frac{5}{4x+9}}{h}$

$= \lim_{h \to 0} \frac{\frac{5}{[4(x+h)+9]} \cdot \frac{(4x+9)}{(4x+9)} - \frac{5}{(4x+9)} \cdot \frac{[4(x+h)+9]}{[4(x+h)+9]}}{h}$

$= \lim_{h \to 0} \frac{5(4x+9) - 5[4(x+h)+9]}{[4(x+h)+9] \cdot (4x+9)} \cdot \frac{1}{h}$

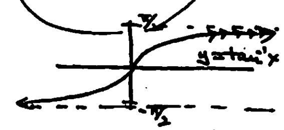
$= \lim_{h \to 0} \frac{\cancel{20x} + \cancel{45} - \cancel{20x} - 20h - \cancel{45}}{[4(x+h)+9] \cdot (4x+9) \cdot h}$

$= \lim_{h \to 0} \frac{-20\cancel{h}}{[4(x+h)+9] \cdot (4x+9) \cdot \cancel{h}} \quad (h \neq 0)$

$= \lim_{h \to 0} \frac{-20}{[4(x+\cancel{h})+9] \cdot (4x+9)} = \frac{-20}{(4x+9)^2}$

5.) a.) $\lim_{x \to \infty} (\tan^{-1} x) = \frac{\pi}{2}$

4 PTS



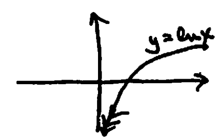
b.) $\lim_{x \to 5} \frac{3(x^2 - 5x)}{(x-5)(x+1)} = \lim_{x \to 5} \frac{3(x)\cancel{(x-5)}}{\cancel{(x-5)}(x+1)} \quad (x \neq 5)$

6 PTS

$= \lim_{x \to 5} \frac{3x}{x+1} = \frac{15}{6} = \frac{5}{2}$

c.) $\lim_{x \to 0^+} (\ln x) =$ D.N.E. or $-\infty$

4 PTS



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6.) $f(x) = \begin{cases} 5 & x \le -2 \\ 3x+1 & -2 < x < 0 \\ 7-x^2 & x \ge 0 \end{cases}$

$(x \le -2)$

$y = 5$

x	y
-2	5
-3	5
-4	5

constant

$(-2 < x < 0)$

$y = 3x+1$

x	y
-2	-5
-1	-2
0	1

DELETE (the rows for $x = -2$ and $x = 0$)

linear; m=3; y-int=1

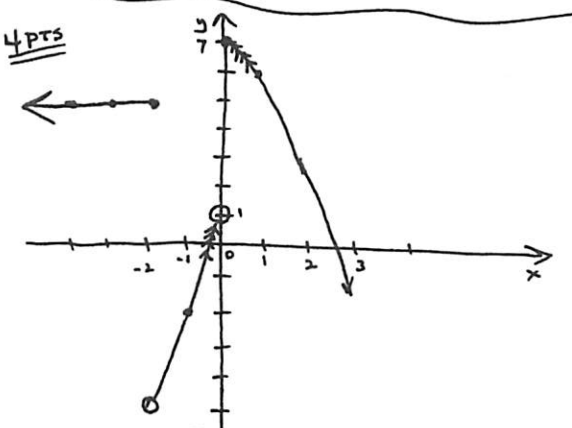
$(x \ge 0)$

$y = 7-x^2$

x	y
0	7
1	6
2	3

parabola; opens down

4 PTS



$\lim_{x \to 0^-} f(x) = 1$

$\lim_{x \to 0^+} f(x) = 7$

5 PTS

$\lim_{x \to 0} f(x) =$ DOES NOT EXIST

¿ continuous at $x = -2$?

5 PTS

① $f(-2)$ exists? yes, $f(-2) = 5$

② $\lim_{x \to -2} f(x)$ exists? no, $\lim_{x \to -2} f(x)$ D.N.E.

$\left\{ \begin{array}{l} \lim_{x \to -2^+} f(x) = -5 \\ \lim_{x \to -2^-} f(x) = 5 \end{array} \right\}$

$\therefore$ DISCONTINUOUS

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7.) average rate of change; $x = 1$ to $x = 4$:

6 PTS

$f(x) = 6x^2 - 5x + 2$

$m_{sec} = \dfrac{f(4) - f(1)}{4 - 1} = \dfrac{78 - 3}{3} = \dfrac{75}{3} = 25$

$f(4) = 6(4)^2 - 5(4) + 2 = 96 - 20 + 2 = 78$

$f(1) = 6(1)^2 - 5(1) + 2 = 6 - 5 + 2 = 3$

instantaneous rate of change at $x = 3$

$f(x) = 6x^2 - 5x + 2$

$f'(x) = 12x - 5$ (by DEF 'OF DERIV OR SHORTCUT)

$f'(3) = 12(3) - 5 = 36 - 5 = 31$

The Alma Mater of NC State

Where the winds of Dixie softly blow o'er the fields of Caroline,
There stands ever cherished, N.C. State, as thy honored shrine
So lift your voices! Loudly sing from hill to oceanside!
Our hearts ever hold you, N.C. State in the folds of our love and pride

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Words by Alvin Fountain : Class of '22
Music by Bonnie Norris: Class of '23

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