

MA141 - DOS

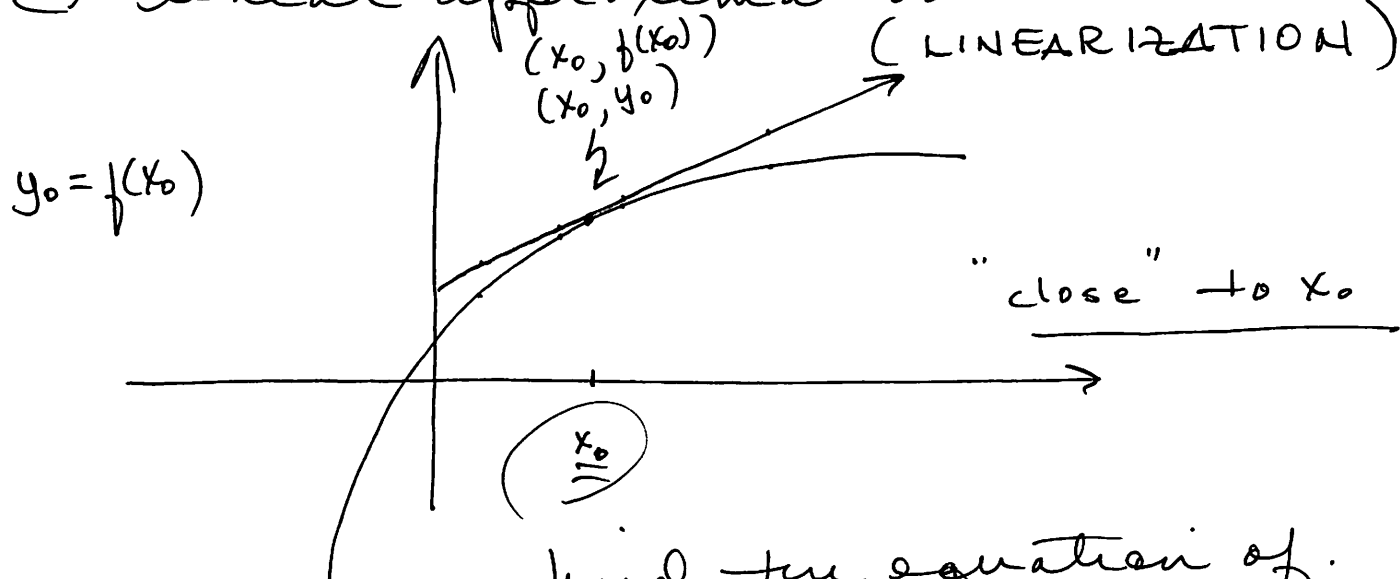
①

Monday, February 25

- TEST #2 (first hour)
- begin ch 3 (second hour)

3.1:

① linear approximation



find the equation of
the tangent line

$$y - y_0 = m(x - x_0)$$

$$y - \underline{f(x_0)} = \underline{m}(x - \underline{x_0})$$

$$m = m_{\text{TAN}} = f'(x) = \underline{f'(x_0)}$$

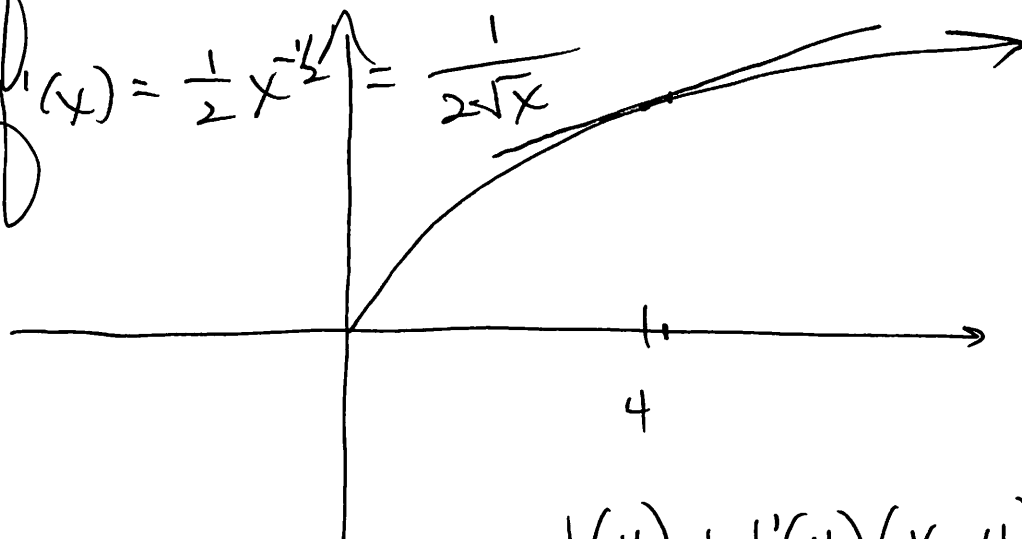
$$y - \underline{f(x_0)} = \underline{f'(x_0)}(x - \underline{x_0})$$

$$\boxed{y = \underline{f(x_0)} + \underline{f'(x_0)}(x - \underline{x_0})}$$

we want
 $\sqrt{4.3}$

$$f(x) = \sqrt{x} = x^{\frac{1}{2}}$$

$$f'(x) = \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$$



$$y = f(4) + f'(4)(x-4)$$

y-value
on
 $\sqrt{x}$

$$\begin{aligned}\sqrt{x} &\approx f(4) + f'(4)(x-4) \\ &\approx 2 + \frac{1}{4}(x-4) \\ &\approx 2 + \frac{1}{4}x - 1\end{aligned}$$

$$\sqrt{x} \approx \frac{1}{4}x + 1$$

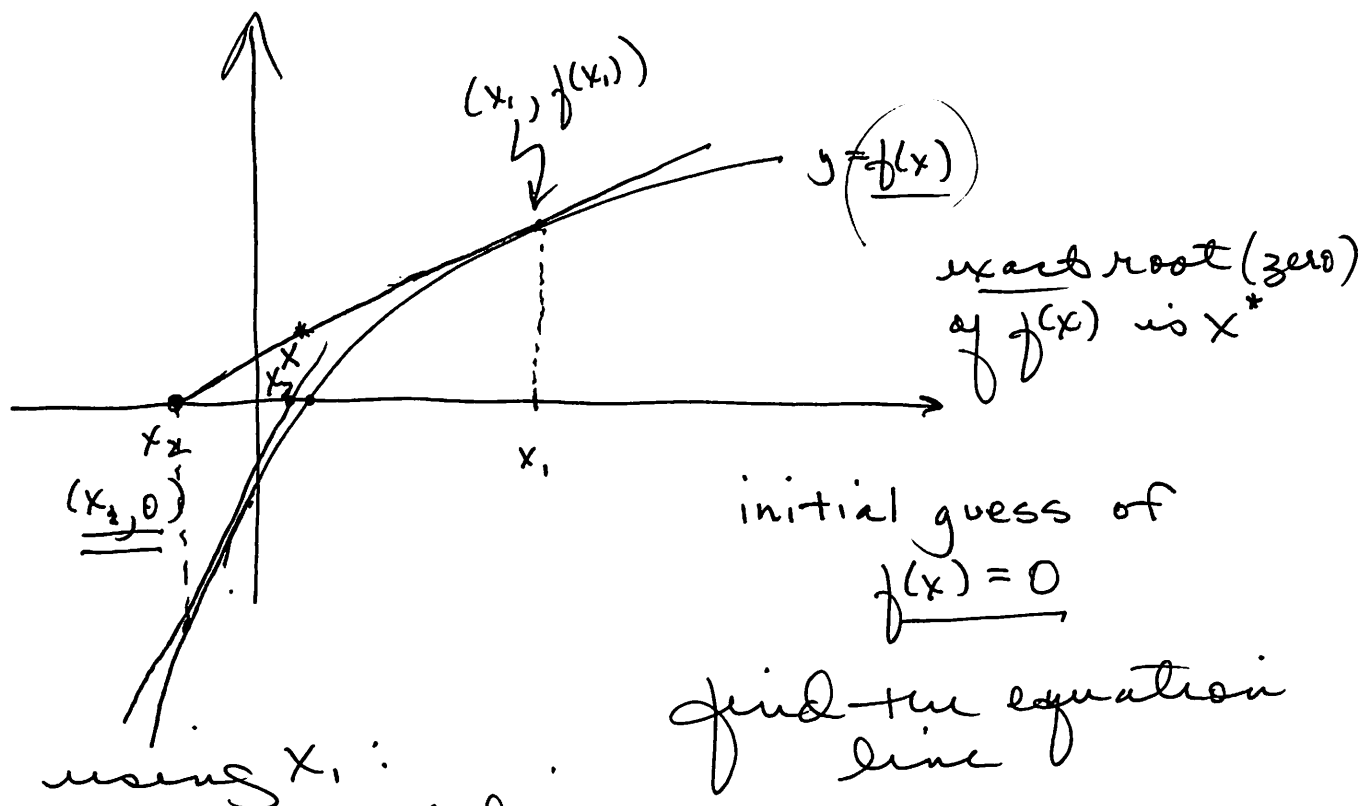
(for values of x
"near" 4)

$$\begin{aligned}\sqrt{4.3} &\approx \frac{1}{4}(4.3) + 1 \\ &\approx 1.075 + 1 \\ \sqrt{4.3} &\approx 2.075\end{aligned}$$

from a
calculator:

$$\sqrt{4.3} \approx 2.074$$

3.1: NEWTON'S METHOD



e.g. of tangent line

$$y - y_1 = m(x - x_1)$$

$$y - f(x_1) = m(x - x_1)$$

$$y - f(x_1) = f'(x_1)(x - x_1) \quad \text{plug in } (x_2, 0)$$

$$0 - f(x_1) = f'(x_1)(x_2 - x_1)$$

solve for $x_2 \dots$

$$\frac{-f(x_1)}{f'(x_1)} = x_2 - x_1$$

$$x_1 = \frac{-f(x_1)}{f'(x_1)} = x_2$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$$

(4)

in general ... (NEWTON'S METHOD)

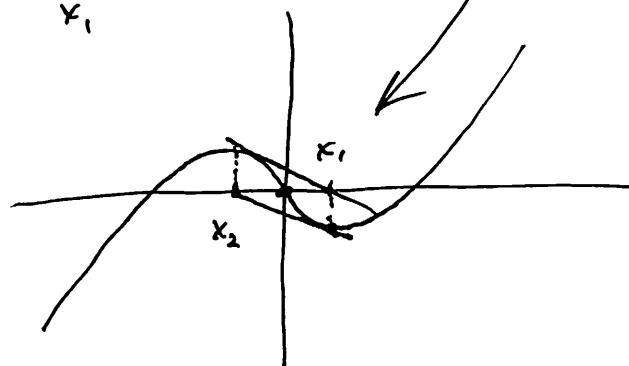
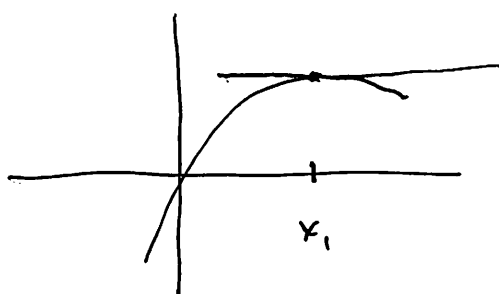
$$X_{n+1} = X_n - \frac{f(X_n)}{f'(X_n)}$$

*

FAIL:

① $f'(x) = 0$

② loop $x_1 \dots x_2 \dots x_i$



(5)

use NEWTON'S METHOD to find
the positive root of $f(x) = x^3 + 5x - 3 = 0$

$$f(0) = -3$$

$$f(1) = 3$$

IVT (contin)

$$f'(x) = 3x^2 + 5$$

$$x_1 = .8$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = .8 - \frac{(.8^3 + 5(.8) - 3)}{(3(.8)^2 + 5)}$$

$$x_2 = .8 - \frac{1.512}{6.92} \approx .8 - .218 \approx .582$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = .582 - \frac{(.582^3 + 5(.582) - 3)}{(3(.582)^2 + 5)}$$

$$x_3 \approx .582 - \frac{.107}{6.016} \approx .564$$

$$(.8) \dots (.582) \dots (.564) \dots$$

4

$$f(x) = x^2 - 7 \quad (\text{root is } \sqrt{7})$$

$$x_1 = 2.5$$

$$f'(x) = 2x$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_2)}$$

$$x_2 = 2.5 - \frac{(2.5)^2 - 7}{2(2.5)}$$

⋮

until 4 decimal
places CONSISTENT