Wednesday, February 20

- more log. diff. (2nd DERIV; $y = x^x \ldots$)
- related rates examples (2.7)

TEST #2: MONDAY (FEB. 25th) CHAP. 2

$$\longrightarrow \frac{dy}{dx} = y\left[\frac{7(2x+1)}{(x^2+x+3)} + \frac{8}{5(8x+1)} - \frac{3(42)}{(42x+3)}\right]$$

$$\longrightarrow \frac{dy}{dx} = \frac{[x^2+x+3]^7 \cdot \sqrt[5]{8x+1}}{(42x+3)^3}\left[\frac{7(2x+1)}{x^2+x+3} + \frac{8}{5(8x+1)} - \frac{3(42)}{(42x+3)}\right]$$

$y = x^x$ ① $y = x^5$ $y' = 5 \cdot x^4$

no existing rule ② $y = 5^x$ $y' = 5^x \cdot \ln 5$

$$y = (\sin 3x)^{11x+2}$$

$\ln a^r = r \cdot \ln a$

LOG. DIFF.

$$\ln y = \ln\left[(\sin 3x)^{11x+2}\right]$$

$$\ln y = (11x+2) \cdot \ln(\sin 3x)$$

(take DERIV ...) prod. rule

$$\frac{1}{y}\cdot\frac{dy}{dx} = \left[(11x+2)\cdot\left(\frac{1}{\sin 3x}\cdot\cos 3x\cdot 3\right)\right.$$

$$\left. + \ln(\sin 3x)\cdot[11]\right]$$

$$\frac{dy}{dx} = y\left[\frac{3\boxed{(\cos 3x)}(11x+2)}{\boxed{(\sin 3x)}} + 11\cdot\ln(\sin 3x)\right].$$

$$\frac{dy}{dx} = (\sin 3x)^{11x+2}\left[3(\cot 3x)(11x+2) + 11\cdot\ln(\sin 3x)\right]$$

$$\boxed{5x^2 + (11x^3)(y^5) = 14}$$

DIFF. IMPLICITLY . . .

find $\frac{d^2y}{dx^2}$.

find $\frac{dy}{dx}$: $\quad 10x + \left[(11x^3)\left[5y^4\cdot\frac{dy}{dx}\right] + (y^5)\cdot[33x^2]\right] = 0$

$$10x + 55x^3y^4\cdot\frac{dy}{dx} + 33x^2y^5 = 0$$

$$\frac{\cancel{55x^3y^4}\cdot\frac{dy}{dx}}{\cancel{55x^3y^4}} = \frac{-10x - 33x^2y^5}{55x^3y^4}$$

$$\frac{dy}{dx} = \boxed{\frac{-10x - (33x^2)(y^5)}{(55x^3)(y^4)}}$$

(take DERIV ...)

$$\boxed{\frac{d^2y}{dx^2}} = \frac{(55x^3y^4)\cdot\left[-10 - \left[(33x^2)\cdot\left(5y^4\cdot\frac{dy}{dx}\right) + (y^5)\cdot(66x)\right]\right] - (-10x - 33x^2y^5)\left[(55x^3)\left(4y^3\cdot\frac{dy}{dx}\right) + (y^4)(165x^2)\right]}{(55x^3y^4)^2}$$

related rates: (rates of change
instantaneous

find: $\frac{dc}{dt} = ?$

$$a^2 + b^2 = c^2$$

GIVEN:

$\frac{da}{dt} = 2 \frac{cm}{sec}$

$\frac{db}{dt} = 5 \frac{cm}{sec}$

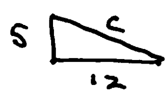
DERIV. WITH RESPECT TO "t"

$$\boxed{2(a)' \cdot \frac{da}{dt} + 2(b)' \cdot \frac{db}{dt} = 2(c)' \cdot \frac{dc}{dt}}$$

rate of change of a

rate of change of b

AT THIS INSTANT:
when a = 12 cm
b = 5 cm



c = 13

$$a \cdot \frac{da}{dt} + b \cdot \frac{db}{dt} = c \cdot \frac{dc}{dt}$$

$$(12)(2) + (5)(5) = (13)\frac{dc}{dt}$$

$$24 + 25 = 13\frac{dc}{dt}$$

$$+\frac{49}{13} \frac{cm}{sec} = \frac{dc}{dt}$$

2 $V = \underbrace{\frac{4}{3}\pi} \cdot r^3$

$\frac{dr}{dt} = -2 \frac{cm}{week}$

$\frac{dV}{dt} = ??$

AT THIS INSTANT:

$r = 15$ cm

$V = \frac{4}{3}\pi(15)^3$ cm^3

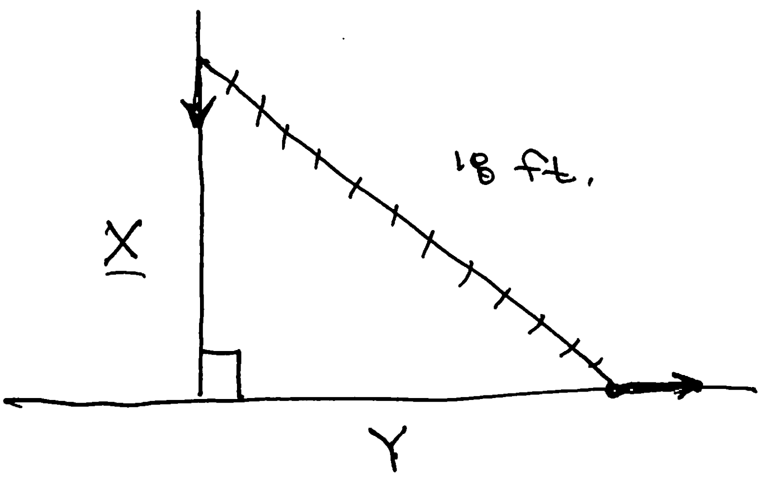
DERIV. WITH ~~RESPECT~~ TO "t"

$$\boxed{\frac{dV}{dt} = \left(\frac{4}{3}\pi\right)\left(3r^2 \cdot \frac{dr}{dt}\right)}$$

$$\frac{dV}{dt} = \left(\frac{4}{\cancel{3}}\pi\right)(\cancel{3})(15)^2(-2)$$

$$\frac{dV}{dt} = -1800\pi \ \frac{cm^3}{week}$$

5

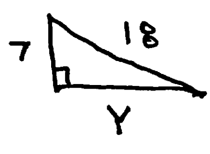


$\frac{dy}{dt} = 2 \frac{ft}{sec}$

$\frac{dx}{dt} = ??$

AT THIS INSTANT:

$x = 7$ ft



$7^2 + y^2 = 18^2$

$y = \sqrt{18^2 - 7^2}$

$$X^2 + Y^2 = 18^2$$

$$2x \cdot \frac{dx}{dt} + 2y \cdot \frac{dy}{dt} = 0$$

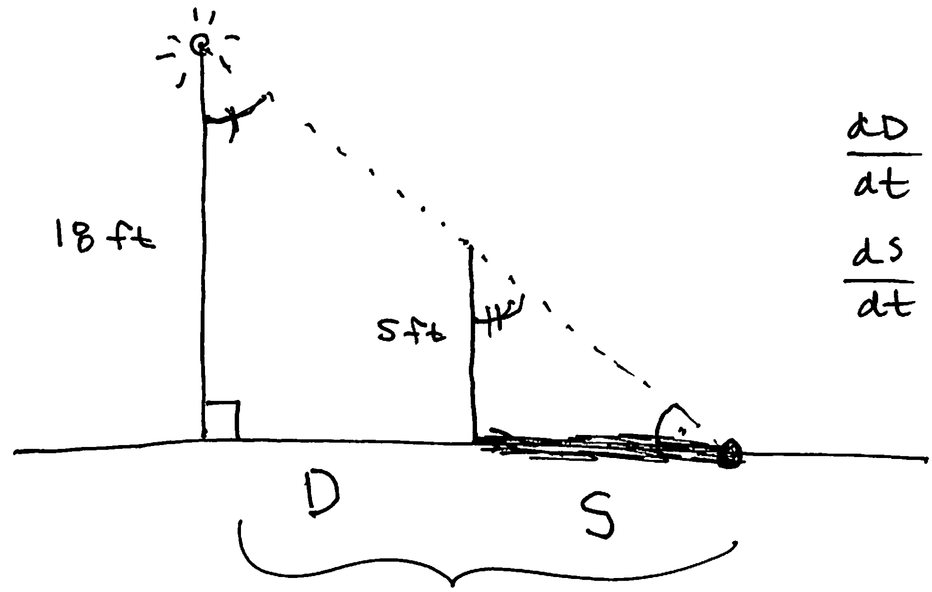
$$x \cdot \frac{dx}{dt} + y \cdot \frac{dy}{dt} = 0$$

$$(7)\left(\frac{dx}{dt}\right) + \left(\sqrt{18^2-7^2}\right)(2) = 0$$

$$\frac{\cancel{7}\,\frac{dx}{dt}}{\cancel{7}} = \frac{-2\left(\sqrt{18^2-7^2}\right)}{7}$$

$$\frac{dx}{dt} \approx -4.74 \ \frac{ft}{sec}$$

15.)

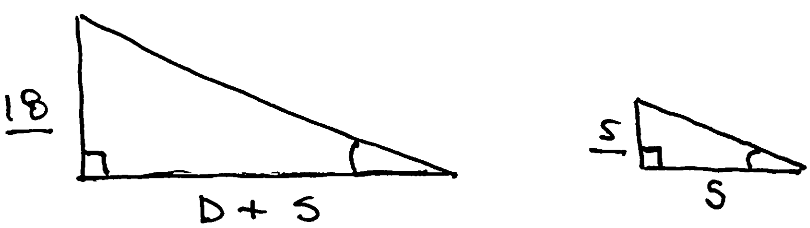


$$\frac{dD}{dt} = 4 \ \frac{ft}{sec}$$

$$\frac{dS}{dt} = ??$$

AT THIS INSTANT:

D = 20 ft.



Δ's are similar

→ sides are proportional

$$\frac{18}{5} = \frac{D+S}{S}$$

$$18S = 5(D+S)$$

$$18S = 5D + 5S$$

$$\boxed{13S = 5D}$$

$$13 \cdot \frac{dS}{dt} = 5 \cdot \frac{dD}{dt}$$

$$(13)\frac{ds}{dt} = 5(4)$$

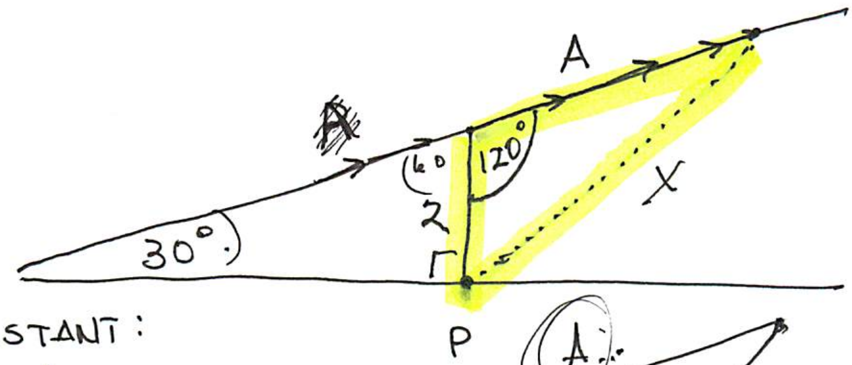
$$\frac{ds}{dt} = \frac{20}{13} \ \frac{ft}{sec}$$

"tip":

$$\frac{dD}{dt} + \frac{dS}{dt}$$

$$\left(4 + \frac{20}{13}\right)\frac{ft}{sec}$$

17.)



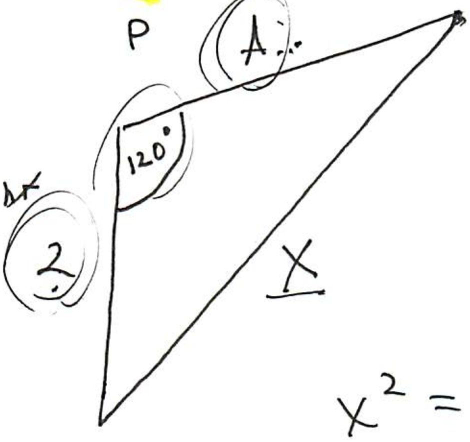
$$\frac{dA}{dt} = 360\ \frac{mi}{hr}$$

$$\frac{dx}{dt} = ??$$

AT THIS INSTANT:

one-minute later:

$\boxed{A = 6\ mi} = 360\ \frac{mi}{hr} \cdot \frac{1}{60}\ hr$



LAW OF COSINES:

$$x^2 = 4 + (6)^2 + 2(6)$$

$$x^2 = 4 + 36 + 12$$

$$x = \sqrt{52}\ mi$$

$$x^2 = 2^2 + A^2 - 2(A)(2)\cos 120°$$

$$x^2 = 4 + A^2 - 4A\left(-\tfrac{1}{2}\right)$$

$$x^2 = 4 + A^2 + 2A$$

$$2x\frac{dx}{dt} = 2A\frac{dA}{dt} + 2\cdot\frac{dA}{dt}$$

$$2(\sqrt{52})\left(\frac{dx}{dt}\right) = 2(6)(360) + 2(360)$$

$$\frac{dx}{dt} = \frac{2(6)(360) + 2(360)}{2\sqrt{52}}\ \frac{mi}{hr}$$

2.7.1 Exercises

In each of these exercises give your answer accurate to 3 decimal places.

1. A funnel in the form of a cone is 10 in across the top and 8 in deep. Water is flowing into the funnel at the rate of 12 in^3/sec, and out at the rate of 4 in^3/sec. How fast is the surface of the water rising when the water is 5 in deep?

AT THIS INSTANT: $r = 15$ $V = \frac{4}{3}\pi(15)^3$

2. The radius of a sphere is decreasing slowly at the rate of 2 cm/week. How fast is the volume changing when the radius is 15 cm?

$\frac{dV}{dt} = ??$ $\frac{dr}{dt} = -2 \frac{cm}{week}$ $V = \frac{4}{3}\pi \cdot r^3$

3. The area of an equilateral triangle is decreasing at a rate of 4cm^2/min. Find the rate at which the length of a side is changing when the area of the triangle is 200 cm^2.

4. A stone dropped into a lake causes circular ripples to emanate from its point of entry. The radius of the circular waves are increasing at a constant rate of 0.5 m/sec. At what rate is the circumference of a wave changing when its radius is 3 m?

5. An 18 ft ladder leans against a vertical building. If the bottom of the ladder slides away from the building at the rate of 2 ft/sec, how fast is the ladder sliding down the building when the top of the ladder is 7 ft from the ground?

6. Gas is being pumped into a spherical balloon at the rate of 6 ft^3/min. Find the rate at which the radius is changing when the diameter is 15 in. (Watch out for units.)

7. A boy flying a kite releases string at the rate of 2 ft/sec as the kite moves horizontally at a height of 100 ft. Assuming no sag in the string, find the rate at which the kite is moving when 125 ft of string has been released.

8. The ends of a water trough 10 ft long are equilateral triangles whose sides are 3 ft long. If water is pumped into the trough at a rate of 5 ft^3/min, find the rate at which the water level is rising when the depth is 1 ft.

9. Claire starts at point A and runs east at a rate of 10 ft/sec. One minute later, Anna starts at A and runs north at a rate of 8 ft/sec. At what rate is the distance between them changing after another minute?

10. A man on a dock is pulling in a boat by means of a rope attached to the bow of the boat at a point that is 1 ft above water level. The rope goes from the bow of the boat to a pulley located at the edge of the dock 8 ft above water level. If he pulls in the rope at a rate of 2 ft/sec, how fast is the boat approaching the dock when the point of attachment is 15 ft from the dock?

11. A weather balloon is rising vertically at the rate of 3 ft/sec. An observer is situated 100 yards from a point on the ground directly below the balloon. At what rate is the distance between the balloon and the observer changing when the height of the balloon is 500 ft?

12. Gas is escaping from a spherical balloon at the rate of 10 ft^3/hr. At what rate is the radius of the balloon changing when the volume is 400 ft^3?

13. A softball diamond has the shape of a square with sides 60 ft long. If a player is running from second base to third base at a speed of 25 ft/sec, at what rate is her distance from home plate changing when she is 15 ft from third base?

14. As sand leaks out of a hole in a container, it forms a conical pile whose altitude is always the same as its radius. If the height of the pile is increasing at the rate of 6 in/min, find the rate at which the sand is leaking out when the height is 10 in.

15. A streetlight is on a pole 18 ft tall. A boy 5 ft tall walks away from the pole at a rate of 4 ft/sec. At what rate is his shadow lengthening when he is 20 ft from the pole? At what rate is the tip of his shadow moving?

16. A point $P(x, y)$ moves along the graph of the equation $y = x^3 + x^2 + 1$, the x-values are changing at the rate of 2 units per second. How fast are the y-values changing at the point $Q(1, 3)$?

17. An airplane, flying at a constant speed of 360 mi/hr and climbing at a 30 degree angle, passes over a point P on the ground at an altitude of 10,560 ft. Find the rate at which its distance from P is changing one minute later.

18. Sand is being dropped at the rate of 10 ft^3/min into a conical pile. If the height of the pile is always twice the base radius, at what rate is the height increasing when the pile is 8 ft high?

19. Water is flowing at the rate of 8 ft^3/min into a tank that is in the shape of a right circular cylinder whose base radius is 2 ft. How fast is the water level rising?

20. A car traveling at a rate of 30 ft/sec is approaching an intersection. When the car is 120 ft from the intersection, a truck traveling at the rate of 40 ft/sec crosses the intersection. The car and truck are on roads which meet at a right angle. How fast are the car and truck separating 2 sec after the truck leaves the intersection?

21. A camera televising the return of the opening kickoff of a football game is located 5 yd from the east edge of the field on the goal line extended. The ball carrier runs down the east sideline (just in bounds) for a touchdown. When he is 10 yd from the goal line, the camera is turning at a rate of .5 radian/sec. How fast is the player running?

